Modulation Instability Induced by Cross-Phase Modulation

Govind P. Agrawal
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
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Modulation instability that leads to breakup of intense cw radiation into a train of ultrashort pulses during propagation in optical fibers occurs only in the presence of anomalous group-velocity dispersion. It is shown that a new kind of modulation instability can occur even in the normal-dispersion regime when two copropagating optical fields interact with each other through cross-phase modulation initiated by the nonlinearity. The quantitative aspects of this cross-phase–modulation–induced modulation instability are discussed and illustrated by use of a realistic experimental example.

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Many nonlinear dispersive systems exhibit an instability, known as the modulation instability, that has been discovered in several branches of physics.[1-8] Modulation instability is a general characteristic of wave propagation in nonlinear dispersive media and has been studied in such diverse fields as fluid dynamics,[1] nonlinear optics,[2] and plasma physics.[3] It refers to a process in which weak perturbations from the steady state grow exponentially as a result of an interplay between the nonlinearity and the group-velocity dispersion. In the context of optical fibers, modulation instability requires anomalous dispersion and manifests itself as breakup of the cw or quasi-cw radiation into a train of ultrashort pulses.[4-8] Anomalous dispersion is also necessary for solitons[9,10] which result from a balance between the nonlinearity-induced self-phase modulation (SPM) and the group-velocity dispersion. Although the observation of modulation instability with cw beams is hampered by the competing nonlinear effects (such as stimulated Brillouin scattering), it has recently been observed[7,8] under quasi-cw conditions. These experiments were performed in the infrared region beyond 1.3 μm in order to operate in the anomalous-dispersion regime of the silica fiber.

This Letter shows that a new kind of modulation instability can occur even in the normal-dispersion regime when two or more optical fields copropagate inside the fiber. The physical mechanism behind this novel phenomenon is cross-phase modulation (XPM) which refers to the nonlinear phase change of an optical field induced by other copropagating fields. The XPM-induced modulation instability is of fundamental importance as it suggests the possibility of soliton formation in the normal-dispersion regime. At the same time, it has practical implications for the propagation of visible radiation in optical fibers.

To present the main results as simply as possible, we consider the case of two optical fields copropagating in a single-mode fiber. In the slowly-varying-envelope approximation, the field amplitudes A_1 and A_2 satisfy the nonlinear Schrödinger equation[9,10] modified to account for XPM by the addition of a cross-coupling term:

$$i(\partial A_j/\partial z + v_{gj}^{-1}\,\partial A_j/\partial t + \tfrac{1}{2}\alpha_j A_j) = \tfrac{1}{2}\beta_j\,\partial^2 A_j/\partial t^2 - \gamma_j(|A_j|^2 + 2|A_{3-j}|^2)A_j, \quad (1)$$

where $j=1$ or 2, v_{gj} is the group velocity, α_j is the absorption coefficient, β_j is the dispersion coefficient ($\beta_j = dv_{gj}^{-1}/d\omega < 0$ for anomalous dispersion), and

$$\gamma_j = n_2\omega_j/cA_{\text{eff}} \quad (2)$$

accounts for the fiber nonlinearity responsible for both SPM and XPM. In Eq. (2), A_{eff} is the effective core area and $n_2 = 3.2\times10^{-16}$ cm^2/W for silica fibers.[10] The last term in Eq. (1) is due to XPM and couples the two waves. It is the XPM-induced coupling that gives rise to modulation instability in the normal-dispersion regime where $\beta_j > 0$ for both waves.

To simplify the discussion, we neglect the fiber loss by setting $\alpha_j = 0$. The inclusion of fiber loss does not change the basic conclusions of this paper. The steady-state solution of Eq. (1) is given by

$$\bar{A}_j = \sqrt{P_j}\exp(i\phi_j), \quad j=1,2, \quad (3)$$

where P_j is related to the optical power and the phase

$$\phi_j = \gamma_j(P_j + 2P_{3-j})z. \quad (4)$$

The stability of the steady state is examined by assuming

$$A_j = (\sqrt{P_j} + a_j)\exp(i\phi_j), \quad (5)$$

where a_j is a weak perturbation. Linearizing Eq. (1) in a_1 and a_2, one obtains

$$i(\partial a_j/\partial z + v_{gj}^{-1}\,\partial a_j/\partial t) = \tfrac{1}{2}\beta_j\,\partial^2 a_j/\partial t^2 - \gamma_j P_j(a_j + a_j^*) - 2\gamma_j(P_1P_2)^{1/2}(a_{3-j} + a_{3-j}^*), \quad (6)$$

where $j=1,2$. If we assume a general solution of the form

$$a_j = u_j \cos(Kz - \Omega t) + i v_j \sin(Kz - \Omega t), \tag{7}$$

where K and Ω are the wave number and the frequency of modulation, Eq. (6) provides a set of four homogeneous equations for u_1, u_2, v_1, and v_2. This set has a nontrivial solution only when K and Ω satisfy the dispersion relation

$$[(K-\Omega/v_{g1})^2 - f_1][(K-\Omega/v_{g2})^2 - f_2] = C^2, \tag{8}$$

where

$$f_j = \tfrac{1}{2}\beta_j \Omega^2 (\tfrac{1}{2}\beta_j \Omega^2 + 2\gamma_j P_j), \tag{9}$$

and the coupling parameter C is given by

$$C = 2\Omega^2 (\beta_1 \beta_2 \gamma_1 \gamma_2 P_1 P_2)^{1/2}. \tag{10}$$

If K has an imaginary part for some values of Ω, the steady state is unstable since a_1 and a_2 experience an exponential growth along the fiber length. This phenomenon is referred to as modulation instability[1-8] since it leads to the modulation of the steady-state amplitude.

In the absence of XPM, $C=0$, and Eq. (8) has the solution $K=\Omega/v_{gj} \pm \sqrt{f_j}$ for $j=1,2$. This is the dispersion relation obtained previously[4-8] for the case of a single field. In the normal-dispersion regime, $\beta_j > 0$, and as a result $f_j > 0$ from Eq. (9). It then follows that K is always real, and modulation instability does not occur in the normal-dispersion regime.

The situation is entirely different when XPM couples the two optical fields. Equation (8) shows that K becomes complex for specific Ω values even when both β_1 and β_2 are positive. This is seen most readily if we neglect the group-velocity mismatch and assume $v_{g1} = v_{g2}$ for the time being. It follows from Eq. (8) that

$$(K-\Omega/v_{g1})^2 = \tfrac{1}{2}\{(f_1+f_2) \pm [(f_1+f_2)^2 + 4(C^2 - f_1 f_2)]^{1/2}\}. \tag{11}$$

Clearly K becomes complex if the condition $C^2 > f_1 f_2$ is satisfied. With use of Eqs. (9) and (10), this condition becomes

$$\Omega^2 < \Omega_c^2 = \tfrac{1}{2}\{[(b_1+b_2)^2 + 12 b_1 b_2]^{1/2} - (b_1+b_2)\}, \tag{12}$$

where

$$b_j = (4\gamma_j/\beta_j) P_j. \tag{13}$$

Thus, for frequencies such that $|\Omega| < \Omega_c$, a weak modulation of the steady state experiences the gain given by

$$g(\Omega) = 2\,\mathrm{Im}(K), \tag{14}$$

where Im stands for the imaginary part.

Figure 1 shows the gain spectra for several values of the power ratio P_2/P_1 after taking $P_1 = 100$ W. [Since the gain spectrum is symmetric with $g(-\Omega) = g(\Omega)$, only the positive-frequency part is shown.] The fiber parameters $\beta_j = 0.06$ ps^2/m and $\gamma_j = 0.015$ W^{-1}/m correspond to a realistic situation of propagation in silica fibers in the visible region near 0.53 μm. The wavelengths of the two fields are assumed to differ only slightly so that β_j, γ_j, and v_{gj} are nearly the same for $j=1$ and 2. For this case, g can be evaluated in a closed form and is given by

$$g(\Omega) = \beta_1 |\Omega| (\Omega_c^2 - \Omega^2)^{1/2}, \tag{15}$$

where Ω_c is given by Eq. (12). The maximum gain occurs at $\Omega = \Omega_c/\sqrt{2}$ and has a value of $\beta_1 \Omega_c^2/2$. Both the frequency range $|\Omega| < \Omega_c$ and the maximum gain increase with an increase in the power levels P_1 and P_2. Since the gain is due to XPM, it vanishes when either P_1 or P_2 is zero; i.e., both optical fields have to be present for modulation instability to occur.

From a practical point of view, it is necessary to account for the group-velocity mismatch represented by the parameter

$$\delta = |v_{g1}^{-1} - v_{g2}^{-1}|. \tag{16}$$

Figure 2 shows the gain spectra for several values of δ obtained by use of Eqs. (8) and (14) for the case of equal powers ($P_1 = P_2 = 100$ W). Other parameters are identical to those used for Fig. 1. As δ increases, the gain spectrum narrows, shifts to higher frequencies, and

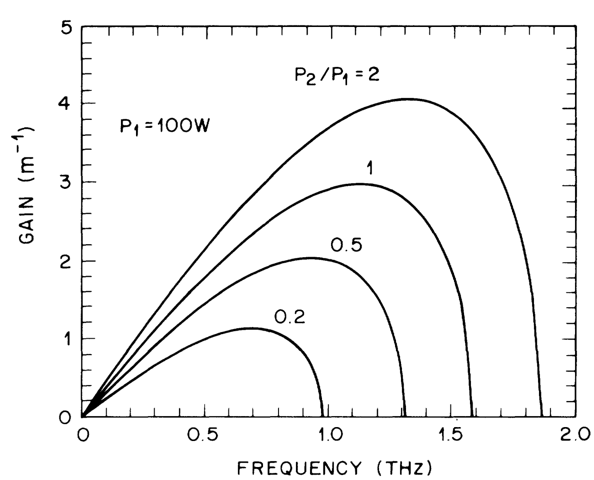

FIG. 1. Gain spectra $g(\Omega)$ for several power levels P_1 and P_2 of the two beams. Only half of the spectrum is shown since $g(-\Omega) = g(\Omega)$. The fiber parameters are $\beta_j = 0.06$ ps^2/m and $\gamma_j = 0.015$ W^{-1}/m for $j=1,2$. The group-velocity mismatch is neglected.

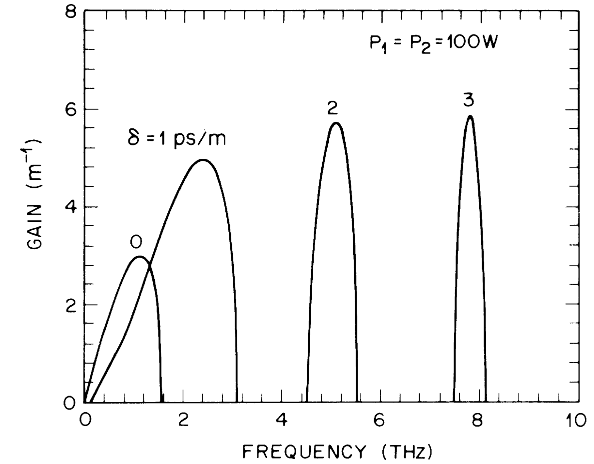

FIG. 2. Effect of group-velocity mismatch δ on the gain spectra for the case of $P_1 = P_2 = 100$ W. Other parameters are identical to those used for Fig. 1.

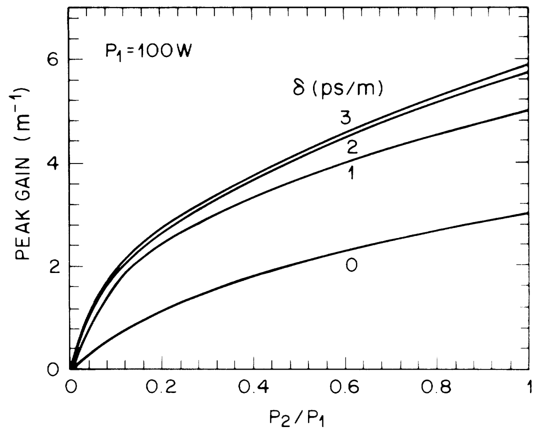

FIG. 3. Variation of peak gain with the power ratio P_2/P_1 for several values of the group-velocity mismatch δ. For $\delta > 2$ ps/m, the peak gain becomes nearly independent of δ.

its peak height approaches a limiting value of about 6 m^{-1}. The main point to note is that the group-velocity mismatch is not detrimental to the existence of modulation instability. For $\delta > 2$ ps/m and $\beta_1 \simeq \beta_2$, the frequency corresponding to the peak gain is well approximated by

$$\nu_m \simeq \delta / 2\pi\beta_1. \qquad (17)$$

Furthermore, the peak gain is not sensitive to the amount of group-velocity mismatch for $\delta > 2$ ps/m. This is shown in Fig. 3 where the peak gain is plotted as a function of the power ratio P_2/P_1 for several values of δ and $P_1 = 100$ W.

The results shown in Figs. 1–3 suggest that XPM-induced modulation instability should be observable under realistic experimental conditions. Consider, for example, the case of two beams in the visible region around 0.53 μm with their wavelengths a few nanometers apart. The parameter $\delta = 2$–5 ps/m for a wavelength difference in the range 5–10 nm. The peak gain of about 6 m^{-1} can be expected for nearly equal powers of 100 W (see Fig. 3). As a result of modulation instability, the optical spectra of both beams should develop modulation sidebands (on both sides) with a frequency separation given by Eq. (17). For $\delta = 2$ ps/m and $\beta = 0.06$ ps^2/m, the frequency separation is about $\nu_m = 5$ THz ($\simeq 5$ nm at 0.53 μm). The modulation sidebands occur as a result of the amplification of the noise input provided by spontaneous emission or vacuum fluctuations. With a peak gain of 6 m^{-1}, the amplification factor is $\exp(6L)$ for a fiber L meters long; significant buildup of the sidebands is thus expected even for a few-meter-long fiber. In the time domain, both beams develop amplitude modulation with a period ν_m^{-1} in the femtosecond range ($\simeq 200$ fs for the example considered above). Thus, modulation instability manifests itself as a breakup of each cw beam into a train of ultrashort pulses with a repetition rate of a few terahertz. Of course, similar to the case of conventional modulation instability,[7,8] experimental observation of XPM-induced modulation instability would require the use of optical pulses to avoid stimulated Brillouin scattering. The overlap of both pulses over the fiber length is ensured if $\tau \gg \delta L$.

So far we have assumed that both beams (or pulses) are incident simultaneously on the optical fiber. An interesting possibility is that the second beam is internally generated through stimulated Raman scattering. For silica fibers, the Stokes shift is about 13 THz (12 nm at $\lambda = 0.53$ μm). The group-velocity mismatch between the pump and the Stokes waves is $\delta \simeq 5$ ps/m. Although the analysis presented here is not directly applicable (it does not include the effect of Raman gain), it is possible that the qualitative features of XPM-induced modulation instability are nonetheless present. Interestingly enough, the modulation frequency estimated from Eq. (17) is also about $\nu_m \simeq 13$ THz for a 0.53-μm pump. Thus, modulation sidebands would appear at multiples of the Stokes shift and may be difficult to identify in the presence of higher-order Stokes lines.

Another possibility that should be considered consists of XPM interaction between the two dominant spectral components of the SPM-broadened spectrum of a single pulse. This is similar to the phenomenon of optical wave breaking,[11] where mixing of two such spectral components results in high-frequency oscillations in the wings of the pulse. At the same time two sidebands appear in the pulse spectrum, suggesting a connection to the analysis presented here. In fact, the experimental observation[11,12] of such sidebands in the normal-dispersion regime can be interpreted in terms of the XPM-induced modulation instability.

In conclusion, XPM that invariably accompanies SPM in the presence of two or more optical fields is shown to be responsible for a novel instability. In contrast to the case of conventional modulation instability, this instability can occur even in the normal-dispersion regime of optical fibers and should be easily observable in the wavelength range of 0.5–1.3 μm. Since the physics of modulation instability and solitons is interrelated, the possibility exists that XPM can sustain solitons in the normal-dispersion regime. An application of the theory may be in the field of squeezing, as evident from the phase-sensitive nature of XPM in Eq. (6). Finally, it should be noted that even though the results are presented in the context of optical fibers, the analysis should be applicable to other nonlinear dispersive systems as well and may find application in different branches of physics.

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