RESTORATION OF STABILITY IN FERROMAGNETIC RESONANCE

H. Suhl
Physics Department, University of California, La Jolla, California
(Received January 30, 1961)

As has long been recognized,[1-5] ferromagnetic resonance is subject to instability when the radio-frequency signal exceeds a certain threshold value. In the more common type of instability, pairs of spin waves with equal and opposite momenta grow far beyond their normal, thermal equilibrium amplitudes at the expense of the uniform precessional motion induced by the signal field. In this paper we show how, by rather simple means, this threshold may be substantially raised. The method is reminiscent (in the time domain) of the alternating gradient stabilization (in the space domain) of charged beams. Studies of large motions of the magnetization, which have proved very fruitful in nuclear resonance, may now become possible.

The method we propose is based on the following observations. The natural frequency of the spin-wave pair prone to unstable growth stands in a very sensitive relation to the signal frequency, as is obvious from energy conservation requirements. If now the natural frequency of these spin waves is varied in time, no pair of waves will satisfy the necessary frequency relation for longer than an instant, and this must inevitably cause an increase in the threshold signal. The extent to which this principle is borne out in practice is described in another Letter in this issue.[6]

In standard form, the equations of motion for a potentially unstable spin-wave pair of wave numbers $(\vec{k}, -\vec{k})$ may be written

$$\dot{a}_{\vec{k}} = i\omega_{\vec{k}} a_{\vec{k}} + P_{\vec{k}} a_0^{\,n} a_{-\vec{k}}^{\,*} - \eta_{\vec{k}} a_{\vec{k}},$$

$$\dot{a}_{-\vec{k}}^{\,*} = -i\omega_{\vec{k}} a_{-\vec{k}}^{\,*} + P_{\vec{k}}^{\,*}(a_0^{\,*})^n a_{\vec{k}} - \eta_{\vec{k}} a_{-\vec{k}}^{\,*}. \qquad (1)$$

Here $\omega_{\vec{k}}$ is the natural frequency of the spin wave; $a_{\vec{k}}$ and a_0, respectively, are the amplitudes of the spin wave and of the uniform precession. $P_{\vec{k}}$ is a coefficient measuring the strength of the nonlinear coupling. $\eta_{\vec{k}}$ is a phenomenological loss constant for the spin wave. n is a positive integer measuring the order of the nonlinear coupling. Only the cases $n=1$, 2 have been observed so far. Equations (1) predict exponential growth of the spin-wave pair when

$$|a_0|^n > \eta_{\vec{k}}/P_{\vec{k}}, \qquad (2)$$

provided the frequency ω which drives a_0 is such that

$$\omega = 2\omega_{\vec{k}}/n. \qquad (3)$$

Equation (3) determines a surface in k space, upon which the right-hand side of Eq. (2) is minimized, to find which spin wave consistent with (3) goes unstable first, and at what minimum power.

We now impose a slow sinusoidal modulation of frequency ν and amplitude h on the applied steady field. ν will ordinarily be much less than ω, or $\omega_{\vec{k}}$. So long as this is so, the equations of motion remain very nearly the same as (1), except that the constants therein become sinusoidal functions of the time. For reasons which will become clear, the time variation of $P_{\vec{k}}$ may be neglected. The time variation of a_0 will be important only in the case $n=2$, when the modulation will tend to sweep the field clear through the resonance line, and then only if the modulating field exceeds the linewidth. In this paper we neglect the time variation of a_0 which is easily taken into account, if desired.

To first order in the modulating field h, we have, for the time-dependent spin-wave frequency,

$$\omega_{\vec{k}}' = \omega_{\vec{k}} + (\partial\omega_{\vec{k}}/\partial H)h\cos\nu t. \qquad (4)$$

A change of variables,

$$a_{\vec{k}} = A_{\vec{k}} \exp\{i[\omega_{\vec{k}} t + (h\,\partial\omega_{\vec{k}}/\nu\partial H)\sin\nu t] - \eta_{\vec{k}} t\},$$

$$A_0 = a_0 \exp(i\omega t),$$

renders Eq. (1) of the form

$$\dot{A}_{\vec{k}} = P_{\vec{k}} A_0^{\,n} \exp[i(n\omega - 2\omega_{\vec{k}})t - 2i(f/\nu)\sin\nu t] A_{-\vec{k}}^{\,*}, \qquad (5)$$

together with the "adjoint" equation. Here we have defined an additional frequency, $f = h\,\partial\omega_{\vec{k}}/\partial H$. Let us now confine further discussion to the case $n=1$, for definiteness, and also because the neglect of the time variation of $A_0 = |a_0|$ is then most certainly justified. The condition most favorable to instability is then, provisionally at any rate, $\omega_{\vec{k}}=\omega/2$. For spin waves satisfying this condition, we have

$$\dot{A}_{\vec{k}}=A_0P_{\vec{k}}A_{-\vec{k}}^* \exp(-2if/\nu)\sin\nu t$$
$$=A_0P_{\vec{k}}A_{-\vec{k}}^* \sum_{m=-\infty}^{+\infty} J_m(2f/\nu)\exp(-im\nu t). \quad (6)$$

The right-hand side of this equation consists of one secular term and a series of nonsecular terms. In the crudest approximation only the secular term contributes to the instability phenomenon. In this approximation, then, we retain only the term with the secular coefficient, $P_{\vec{k}}J_0$. Instead of the instability criterion (2), we then have

$$A_0=\eta_{\vec{k}}/P_{\vec{k}}J_0(2f/\nu).$$

Hence if the modulating field amplitude and the modulating frequency are chosen so that the argument of the zero-order Bessel function in this expression is equal to one of its zeroes, the threshold becomes theoretically infinite. In a more refined approximation the threshold is, generally, finite. The solution of (5) and its adjoint will have the form

$$A_{\vec{k}}=e^{\lambda t}\sum_{r=-\infty}^{+\infty} B_r e^{ir\nu t},$$

where the B's in the summation are constants, satisfying the equation

$$B_n=A_0^2|P_{\vec{k}}|^2\sum_{m=-\infty}^{+\infty} K(n,m;\lambda)B_m, \quad (7)$$

where

$$K(n,m;\lambda)=\sum_{r=-\infty}^{+\infty} J_{n-r}J_{r-m}/(\lambda+ir\nu)(\lambda+in\nu),$$

the argument of the two Bessel functions in this expression being the same as above. Since Eq. (7) leads to a very complicated infinite determinant relation for λ, we instead discuss a completely soluble case: that of square-wave modulation. For the first half-period, $T/2$, we assume that the frequency condition $\omega_{\vec{k}}=\omega/2$ is satisfied exactly; for the second half-period we assume a deviation f. Then the equation for A $[=a_{\vec{k}}\exp(i\omega_{\vec{k}}t)]$ becomes

$$\dot{A}=A_0PA^*, \qquad (0<t<\tfrac{1}{2}T)$$
$$\dot{A}=ifA+A_0PA^*, \qquad (\tfrac{1}{2}T<t<T) \quad (8)$$

where the $\vec{k}$ subscripts have been omitted.

Equations (8) are solved in the two half-periods and fitted together at $\frac{1}{2}T$ by the requirement that A, A^* are continuous there. Next, a linear combination of $A(T)$ and $A^*(T)$ is formed in such a way that it is proportional, with a factor of proportionality $\exp\lambda T$, to the same linear combination of $A(0)$ and $A^*(0)$. Stability is achieved if $\lambda<\eta_{\vec{k}}$. Some algebra shows that λ is given by

$$\lambda T=\cosh^{-1}[\cosh\tfrac{1}{2}P_1T\cosh\tfrac{1}{2}\Omega T + (P_1/\Omega)\sinh\tfrac{1}{2}P_1T\sinh\tfrac{1}{2}\Omega T],$$

where $\Omega^2=P_1^2-f^2$ and where $P_1=A_0P$. A particularly interesting condition arises when the argument of the inverse hyperbolic cosine is less than unity. Then λ is imaginary and the system is stable, no matter how small $\eta_{\vec{k}}$. It is easily verified that this condition of <u>absolute</u> stability is satisfied only when Ω is imaginary, i.e., when the depth of modulation f exceeds A_0P. This is, of course, to be expected from the general threshold conditions (see references 1-5) for spin waves failing to satisfy the frequency condition. For modulation frequencies much smaller than A_0P the absolute stability criterion is particularly easily interpreted. First, for general T, absolute stability is achieved when

$$|\cos(\tfrac{1}{2}\Omega'T-\phi)|\le R^{-1},$$

where

$$R=[\cosh^2(\tfrac{1}{2}P_1T)+(P_1^2/\Omega'^2)\sinh^2(\tfrac{1}{2}P_1T)]^{1/2},$$
$$\Omega'^2=f^2-P_1^2, \quad (>0)$$

and

$$\tan\phi=(P_1/\Omega')\tanh(\tfrac{1}{2}P_1T).$$

For large P_1T we thus require

$$\tfrac{1}{2}\Omega'T>\phi=(n+\tfrac{1}{2})\pi.$$

The interpretation is that for large P_1T only the growing solution of (8) is of importance at the end of the first half-cycle. For $f>P_1$, the solution in the second half-cycle is purely oscillatory and, by judicious choice of T, etc., can always be made to vanish at T, no matter how large its amplitude was at $\frac{1}{2}T$.

All the results in this note apply to instabilities due to parallel pumping as well. It is necessary only to redefine A_0 as the pumping field amplitude itself.

Stabilization can also be achieved by modulating the signal frequency, but this will generally be less convenient.

[1]P. W. Anderson and H. Suhl, Phys. Rev. 100, 1788 (1955).
[2]H. Suhl, Phys. Rev. 101, 1937 (1956).
[3]H. Suhl, J. Phys. Chem. Solids 1, 209 (1957).
[4]E. Schlömann, J. J. Green, and U. Milano, J. Appl. Phys. 31, 386S (1960).
[5]F. R. Morgenthaler, J. Appl. Phys. 31, 95S (1960).
[6]T. S. Hartwick, E. Peressini, and M. T. Weiss, following Letter [Phys. Rev. Letters 6, 176 (1961)].

SUPPRESSION OF SUBSIDIARY ABSORPTION IN FERRITES BY MODULATION TECHNIQUES

T. S. Hartwick, E. R. Peressini, and M. T. Weiss
Components Division, Hughes Aircraft Company, Culver City, California
(Received January 30, 1961)

In the preceding paper,[1] Suhl has shown theoretically that the threshold power for subsidiary absorption in ferrites can be increased by superimposing on the dc magnetic field, H_{dc}, a small ac modulation, H_m. Experiments have been performed which confirm the above prediction and show that subsidiary absorption can indeed be suppressed at power levels as high as 10 db above the threshold power.

The experiments were performed on an yttrium iron garnet (YIG) sphere 0.059 inch in diameter with a uniform precession linewidth of 1.2 oe. The sphere was placed near the wall of a cavity resonant at 9250 Mc/sec. Eddy current losses were minimized by using a cavity wall 0.002 mm thick and thus an external modulating coil was suitable. In Fig. 1 a plot is shown of the relative amplitude of H_m required to suppress subsidiary resonance for various values of microwave field, h. For these data h was oriented perpendicular

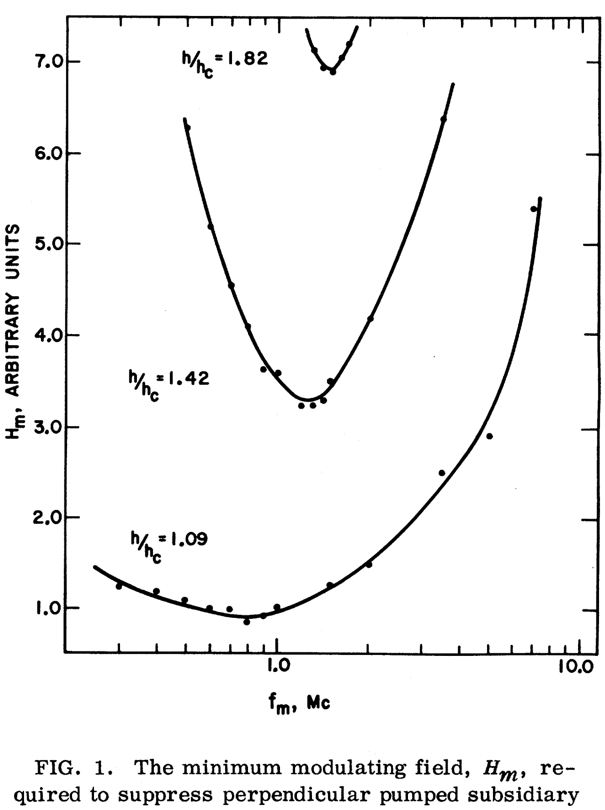

FIG. 1. The minimum modulating field, H_m, required to suppress perpendicular pumped subsidiary resonance as a function of the modulation frequency for various microwave fields.

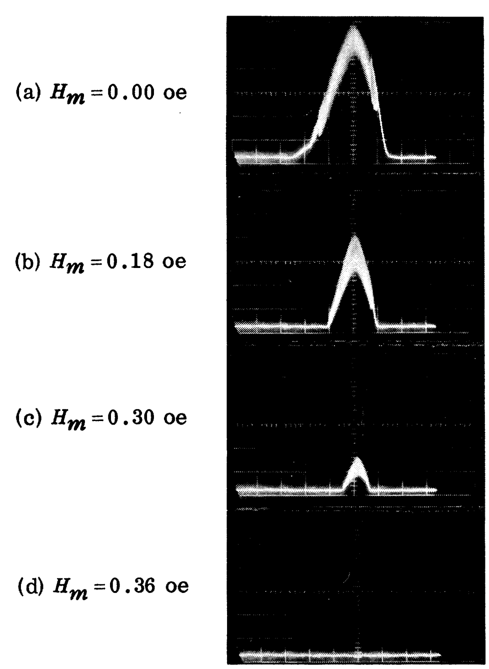

FIG. 2. Absorption is recorded on the vertical axis as H_{dc} is traversed through parallel pumped subsidiary resonance. Horizontal scale is 20 oe/cm. The modulating field is increased from trace (a) through trace (d).