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MATRIX ELEMENTS IN THE FORBIDDEN BETA DECAY OF Ce^{141}

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We have performed measurements on oriented Ce^{141} in which the energy and angular distribution of the beta particles are determined with respect to the nuclear spin orientation. These measurements give information on the relative contribution of the various nuclear matrix elements contributing to the beta transitions.

The 33-day Ce^{141} undergoes beta decay to Pr^{141} via two beta groups according to the schemes

$$\tfrac{7}{2}- \xrightarrow{\beta_1} \tfrac{7}{2}+ \xrightarrow{\gamma} \tfrac{5}{2}+$$

and

$$\tfrac{7}{2}- \xrightarrow{\beta_2} \tfrac{5}{2}+.$$

The 70% abundant β_1 transition with a maximum energy of 435 kev and a $\Delta J=0$ will depend on all six first forbidden (real) matrix elements: $\mathfrak{M}(i\vec{\sigma}\cdot\vec{r})$, $\mathfrak{M}(\gamma_5)$, $\mathfrak{M}(i\vec{r})$, $\mathfrak{M}(\vec{\sigma}\times\vec{r})$, $\mathfrak{M}(\vec{\alpha})$, and $\mathfrak{M}(iB_{ij})$. The 30% abundant ground-state β_2 transition with a maximum energy of 580 kev and a $\Delta J=1$ will depend on only the last four matrix elements, i.e., those of tensor rank one and two.

The energy and angular distribution function, which has been calculated by Morita and Morita[1] and by Bincer,[2] has the form

$$W(\hat{p}\cdot\hat{J})=N_0(E)+N_1(E)\frac{p}{E}f_1P_1(\hat{p}\cdot\hat{J})+N_2(E)\frac{p^2}{E}f_2P_2(\hat{p}\cdot\hat{J})$$
$$+N_3(E)\frac{p^3}{E}f_3P_3(\hat{p}\cdot\hat{J}),$$

where $N_k(E)$ gives the energy dependence of the term of order k and contains products of the reduced matrix elements. The orientation parameter f_k describes the nuclear orientation, given in terms of an average over the population of the magnetic sublevels. The electron momentum and energy are p and E, respectively, and $P_k(\hat{p}\cdot\hat{J})$ is a Legendre polynomial. $N_0(E)$ gives the usual energy distribution for an unoriented beta emitter apart from the statistical factors.

The experimental apparatus and the procedure for the reduction of the data are similar to those used in earlier studies on the angular distribution of beta particles from oriented nuclei.[3] A single crystal of neodymium ethyl sulfate used for both cooling and orienting purposes is mounted with the c axis vertical in the demagnetization apparatus. On the uppermost surface of this crystal is grown a thin surface layer containing the Ce^{141} activity. Located about one centimeter above this is a thin anthracene scintillator which provides the input to a 100-channel pulse-height analyzer for the analysis of the beta spectra. Three 2-in. by 2-in. NaI gamma counters are located equatorially about 15 cm from the axis of the apparatus. These counters monitor the gamma-ray anisotropy which, combined with nuclear hyperfine splitting data for Ce^{141} neodymium ethyl sulfate, allows one to determine the nuclear orientation parameters f_k. The outputs of the gamma-ray counters are also compared for time coincidence with the output of the beta channel by means of a fast-slow coincidence circuit in order to separate the distribution of the 435-kev inner beta group from the 580-kev ground-state transition.

The quantities $N_1(E)/N_0(E)$ and $N_2(E)/N_0(E)$ can be measured independently. The measurements of the ratio $N_2(E)/N_0(E)$ were made by determining the beta distribution when the sample has been aligned by the Bleaney method after cooling the sample by adiabatic demagnetization from a horizontal magnetic field of 23 kilogauss at 1.0°K. After a counting period of about thirty minutes the temperature of the sample was raised to 1°K again, destroying the nuclear alignment. The normalization for the gamma-ray anisotropy depended on a reliable measurement of a uniform gamma-ray distribution. In the case of nuclear alignment f_1 and f_3 are always zero.

The ratio $N_1(E)/N_0(E)$ was determined under similar conditions except that here a magnetic polarizing field of 200 gauss was applied along the c axis of the source-crystal so that the Ce^{141} nuclei were polarized by the Rose-Gorter method. Since this polarizing field was applied adiabatically, the sample temperature was not as low as in the case when no field was applied. By taking different combinations of the ratio of beta counts before and after orientation for two opposite field directions one can obtain values for both N_1/N_0 and N_2/N_0. The accuracy of the latter is poorer than in the alignment experiments where a larger f_2 is achieved. Counting was continued for alternating polarizing field directions for periods up to one hour. At the end of this period the sample was warmed and the orientation destroyed. The contribution of the f_3 term is calculated to be negligible at the temperatures achieved.

The observed distributions were corrected for backscattering effects in the source and for the finite energy and angular resolution of the apparatus; the limits on our experimental values include estimates of errors in these corrections as well as those due to counting statistics.

The experimentally determined distribution function for the ground-state 580-kev transition is

$$W(\hat{p}\cdot\hat{J})/N_0 = 1 - (1.11\pm 0.10)\frac{p}{E}f_1P_1(\hat{p}\cdot\hat{J}) + (0.36\pm 0.10)\frac{p^2}{E}f_2P_2(\hat{p}\cdot\hat{J}),$$

where we have evaluated the coefficients for an average $E = 1.85$ in relativistic units. A rough measurement of the energy distribution $N_1(E)/N_0(E)$ is also obtained.

The interpretation of these results requires that accurate radial Coulomb wave functions for the electron be employed. We have used exact radial wave functions taking into account nuclear finite-size effects. These functions have been calculated in a similar manner to those calculated by Bhalla and Rose.[4]

Each $N_k(E)$ contains products and squares of the matrix elements, each multiplied with energy-dependent factors. In principle one can get three independent nonlinear simultaneous equations for any individual $N_k(E)$ at three energies or for all three $N_k(E)$ at one energy. However, in the first case the energy dependence of $N_k(E)$ on the matrix elements is so insensitive that in practice only a rough limitation on the matrix elements can be determined. There is no evidence for a deviation from a statistical shape of $N_0(E)$ as measured by Freedman and Engelkemier[5] and Jones and Jensen,[6] and of $N_1(E)$ within the accuracy of our experiments.

For the purposes of illustration only, we employ an approximation where the radial functions are expanded in powers of $\xi = \alpha Z/2R$ and retain only those terms involving the highest power of ξ multiplying each matrix element product in each particle parameter (see reference 2). This approximation is useful only when the matrix element ratios are such that energy-independent factors dominate, giving the statistical shapes to the spectra. When we do this we obtain rather simple expressions for the $N_k(E)$, which are evaluated at the average energy $E = 1.85$ and compared with experiment in Figs. 1 and 2. We find that the allowable values of certain matrix element combinations are greatly restricted by experiment. In order to fix the relative values of the individual matrix elements, we must resort to a model to fix at least one ratio theoretically.

Fortunately the ground states of both ${}_{58}Ce_{83}{}^{141}$ and ${}_{59}Pr_{82}{}^{141}$ have configurations that should be adequately described by shell-model wave functions. For Ce^{141} the measured spin and magnetic moment establish the ground state of the eighty-third neutron as an $(f_{7/2})^1$ configuration. For Pr^{141} the measured spin, the magnetic moment of +3.92 nuclear magnetons, and the quadrupole moment of -0.054 barn are consistent with a mixture[7] of configurations $(g_{7/2})^8(d_{5/2})^1$ and $(g_{7/2})^6(d_{5/2})^3$ for the protons outside the closed shell at 50. With these configurations the ratios of certain nuclear matrix

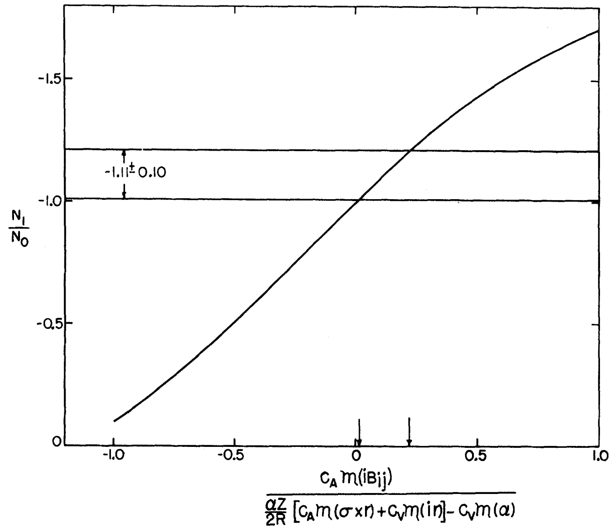

FIG. 1. The quantity N_1/N_0, evaluated at $E=1.85$, as a function of $\{C_A\mathfrak{M}(iB_{ij})\}/\{(\alpha Z)(2R)^{-1}[C_A\mathfrak{M}(\vec{\sigma}\times\vec{r})+C_V\mathfrak{M}(i\vec{r})]-C_V\mathfrak{M}(\vec{\alpha})\}$, using the approximate Coulomb functions described in the text. The range of values allowed by experiment is indicated on the curve.

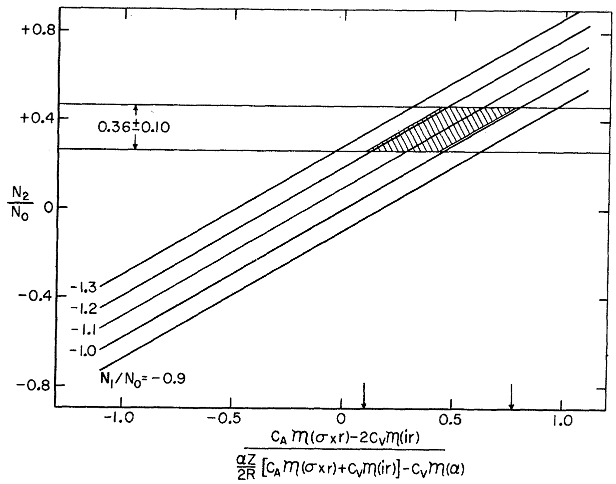

FIG. 2. The quantity N_2/N_0, evaluated at $E=1.85$, as a function of $\{C_A\mathfrak{M}(\vec{\sigma}\times\vec{r})-2C_V\mathfrak{M}(i\vec{r})\}/\{(\alpha Z)(2R)^{-1}\times[C_A\mathfrak{M}(\vec{\sigma}\times\vec{r})+C_V\mathfrak{M}(i\vec{r})]-C_V\mathfrak{M}(\vec{\alpha})\}$, using the approximate Coulomb functions described in the text. The range of values allowed by experiment is indicated on the curve.

elements are predicted uniquely and are, furthermore, completely independent of any details of the nuclear radial wave functions. In addition, admixtures of other configurations such as $(g_{7/2})^4(d_{5/2})^5$, with appropriate initial state proton configurations, do not alter these ratios. Admixtures of higher seniority states involving the rearrangement of the relevant nucleons in the $g_{7/2}$ and $d_{5/2}$ shells will not contribute at all to these matrix element ratios.

We have

$$\mathfrak{M}(i\vec{r})/\mathfrak{M}(\vec{\sigma}\times\vec{r})=-1;\quad \mathfrak{M}(iB_{ij})/\mathfrak{M}(\vec{\sigma}\times\vec{r})=-3(\tfrac{2}{5})^{1/2}.$$

The other ratio $\mathfrak{M}(\vec{\alpha})/\mathfrak{M}(\vec{\sigma}\times\vec{r})$ involves the relativistic operator $\vec{\alpha}$ which normally connects large with small components of relativistic four-component wave functions. Since most nuclear models involve only nonrelativistic wave functions, we must use a nonrelativistic operator as obtained from a canonical transformation[8] yielding $\vec{\alpha}\to i\vec{\nabla}/M$. This ratio is calculated to be

$$\frac{\mathfrak{M}(\vec{\alpha})}{\mathfrak{M}(\vec{\sigma}\times\vec{r})}=-\frac{1}{M}\,\frac{\int r^2u_f(r)^*\left(\frac{d}{dr}+\frac{4}{r}\right)u_i(r)dr}{\int r^3u_f(r)^*u_i(r)dr},$$

where the value depends on the details of the nuclear radial wave functions.

The radial wave functions are not predicted by the shell model without the introduction of a specific potential, so that we treat $\mathfrak{M}(\vec{\alpha})/\mathfrak{M}(\vec{\sigma}\times\vec{r})$ as a parameter to be determined by the experiment after the theoretical values of $\mathfrak{M}(i\vec{r})/\mathfrak{M}(\vec{\sigma}\times\vec{r})$ and $\mathfrak{M}(iB_{ij})/\mathfrak{M}(\vec{\sigma}\times\vec{r})$ have been introduced. In the evaluation of this parameter, we use the exact lepton radial wave functions referred to above. In Fig. 3 we show the theoretical predictions for N_1/N_0 and N_2/N_0 as functions of $[C_V\mathfrak{M}(\vec{\alpha})]/[C_A\mathfrak{M}(\vec{\sigma}\times\vec{r})]$. In this figure we have assumed nuclear radii $R=1.2A^{1/3}\times10^{-13}$ cm and $R=1.3A^{1/3}\times10^{-13}$ cm, and that $C_A=-1.21C_V$. The value of $\mathfrak{M}(\vec{\alpha})/\mathfrak{M}(\vec{\sigma}\times\vec{r})$ allowed by experiment is also given in Table I for the two values of the nuclear radius.

For comparison we have also included the conventional approximation[1,2] for the radial functions

Table I. Experimental values of $\mathfrak{M}(\vec{\alpha})/\mathfrak{M}(\vec{\sigma}\times\vec{r})$ for several evaluations of the lepton radial wave functions.

Lepton radial wave functions employed	
"Exact," $R=1.3A^{1/3}\times10^{-13}$ cm	-29.4 ± 1.4
"Exact," $R=1.2A^{1/3}\times10^{-13}$ cm	-29.5 ± 2.3
"Approx.," $R=1.3A^{1/3}\times10^{-13}$ cm	-36.6 ± 1.0

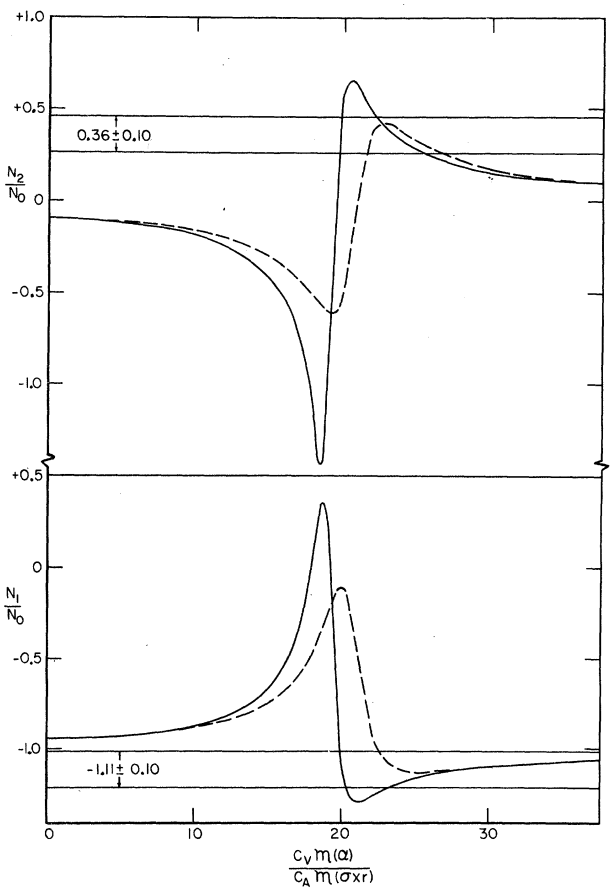

FIG. 3. The quantities N_1/N_0 and N_2/N_0 as functions of $[C_V\mathfrak{M}(\vec{\alpha})]/[C_A\mathfrak{M}(\vec{\sigma}\times\vec{r})]$, using exact Coulomb radial functions. The values of $[C_V\mathfrak{M}(i\vec{r})]/[C_A\mathfrak{M}(\vec{\sigma}\times\vec{r})]$ and $[C_A\mathfrak{M}(iB_{ij})]/[C_A\mathfrak{M}(\vec{\sigma}\times\vec{r})]$ have been fixed by adopting the single-particle values and $C_A=-1.21C_V$. The solid line corresponds to $R=1.3A^{1/3}\times10^{-13}$ cm, and the dashed line corresponds to $R=1.2A^{1/3}\times10^{-13}$ cm.

expanded in powers of ξ for a point nucleus but evaluated at $R=1.3A^{1/3}\times10^{-13}$ cm. It is instructive to see how poor the approximation is compared to the more exact treatment.

Figure 3 indicates that for certain choices of the radius $\mathfrak{M}(\vec{\alpha})/\mathfrak{M}(\vec{\sigma}\times\vec{r})$ may be double valued. The lower value can be rejected as being incompatible with the observed spectrum shape. It is interesting to note that the large coefficient for the P_2 term is consistent with the measured shape. The errors quoted in Table I do not include any uncertainty in the correct ratio for C_A/C_V in this particular transition. It is also interesting to note that a radius much smaller than $R=1.2A^{1/3}\times10^{-13}$ cm would be incompatible with the experimental results.

Several theoretical estimates of $\mathfrak{M}(\vec{\alpha})/\mathfrak{M}(i\vec{r})$ based on various models have been discussed by Rose and Osborn,[8] Ahrens and Feenberg,[9] and Pursey,[10] and our value for $\mathfrak{M}(\vec{\alpha})/\mathfrak{M}(\vec{\sigma}\times\vec{r})$ lies at about the negative mean of these estimates.

The absolute magnitude of the nuclear matrix elements may be fixed by fitting the calculated total transition probability to the observed ft value, viz., $\log ft=7.7$. Information on the radial integrals and configurational mixing is thereby obtained. The observed ft is several orders of magnitude greater than the lower limit that is predicted using our matrix-element values when one assumes a pure single-particle configuration with complete overlap of the radial functions.

Finally, we have also a preliminary determination of the β-correlation function calculated by Morita and Morita[11] for the 435-kev, $\Delta J=0$, transition to the excited state. However, the six contributing matrix elements cannot be uniquely defined by the limited number of parameters thus far determined. We are not presenting our results on this transition until further work is completed, at which time we shall present a more complete exposition of the experimental results.

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