# New Spin-Wave Modes in $^3$He-$^4$He Solution

H. Ishimoto, H. Fukuyama, N. Nishida,[a] Y. Miura,[b] Y. Takano, T. Fukuda, T. Tazaki, and S. Ogawa
*Institute for Solid State Physics, The University of Tokyo, Roppongi, Minato-ku, Tokyo 106, Japan*
(Received 22 June 1987)

cw NMR measurements have been made on $^3$He-$^4$He solutions of $^3$He concentration ($X_3$) between 1.3% and 8.6%, at pressures of 0, 7, or 8 bars and at temperatures down to 0.2 mK. In the collisionless region, the NMR absorption line exhibits several satellite peaks in addition to its narrowing. Numerical analysis of the phenomenon as an excitation of standing spin-wave modes leads to the determination of the Fermi-liquid interaction parameter $\lambda$. Its concentration dependence supports the quasiparticle interaction potential proposed by Murdock, Mountfield, and Corruccini.

PACS numbers: 67.60.−g, 76.50.+g

The existence of spin waves in the degenerate Fermi liquid in a magnetic field was predicted almost thirty years ago by Silin.[1] For a long time the only evidence of its existence in normal $^3$He and $^3$He-$^4$He solutions was the observation by Corruccini *et al.*[2] of the Leggett-Rice effect. They found that the effective spin diffusion had a maximum as a function of the temperature, but were unable to detect spin waves directly. Recently, standing spin-wave modes were observed both in normal $^3$He[3] and in degenerate $^3$He-$^4$He solutions.[4] In pure $^3$He, many peaks were seen in the NMR absorption line, whereas in the $^3$He-$^4$He mixtures, only one resonance peak was observed to shift and to have a maximum in width with variation of temperature. Although this behavior was attributed to the excitation of a spin wave with nonzero wave vector ($k\neq 0$), the absence of the $k=0$ mode was puzzling. One explanation was given by Bowley and Owers-Bradley,[5] but further experiments are obviously desirable. In spin-wave theory, the differences between pure $^3$He and degenerate mixtures are the values of the Fermi velocity $V_F$, the Fermi-liquid parameters ($F_0^a$,$F_1^a$), and the quasiparticle relaxation time $\tau$. One therefore expects to see similar wave modes in the mixtures and in pure $^3$He.

In this Letter we report a study of spin waves in dilute mixtures confined to a rectangular sample cell. The observed spectra are compared with numerical solutions of the equation of motion[6] for the transverse nuclear magnetization [$M^+(r,t)=M_x+iM_y$] under a static magnetic field $H_0(r)$ and an rf field $H_1(r,t)$, with $H_1^+=H_{1x}+iH_{1y}$, as successfully done for the spin wave of pure $^3$He by Masuhara *et al.*[3] In the case of a small variation of the Larmor frequency $\Omega_0(r)=\gamma H_0(r)$ over the sample and $|M^+|\ll M_z$, the equation of motion can be written as

$$iD\nabla^2 M^+ + \Omega_0 M^+ = i\,\partial M^+/\partial t + \gamma M_z H_1^+, \quad (1)$$

where $D$ is the complex diffusion constant given by

$$D=D_0/(1+i\lambda\omega_0\tau). \quad (2)$$

Here $D_0=V_F^2(1+F_0^a)/3$ is the diffusion constant in the hydrodynamic regime, $\omega_0$ is the mean value of $\Omega_0(r)$, and $\lambda=(1+F_0^a)^{-1}-(1+\frac{1}{3}F_1^a)^{-1}$. The substitutions $M^+(r,t)=m^+(r)e^{-i\omega t}$, $H_1^+(r)=h_1^+(r)e^{-i\omega t}$ lead to the dynamic susceptibility averaged over the sample,

$$\chi(\omega)=\chi(0)\textstyle\sum_n C_n\omega_0/(\omega_n-\omega), \quad (3)$$

where $\omega_n$ are the complex eigenfrequencies of Eq. (1) with $H_1^+=0$, and $C_n$ are calculated from the corresponding eigenfunctions and $h_1^+(r)$. The imaginary part of $\chi(\omega)$ is compared with the obtained absorption spectrum.

In our experiment, the mixtures are cooled by use of our single-stage nuclear-demagnetization refrigerator with a total 140 mol of copper. To reduce the heat input entering directly into the mixture, three heat exchangers are employed in series. The main one is made of a composition of 100-Å platinum powder and 400-Å silver powder. The total surface area is about 2400 m$^2$.[7] The sample volume is 40 cm$^3$. Thermometry is by platinum-

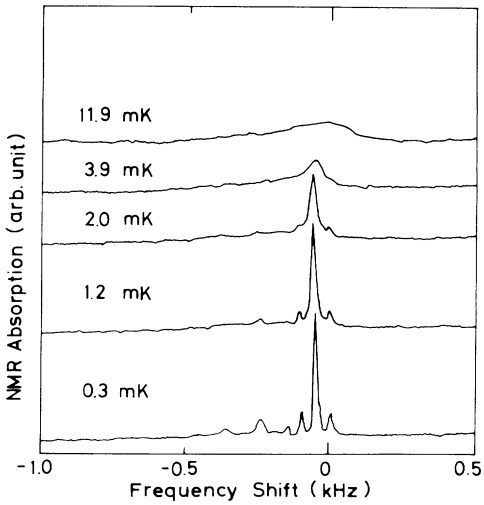


FIG. 1. Temperature dependence of the NMR absorption line for $X_3=6.9\%$ (7 bars) at Larmor frequency of 2040 kHz without an intentional field gradient.

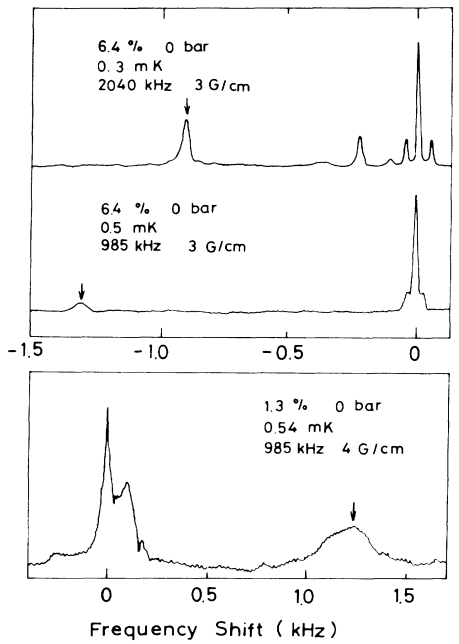


FIG. 2. Absorption lines at the indicated $^3$He concentrations, pressures, temperatures, Larmor frequencies, and applied field gradients. The vertical scale is different in each case.

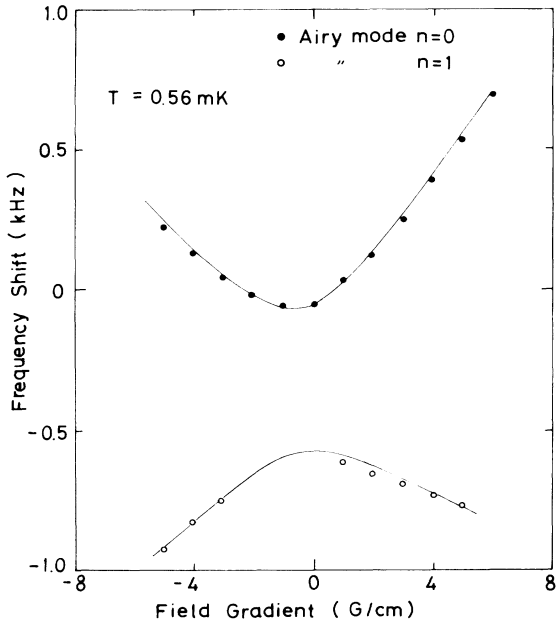


FIG. 3. Frequencies of the Airy-mode spin-wave peaks for $X_3=6.4\%$ (0 bar) as a function of the applied field gradient at Larmor frequency of 2040 kHz. Circles are experimental data and the lines are calculated by use of $V_F=2.9\times10^3$ cm/sec, $F\beta=0.08$, $\tau=2.5\times10^{-11}/T^2$ sec, and $\lambda=0.035$.

wire NMR at 250 kHz, calibrated against the $^3$He melting curve above 15 mK.[8] The platinum wires are thermally connected to the mixture through 100-Å platinum powder with a surface area of 370 m$^2$.[7] The NMR tower, mounted on the main heat exchanger with an indium O ring, consists of a copper base and an L-shaped epoxy tube. The part perpendicular to the axis of the tower has a square hole of $1.39\times1.78$ mm$^2$ and an NMR coil is wound outside of it. A static field is produced along the tower by a superconducting magnet with a niobium shield. A separate set of windings on the main solenoid allows the gradient along the static field to be varied. $^3$He NMR measurements were performed at frequencies of 985 and 2040 kHz. The spectrometer is a simple $Q$-meter circuit, which is tuned at 985 kHz. For 2-MHz experiments, an external inductance was added in parallel to the original coil. Measurements were taken for $^3$He concentrations ($X_3$) of 8.6% (8 bars), 6.4% (0 bar), 6.9% (7 bars), and 1.3% (0 and 7 bars), in the temperature region between 0.2 and 40 mK.

Figure 1 shows experimental absorption lines for $X_3=6.9\%$, $P=7$ bars at 2040 kHz without an intentionally applied field gradient. In the hydrodynamic region of 11.9 mK, the line shape is distorted as a result of a quadratic term of the static field inhomogeneities. As we cool the sample, the absorption line narrows below 10 mK and several side peaks appear below 1 mK, corresponding to the standing spin-wave modes. The frequency of the central line shifts to the low-frequency side, although the magnitude is of the order of 50 Hz. Its half-width monotonically decreases from 10 to 1 mK, and reaches about 20 Hz below 1 mK. These different behaviors compared with those observed by Owers-Bradley *et al.*[4] should be attributed to our boundary condition that both ends of the sample cell are closed. An added field gradient causes another side peak to appear at the low-frequency side, as indicated by an arrow in Fig. 2. Around this peak, small peaks were seen at several field gradients, in the same configuration as those around the central peak. Similar narrowing of the absorption line and appearance of the side peaks have been observed for $X_3=6.9\%$ (7 bars) and 8.6% (8 bars). On the other hand, for $X_3=1.3\%$ (0 and 7 bars) at 985 kHz, the side peak indicated by the arrow appears on the high-frequency side of the central peak, as shown in Fig. 2. This indicates that the interaction parameter $\lambda$ for 1.3% has a sign different from that for $X_3=6.4\%$ and 8.6%.

To compare these experimental absorption lines with theoretical ones, the sample was assumed to be a rectangular prism $1.39\times1.783\times9.0$ mm$^3$, thus ignoring the complicated shape of the vertical channel at one end. For the static field, we used the same assumption as Masuhara *et al.*,[3]

$$\Omega_0(x,y,z)=A+Bz+C(z^2-\tfrac{1}{2}x^2-\tfrac{1}{2}y^2), \qquad (4)$$

where $z$ is the static field direction and $x$ is the rf coil

axis. The parameters $A$, $B$, and $C$ for both the main and gradient coils were determined by the high-temperature line shape and the field-gradient dependence of the observed side peaks. During the analyses, it was confirmed that the central peak and the side peak indicated by the arrow corresponded to Airy modes along the $z$ direction, $n=0$ and $n=1$, respectively, confined between the top and bottom surfaces of the sample cell, while the other small side peaks were due to the modes excited along the rf field. The frequencies of the Airy modes for $X_3=6.4\%$ (0 bar) at 2040 kHz as a function of the applied field gradient are compared with calculated ones in Fig. 3. A small asymmetric dependence on the applied field gradient is attributed to the parameter $A$ produced by the gradient coils. The best fit is obtained for a zero-spin-current boundary condition $\nabla M^+=0$ and resulted in the interaction parameter $\lambda=0.035\pm0.005$ when the values of $V_F=2.9\times10^3$ cm/sec and $F_0^a=0.08$ (Ref. 2) were used. The predicted peak separation of the Airy modes at 985 kHz by use of the obtained $\lambda$ at 2040 kHz is verified in Fig. 2, confirming our analyses.

Similar derivations of $\lambda$ for the other $^3$He concentrations and pressures depends on our knowledge of $V_F$ and $1+F_0^a$. No experimental results for these values are available except for $X_3=1.3\%$ (0 bar).[9] For the calculation of $V_F$ at high pressures of 7 or 8 bars, we need the effective mass $m^*$. Values have been obtained from specific-heat measurements by several groups.[9-11] Although the data agree well with each other at 0 bar, those at elevated pressures have large discrepancies.[10,11] At the present stage we tentatively estimated $m^*$ at 7 or 8 bars[12] from the $X_3$ dependence of $m^*$ at 0, 10, and 20 bars reported by Murdock, Mountfield, and Corruccini.[13] Here we assumed a smooth variation of $m^*$ with pressure. The values of $F_0^a$ were inferred from the calculated values of Ref. 13 to be 0.078, 0.0, and $-0.02$ for $X_3=1.3\%$ (7 bars), 6.9% (7 bars), and 8.6% (8 bars), respectively. Another estimation of $F_0^a$ from the experimental values at 0, 10, and 20 bars reported by Ahonen *et al.*[14] gives 0.115, 0.02, and $-0.02$, for the above $X_3$ and pressures. In spite of their large uncertainty, exact values are not so crucial for the determination of $\lambda$, because the absolute magnitudes of $F_0^a$ are small compared with 1. With use of $V_F$ and $F_0^a$ obtained in the above way, the corresponding $\lambda$ value was deduced from a fitting similar to that for $X_3=6.4\%$. The results are shown in Fig. 4 at 0 and 7 bars.

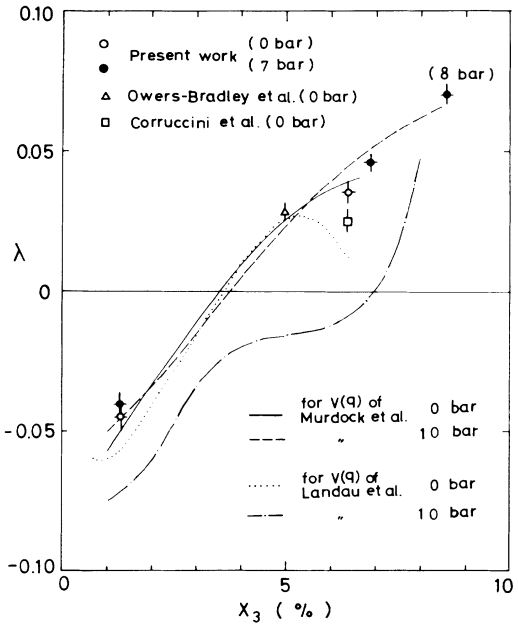


FIG. 4. Variation of $\lambda$ with $^3$He concentration. The solid and broken lines are calculated for $V(q)$ of Murdock, Mountfield, and Corruccini (Ref. 13) at 0 and 10 bars. The dotted and dash-dotted lines are for $V(q)$ of Landau *et al.* (Ref. 15) at 0 and 10 bars. The sign of $\lambda$ from Corruccini *et al.* (Ref. 2) was assumed to be positive.

Theoretical curves at 0 and 10 bars in Fig. 4 were evaluated in the Hartree-Fock approximation from the quasiparticle interaction potential $V(q)$, where $q$ is the momentum transfer.[5] Two different potentials were chosen, one given by Murdock, Mountfield, and Corruccini,[13] and the other by Landau *et al.*[15] The $\lambda(X_3)$ obtained here has little pressure dependence and the $V(q)$ of Ref. 13 gives a better fit to the present data. In terms of weak-coupling BCS theory, the potential of Ref. 13 predicts a superfluid transition of $^3$He in the mixtures at temperatures below 1 $\mu$K. This is consistent with our results which show no evidence of such a transition in $^3$He-$^4$He mixtures at temperatures down to 212 $\mu$K for 8.6% (10 bars), and 237 $\mu$K for 6.4% (3.5 bars).

In summary, standing spin-wave modes were observed in the cw NMR spectrum of $^3$He-$^4$He solutions. From the numerical analyses of such modes, the interaction parameter was found to change its sign between 1.3% and 5%. The $^3$He concentration and pressure dependence of $\lambda$ seems to support the quasiparticle interaction potential $V(q)$ proposed by Murdock, Mountfield, and Corruccini.[13]

We would like to thank Professor K. Ôno for his encouragement during the experiment. This work was supported in part by a Grant-in-Aid for Special Project Research from the Ministry of Education, Science, and Culture of Japan.

[a]Present address: Department of Physics, Tokyo Institute of Technology, Ookayama, Tokyo 152, Japan.

[b]Present address: Department of Physics, Nagoya University, Chikusa-ku, Nagoya 464, Japan.

[1]V. P. Silian, Zh. Eksp. Teor. Fiz. **33**, 726 (1958) [Sov. Phys. JETP **6**, 945 (1958)].

[2]L. R. Corruccini, D. D. Osheroff, D. M. Lee, and R. C.

Richardson, J. Low Temp. Phys. **8**, 229 (1972).

[3]M. Masuhara, D. Candela, D. O. Edwards, R. F. Hoyt, H. N. Scholz, and D. S. Sherrill, Phys. Rev. Lett. **53**, 1168 (1984); D. Candela, M. Masuhara, D. S. Sherrill, and D. O. Edwards, J. Low Temp. Phys. **63**, 369 (1986).

[4]J. R. Owers-Bradley, H. Chocholacs, R. M. Mueller, Ch. Buchal, M. Kubota, and F. Pobell, Phys. Rev. Lett. **51**, 2120 (1983).

[5]R. M. Bowley and J. R. Owers-Bradley, J. Low Temp. Phys. **63**, 331 (1986).

[6]A. J. Leggett, J. Phys. C **3**, 448 (1970).

[7]The experimental details will be described elsewhere.

[8]D. S. Greywall, Phys. Rev. B **33**, 7520 (1986).

[9]A. C. Anderson, D. O. Edwards, W. R. Roach, R. E. Sarwinski, and J. C. Wheatley, Phys. Rev. Lett. **17**, 367 (1966).

[10]H. C. Choholacs, R. M. Mueller, J. R. Owers-Bradley, Ch. Buchal, M. Kubota, and F. Pobell, in *Proceedings of the Seventeenth International Conference on Low-Temperature Physics,* edited by U. Eckern, A. Schmid, W. Weber, and H. Wühl (North-Holland, Amsterdam, 1984), p. 1247.

[11]E. Polturak and R. Rosenbaum, J. Low Temp. Phys. **43**, 477 (1981).

[12]The effective mass $m^*$ for $X_3=1.3\%$ (7 bars) obtained in the text agrees well with that estimated from Ref. 10. There are few experimental values of $m^*$ above 5% at elevated pressures, and the interpolation to the desired $X_3$ and pressures has large ambiguities. A rough estimation of $m^*$ for $X_3=6.9\%$ (7 bars) and 8.6% (8 bars) from Ref. 10 gives values lower by about 10% and 20% than those used in the text, respectively. Such a change of $m^*$ causes an increase of $\lambda$ by about 20% and 40% in each case, but does not affect our discussion seriously.

[13]E. S. Murdock, K. R. Mountfield, and L. R. Corruccini, J. Low Temp. Phys. **31**, 581 (1978).

[14]A. I. Ahonen, M. A. Paalanen, R. C. Richardson, and Y. Takano, J. Low Temp. Phys. **25**, 733 (1976).

[15]J. Landau, J. T. Tough, N. R. Brubaker, and D. O. Edwards, Phys. Rev. A **2**, 2472 (1970).