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INTERACTION OF AN ELECTRON BEAM WITH A FULLY IONIZED PLASMA*

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The amplification of microwave signals on an electron beam which passes through a plasma has been observed by Boyd et al.[1] and Bogdanov et al.[2] and discussed in many theoretical papers. However, in the past, because of the nature of the mercury vapor discharge in which these experiments were carried out, it was difficult to obtain a close comparison between theory and experiment. Preliminary experiments using an almost fully ionized thermal cesium plasma have been carried out with encouraging results, and are reported here.

A schematic diagram of the apparatus which was used is shown in Fig. 1. An electron beam passes through a helix on which a microwave signal is induced. The modulated electron beam then passes through a 2.2-cm length of cesium plasma, which is formed from two hot tungsten spiral filaments immersed in cesium vapor. Ions are formed at the filaments by resonance ionization; the electrons are produced by thermionic emission. Both the cesium plasma and the electron beam are confined radially by means of the externally applied magnetic field. The microwave signal on the electron beam is amplified after passing through the plasma and the consequent enhanced modulation is detected on the second helix. Finally, the electron beam is collected on a collector. A water jacket is provided around the plasma region to control the temperature of the cesium vapor, and hence, the vapor pressure and rate of ion emission. Half-wave rectified ac was used on the heaters, and all measurements of interest were carried out during the off-period of the heating cycle. This method of producing a plasma has been shown[3]

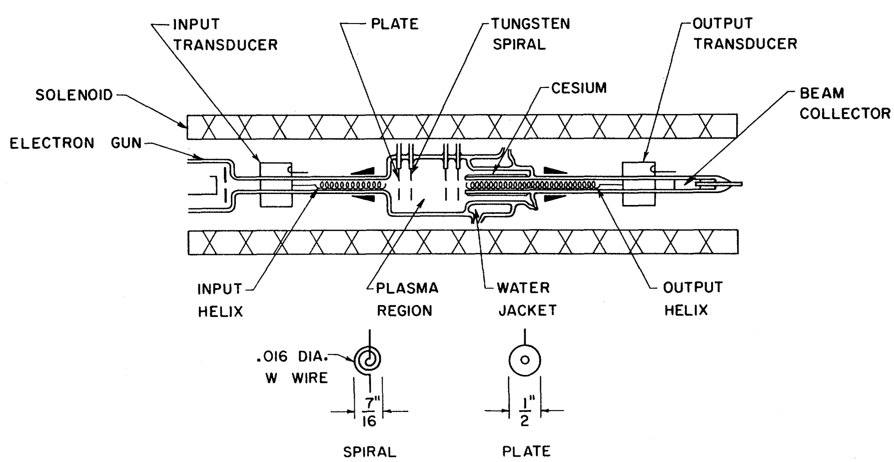

FIG. 1. Cross section of beam-plasma tube shown schematically.

to yield a plasma characterized by a very high percentage of ionization (greater than 10%), with temperatures of the ions and electrons at approximately that of the heater (2300°K). The dominant collision process present is that between electrons and ions.

By measuring the gain of the system, from the input to the output transducers, with the plasma on and the plasma off, it was possible to determine the gain due to the plasma alone. The parameters of the system which could be varied were signal frequency, plasma density, and beam current. Assuming that for a given plasma density the gain was maximum at the plasma frequency, then the maximum gain measured with approximately 1 ma in the beam varied from 10 db at a plasma frequency of 1000 Mc/sec to over 20 db at a plasma frequency of 3000 Mc/sec. At higher plasma frequencies, the gain was so high that the tube oscillated at that value of beam current.

A theoretical analysis has been performed in order to determine the rf gain for a finite electron beam in a finite magnetic field passing through a plasma. By making the assumption of slow waves, it is found that the fields inside and outside the beam have transverse propagation constants T_1 and T_2, respectively, where

$$T_1^2 = -k^2 \frac{1 - (\omega_p^2/\omega^2) - [\omega_{pb}^2/(\omega - ku_0)^2]}{1 + [\omega_p^2/(\omega_c^2 - \omega^2)] + \{\omega_{pb}^2/[\omega_c^2 - (\omega - ku_0)^2]\}}, \quad (1)$$

$$T_2^2 = -k \frac{1 - (\omega_p^2/\omega^2)}{1 + [\omega_p^2/(\omega_c^2 - \omega^2)]}, \quad (2)$$

and where u_0 is the electron beam velocity. All fields are assumed to vary as $\exp[i(kz - \omega t)]$; ω_p and ω_{pb} are the plasma frequencies of plasma and beam, respectively; and ω_c is the electron cyclotron frequency for the applied magnetic field. The assumption was made that the plasma frequency of the beam is much less than the cyclotron frequency, and hence, the third term in the denominator of Eq. (1) is negligible. However, with the normal values of magnetic field used in this type of experiment, such an assumption is certainly not justified for the plasma frequency of the plasma electrons. Thus, it is felt that the theoretical results of Bogdanov et al. which rely on this assumption are not applicable to our experiment. The appropriate matching conditions of the fields at the edge of the beam were used to determine that for $\omega_c < \omega < \omega_p$, T_1 is real and T_2 is imaginary. It follows that there should be gain in this range of frequencies which becomes infinite at the plasma frequency. However, because of the nature of the Bessel functions involved, for a finite cylindrical beam the gain approaches infinity not as $(1 - \omega_p^2/\omega^2)^{-1/2}$ as in the case of the infinite beam, but only as $\ln(1 - \omega_p^2/\omega^2)$. Consequently, any effect in the plasma, such as nonuniformity in the density, finite temperature, or collision losses, causes the measured gain to be finite near $\omega = \omega_p$. Because of the logarithmic behavior of the gain parameter, the actual nature of the loss mechanism need not be known to great quantitative accuracy.

In the range $\omega_p < \omega < (\omega_c^2 + \omega_p^2)^{1/2}$ the propagation constant T_2 is real, with the result that, if the beam is finite and small in diameter compared with the wavelength, then there are values of frequency in this range over which the plasma presents an inductive impedance to the beam, and gain would be expected. The nature of the plasma is not known precisely; thus, it can be said only that there is gain within this range, and that its magnitude as a function of frequency does not vary monotonically with frequency but has local maxima and minima. For $\omega > (\omega_c^2 + \omega_p^2)^{1/2}$, the predicted gain is zero.

The theoretical curve for gain in the range $\omega_c < \omega < \omega_p$ for a given ω_p is plotted with an experimental curve in Fig. 2. The effect of conduction currents due to cesium collecting on the

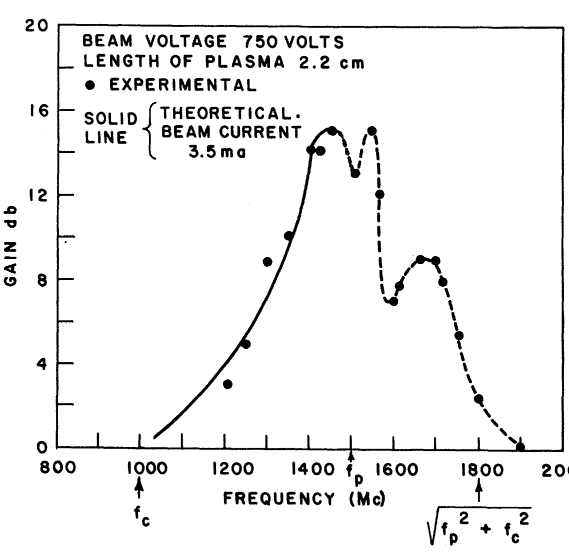

FIG. 2. Gain due to beam-plasma interaction plotted as a function of frequency.

walls of the tube prevented an accurate measurement of beam current being made. The theoretical curves are for an assumed value of 3.5 ma. It is noted that gain is obtained roughly from ω_c to $(\omega_p^2+\omega_c^2)^{1/2}$.

Finally, it is noted that there is an important conclusion to be drawn from finite-beam theory, which in the range $\omega_p<\omega<(\omega_p^2+\omega_c^2)^{1/2}$ gives a propagation constant T_2 which is not only real but may be small. This conclusion is, that it is possible to obtain gain when the fields radiate outwards from the beam, and the wavelength in the radial direction is comparable to the wavelength in free space. Thus, if the plasma ends abruptly, it is possible for such fields to radiate directly into free space. This mechanism might explain how radiation may be emitted from a stellar atmosphere. It may also point to possible methods for detecting rf radiation from a plasma in the laboratory.

Further experiments are being carried out with a modified cesium plasma tube, and it is intended to give a full description of our experimental and theoretical results in a forthcoming paper. The authors are indebted to Sylvania for providing parts of a traveling-wave tube and to Mr. D. L. Masterson for his part in the construction of the experimental tube.

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[1]G. D. Boyd, L. M. Field, and R. W. Gould, Phys. Rev. 109, 1393 (1958).

[2]E. V. Bogdanov, V. A. Kislov, and Z. S. Chernov, Radiotekh. Elektron. 5, 229 (1960) (translated by Pergamon Press, New York).

[3]M. A. Allen, G. S. Kino, and J. D. Lawson, presented at the Conference on Plasma Physics, American Physical Society, Gatlinburg, November, 1960.

AN ERROR IN THE THERMAL CONDUCTIVITY FOR A FULLY IONIZED GAS

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The existence of a discrepancy between the thermal conductivities for a fully ionized gas perpendicular to a magnetic field as calculated by Marshall[1] and by Rosenbluth and Kaufmann[2] has been widely realized for some time. Because of the current interest in transport phenomena and the fact that the presence of such a discrepancy may lead to doubts concerning the validity of the underlying principles of these different calculations, we feel it important to point out the source of this error.

Rosenbluth and Kaufmann compute transport coefficients for large $\omega\tau$, where ω and τ^{-1} denote gyration and collision frequencies, respectively. In particular they find that, assuming $m_i \gg m_e$, the dominant contribution to the thermal conductivity K perpendicular to the magnetic field comes from ion-ion collisions. This result is at variance with the work of Marshall, where in the limit of large $\omega\tau$, all types of collisions contribute significantly to the thermal conductivity. The other transport coefficients are found to be in agreement.

We used the collision integrals listed on page 81 of Marshall[1] to recompute the moments of f_{11} of Rosenbluth and Kaufmann.[2] As expected, the first moment leads to Eq. (10) of Rosenbluth and Kaufmann but the second moment does not give their K. The discrepancy is found to arise through an error in the last collision integral of Marshall.[1] This should read

$$[w_{-2}S_{3/2}^{1}(w_2^2),\ w_{-2}S_{3/2}^{1}(w_2^2)]_2=\sqrt{2}M_1^{1/2}\phi(1-\delta),$$

using Marshall's notation. On inserting this in Eq. (5.4) on page 41, we found that a_1^0 and a_1^1 were unchanged, but that

$$a_2^1=-\tau\left(\frac{2kT}{m_1}\right)^{1/2}\frac{3M_1^{1/2}+0.566+i\omega\tau M_1^{1/2}}{\omega^2\tau^2M_1+9M_1+3.394M_1^{1/2}+0.32}.$$

This leads to

$$\lambda'\text{II}=1.25\,\frac{kn\tau kT}{m_1}\left[\frac{1.866\omega^2\tau^2+0.966}{\omega^4\tau^4+6.282\omega^2\tau^2+0.933}+\frac{3M_1+0.566M_1^{1/2}}{\omega^2\tau^2M_1+9M_1+3.394M_1^{1/2}+0.32}\right],$$