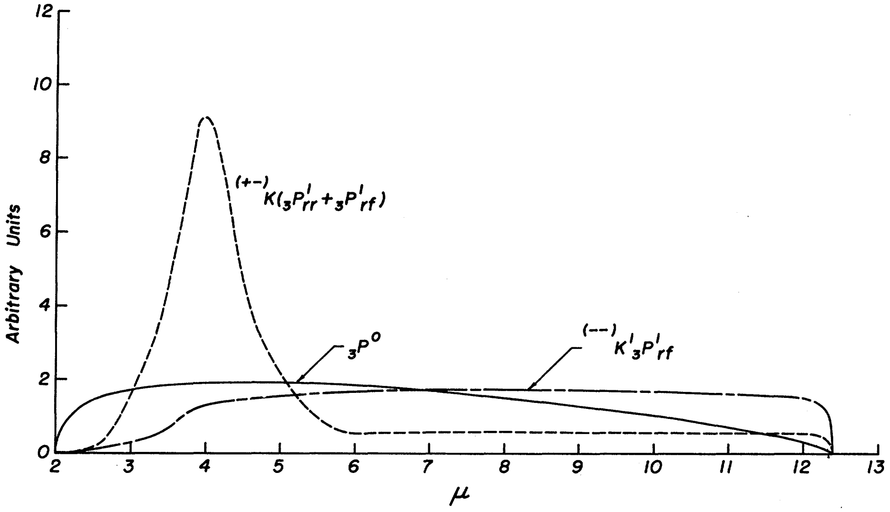

FIG. 1. Pion mass squared distributions for $\bar{p}+n\rightarrow\pi^- + \pi^- + \pi^+$.

space sufficiently to account for the suppression of this mode.

Even if the number of such annihilations is small compared to the number of events with one and two neutral pions, the poorer statistics should be compensated for by the pronounced peaking of the effective mass, which would tend to be suppressed for higher multiplicities.

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TWO-PION EXCHANGE INTERACTION*

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There is evidence for the one-pion exchange (OPE) interaction in nucleon-nucleon scattering.[1-3] The agreement of the coupling constant g with that from pion physics is satisfactory, and tests of the form of the OPE potential (OPEP) performed by adding a central field term and by varying proportions of spin-spin and tensor force terms have not indicated[4] a significant deviation from expectation for angular momentum $L>4$. The inclusion of phase shift K_4 indicated[4] a statistically significant coefficient of a central force addition to OPEP and it has been stated[4] that this probably indicates the setting in of two-pion exchange (TPE) effects. The statement was based on comparisons of Gupta's TPE potential[5] (TPEP) with the OPEP at the classical turning point for $L=4$. Results of further comparisons and of employing dispersion relations[6] are reported below.

Gupta's potential has a direct relationship to Dyson's scattering matrix S in terms of free-particle wave functions. In this respect the employment of field theory is similar to that of dispersion relations. The location of singularities does not enter the application of field theory, but unknown contributions[7] of higher powers of g^2 interfere with applicability of theory except at high L. The numerical values[5] of the TPEP use nonrelativistic approximation and do not include some additional effects.[8]

Since the TPE as well as the sum of OPE and TPE effects on phase shifts in states with $L > 1$ is in most cases < 0.2, accurate distinctions between a phase shift δ, and $\sin\delta$ or $\tan\delta$ do not enter and effects of second and fourth order potentials V_2 and V_4 were added employing Taylor's formula for each.[9] Effects of the order of a percent corresponding to $\delta = 0.2$ are of no interest here. The effect of V_4 was obtained with the aid of numerical values in Table I of reference 5. The integrand of the phase shift formula has a maximum close to the classical turning point, and for V_4 the relative importance of small r is higher than for V_2. The effect of V_2 was calculated as in reference 9. On account of the meaning of V_4, questions of wave function distortion do not arise and the integral converges.

According to Grashin,[10] in triplet states with total angular momentum $J = L + 1$ the energy dependence of the OPE phase shift is anomalously rapid at small energies, E. This may be seen as a cancellation of dominant terms employing the formulas in reference 9. These states are thus especially suitable for TPE detection. The $^3P_2$ phase parameter $\theta^{P}{}_{2}$ is higher than its OPE value for all recently published fits[11] which fall in a band narrow compared with the distance from OPE. For $^3F_4$ only the rather improbable fit YRB3 crosses the OPE curve and falls below it above 180 Mev. All other fits give values of $\theta^{F}{}_{4}$ greater than those for OPE. Similarly for the n-p phase parameter[12] $\theta^{D}{}_{3}$ the OPE value is below all empirical fits and has the wrong sign. An exception to these relationships is $\theta^{G}{}_{5}$ for which most fits give values below those of OPE. But one of the empirically probable fits falls above OPE at low E, crosses it at 180 Mev, and differs from the sum of OPE and TPE at 274 Mev on the limit of error.

In Table I the contributions of V_2 and V_4 are shown for T = 1. For $\theta^{P}{}_{2}$, V_4 definitely overcorrects the difference between OPE and the fits, the standard deviation being small and all searches giving nearly the same values at the energies considered. Evidence below shows that this is probably due to short-distance effects. The disagreement of calculation with YLAM values of $\delta^{P}{}_{0}$ and $\delta^{P}{}_{1}$ is more marked and similarly in the case of K_1. For K_2 the agreement of calculated and YLAM values is good at the two lower energies and is much better than between OPE and YLAM. At 274 Mev, V_4 overcorrects OPE, presumably because of short-distance effects. For $\theta^{F}{}_{2}$ the accuracy of YLAM and the agreement among searches are poor.[11] At the two higher energies calculation and YLAM disagree markedly, but on account of the poor accuracy, a conclusion is difficult. An additional uncertainty may enter on account of coupling of P_2 to F_2. The disagreement is therefore not

Table I. Comparison of combined effect of $V_2 + V_4$ (in radians) with fit YLAM (T = 1).

Phase parameter	E (Mev)	V_2	V_4	$V_2 + V_4$ - YLAM	Standard deviation
$\theta^{P}{}_{2}$	68.5	0.026	0.23	0.12	0.011
	137	0.05	0.48	0.30	0.009
K_2	68.5	0.025	0.018	0.000	0.006
	137	0.035	0.067	0.006	0.003
	274	0.044	0.176	0.055	0.007
$\theta^{F}{}_{2}$	68.5	0.0096	0.0014	0.0040	0.009 (0.0025)
$\delta^{F}{}_{3}$	68.5	-0.020	0.001	0.002	
	137	-0.043	0.011	0.011	
	274	-0.073	0.043	0.035	0.006
$\theta^{F}{}_{4}$	68.5	0.002	0.001	0.002	
	137	0.006	0.011	0.011	
	274	0.015	0.043	0.031	0.006

Table II. Comparison of combined effect of V_2+V_4 (in radians) with fit YLAN3M (T = 0).

Phase parameter	E (Mev)	V_2	V_4	V_2+V_4 - YLAN3M	Standard deviation
θ^D_1	68.5	-0.132	0.025	0.066	0.011
	137	-0.265	0.094	0.084	0.010
	274	-0.467	0.252	0.164	0.03
δ^D_2	68.5	0.183	0.025	-0.07	0.018
	137	0.33	0.09	0.04	0.013
θ^D_3	68.5	-0.021	0.025	-0.010	0.010
	137	-0.050	0.094	0.010	0.006
	274	-0.10	0.25	0.07	0.022
K_3	68.5	-0.031	0.002	0.002	
	137	-0.056	0.014	0.015	
	274	-0.073	0.059	0.045	0.030

surprising. The F_3 search YLAM used the OPE value below 160 Mev. The disagreement at 137 Mev is, therefore, not significant and δ^F_3 was probably forced towards small values by the enforcement of OPE. This view is supported by the almost exact agreement of the calculated value with fit YLA at 137 Mev and the location of YLA nearly halfway between YLAM and YRB1. Similarly for θ^F_4, the 137-Mev value falls on YRB2 and overshoots YRB1 only slightly. Possibly in both cases YLAM has underestimated the values at 137 Mev.

In Table II similar comparisons are made with fit YLAN3M to T = 0 states.[12] Effects of V_4 are in the wrong direction to give agreement for ${}^3D_1$. Since ${}^3D_1$ is coupled to ${}^3S_1$ to which the application of V_2+V_4 is inadequate, this disagreement is in the same class as that for θ^P_2. For δ^D_2 the addition of the V_4 effects is helpful at 68.5 Mev and overshoots YLAN3M at 137 Mev. A number of other fits fall in between the calculated value and YLAN3M, however. There appears to be agreement on the whole in this case. For θ^D_3 the OPE value has the wrong sign but TPE brings about agreement at 68.5 and 137 Mev on the limit of error. At 274 Mev the TPE overshoots again. For K_3 the YLAN3M used OPE values in $0<E<175$ Mev. The calculated value exceeds YLAN3M by half the standard deviation but YLAN3M would be higher had OPE not been enforced below 175 Mev. There is agreement on the whole and deviations from V_2+V_4 follow similar patterns for T = 0 and T = 1.

Tables III and IV show comparisons of phase parameters for T = 1 and T = 0, respectively, according to the dispersion relations (DR) treatment of Galanin et al.[6] and of Grashin and Kobsarev[6] employing their tabulated ratios of values of TPE to OPE. The TPE gives improvement for ${}^3P_1$ and ${}^3P_2$, but not for ${}^3P_0$. The critical quantity $(L/k)(m_\pi c/\hbar)=L\mu/k$ is only 1.5 at 100 Mev and is hardly large enough to expect agreement. For K_2, i.e., ${}^1D_2$, the potential gives by far the better agreement; for F_2 the DR are somewhat the better; for F_3 and F_4 the DR give the better fit to YLAM but the potential is conceivably the better taking YRB1, YLA into account. For T = 0 the DR are much the better for K_1, i.e., ${}^1P_1$, and also for ${}^3D_1$, but the potential is the better below 200 Mev for ${}^3D_2$ and much the better up to 300 Mev for ${}^3D_3$. For K_3 the DR agree best with YLAN3 but considering the enforcement of OPE below 175 Mev for this fit the potential may be actually the better. For G_3, G_4, G_5 the two ways of treating TPE do not appear significantly different considering the uncertainties of the fits to data.

In the calculations of the effect of V_4, the maximum of the integrand falls at $x_m>1$ $(x=rm_\pi c/\hbar)$ only for $L\geqslant 2$ at the three energies used. At 274 Mev, $x_m>1$ only for $L>2.2$, and for $L=2$ this criterion is barely satisfied at 137 Mev. The disagreement of the potential with the empirical fits for P terms is thus understandable and so are the poorer values for $L=3$ at 274 Mev. The reasonableness of this situation combined with

Table III. Comparison of combined effect YLAM+Δ of one-pion and two-pion exchange effects (in radians) according to Galanin et al. and Grashin and Kobsarev,[a] with fit YLAM for T = 1.

State	E (Mev)	OP	TP	Δ
$^3P_1$	10	-0.052	0.004	0.002
	40	-0.176	0.018	-0.016
	100	-0.341	0.051	-0.056
$^3P_2$	10	0.002	0.002	-0.009
	40	0.014	0.028	-0.045
	100	0.040	0.080	-0.071
$^1D_2$	40	0.016	0.0002	-0.007
	100	0.032	0.008	-0.033
	200	0.041	0.020	-0.071
$^3F_2$	40	0.0042	0.0002	0.0013
	100	0.0160	0.0010	0.16
$^3F_3$	40	-0.094	0.002	0.002
	100	-0.031	0.0012	0.0012
	200	-0.059	0.005	0.0026
$^3F_4$	40	0.0008	0.0002	0.0002
	100	0.0041	0.0016	0.0016
	200	0.0104	0.0073	0.0049
	300	0.0158	0.0158	-0.0008

[a] See reference 6.

the striking improvements for K_2, D_2, D_3 caused by V_4 indicate the reality of the TPE. This conclusion is strengthened by the DR calculations. The disagreements of the potential with data at low L and the higher E indicate the presence of additional effects in the general region $x < 1$ and closer examination shows the presence of spin-orbit like effects with an "inverted" order.

Table IV. Comparison of combined effect YLAN3M +Δ of one-pion and two-pion exchange effects (in radians) according to Galanin et al. and Grashin and Kobsarev,[a] with fit YLAN3M for T = 0.

State	E (Mev)	OP	TP	Δ
$^1P_1$	10	-0.060	0.009	-0.033
	40	-0.158	0.032	-0.034
	100	-0.215	0.043	0.034
$^3D_1$	40	-0.070	0.001	0.035
	100	-0.197	-0.010	0.010
	200	-0.364	-0.026	-0.078
$^3D_2$	40	0.103	0.003	-0.061
	100	0.258	0.008	-0.074
	200	0.445	0.018	0.063
$^3D_3$	40	-0.009	0.005	-0.010
	100	-0.034	0.028	-0.029
	200	-0.075	0.060	-0.067
$^1F_3$	100	-0.0424	0.0017	0.0015
	200	-0.065	0.0046	0.0032
	300	-0.076	0.0076	-0.014
$^3G_3$	100	-0.149	-0.0001	0.0000
	200	-0.038	-0.0011	0.0067
	300	-0.059	-0.003	0.054
$^3G_4$	100	0.037	0.0004	0.0005
	200	0.082	0.0016	0.0077
	300	0.120	0.0024	0.044
$^3G_5$	100	-0.0044	0.0009	0.0009
	200	-0.0135	0.0041	0.0047
	300	-0.0226	0.0068	0.0103

[a] See reference 6.

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