Pole Structure of the $J^{\pi}=\frac{3}{2}^{+}$ Resonance in ^{5}He

G. M. Hale, Ronald E. Brown, and Nelson Jarmie
Los Alamos National Laboratory, Los Alamos, New Mexico 87545
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The S-matrix pole structure of the $J^{\pi}=\frac{3}{2}^{+}$ resonance in ^{5}He is obtained from R-matrix parameters for the system. Two poles are found on different unphysical Riemann sheets. One is a conventional resonance that is primarily responsible for structure in the n-α total cross section. The other is a "shadow" pole that contributes to the large values of the cross section for the reaction ^{3}H$(d,n)^{4}$He at low energies. It is the first experimental evidence for the existence of shadow poles in nuclear and particle physics. The two-pole structure of the resonance accounts for different peak positions and widths in the amplitudes for the n-α total cross section and for the cross section for the reaction ^{3}H(d,n).

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The $J^{\pi}=\frac{3}{2}^{+}$ resonance in ^{5}He located at $E=60$ keV (center-of-mass energy) above the d-t threshold is one of the most famous in nuclear physics. It is responsible for the pronounced bump that makes the low-energy cross section for the reaction ^{3}H$(d,n)^{4}$He larger than that of any other thermonuclear process, and it causes a similar feature in the n-α total cross section near 22-MeV incident neutron energy. The most recent resonance parameter compilation[1] for light systems assigns it a total width $\Gamma=100$ keV, with partial widths $\Gamma_n=\Gamma_d=50$ keV. These parameters are presumed to correspond with the position and residue of a pole in the S matrix that gives asymptotically the relative amplitudes of outgoing to incoming waves for the reactions in the ^{5}He system.

We report here the S-matrix pole structure for the $J^{\pi}=\frac{3}{2}^{+}$ resonant state resulting from R-matrix parametrizations of the ^{5}He reactions. This structure has been properly derived from the complex poles of the outgoing-wave Green's function, rather than the from the real poles of the principal-value Green's function (i.e., Heitler's K matrix) that are often used. We find two poles located on different unphysical sheets of the two-channel Riemann energy surface, both of which contribute to the experimental anomalies associated with the $J^{\pi}=\frac{3}{2}^{+}$ resonance.

One of these poles is clearly responsible for much of the resonant behavior of the cross sections. Although its position and residue give resonance parameters that differ somewhat from the expected values, it is the type of pole described in the conventional lore surrounding low-energy resonances. The other pole has properties uncharacteristic of a conventional resonance, although its effect on the reaction cross section, especially, is quite evident. It appears to be what Eden and Taylor[2] termed a "shadow" pole in their paper concerning elementary-particle resonances, and is the first experimental evidence of such phenomena.

We will first describe a correct procedure for obtaining poles and residues of the S matrix from R-matrix parameters and relating them to resonance parameters. Then we will give $\frac{3}{2}^{+}$ resonance parameters for the S-matrix poles found in two different R-matrix descriptions of ^{5}He reactions (one simplified and one complete) and compare them with previous values. The paper concludes with a discussion of the interpretation of the pole properties and the evidence for them in the measurements.

To obtain S-matrix pole positions from R-matrix eigenenergies E_λ and reduced-width amplitudes $\gamma_{c\lambda}$ for boundary conditions B_c, one finds an energy $E=E_0$ such that at least one eigenvalue of the complex "energy-level" matrix,

$$\mathcal{E}_{\lambda'\lambda}=E_\lambda\delta_{\lambda'\lambda}-\sum_c \gamma_{c\lambda'}[L_c(E)-B_c]\gamma_{c\lambda}, \tag{1}$$

is the same as E_0. In that case

$$E_0=E_r-i\Gamma/2 \tag{2}$$

is a pole of S, with E_r the "resonance energy" and Γ the "total width" of the resonance. In Eq. (1), the outgoing-wave logarithmic derivatives at channel radii $r_c=a_c$,

$$L_c(E)=r_c(\partial\mathcal{O}_c/\partial r_c)\mathcal{O}_c^{-1}|_{r_c=a_c}, \tag{3}$$

depend on the channel momenta k_c through the outgoing spherical (Coulomb) waves $\mathcal{O}_c$, thereby placing E_0 on some definite Reimann sheet of the energy surface.

The residue of S at the pole is a rank-one matrix, $i\rho_0\rho_0^T$, in which the vector ρ_0 has channel elements

$$\rho_{0c}=(2k_{0c}a_c/N)^{1/2}\mathcal{O}_c^{-1}(k_{0c}a_c)\sum_\lambda \gamma_{c\lambda}(\lambda\,|\,\mu_0) \tag{4}$$

that depend on the components $(\lambda\,|\,\mu_0)$ of the eigenvector $|\mu_0)$ belonging to the eigenvalue E_0. The complex quantity

$$N=\sum_{\lambda'\lambda}(\lambda\,|\,\mu_0)(\lambda'\,|\,\mu_0)\left[\delta_{\lambda'\lambda}+\sum_c \gamma_{c\lambda'}\frac{\partial L_c}{\partial E}\bigg|_{E=E_0}\gamma_{c\lambda}\right] \tag{5}$$

in the denominator of Eq. (4) corresponds to the normalization of the complex-energy state $|\mu_0)$ over all space. The first term, $\sum_\lambda(\lambda\,|\,\mu_0)^2$, is the normalization inside

TABLE I. R-matrix parameters for the $J^{\pi}=\frac{3}{2}^{+}$ states of ^{5}He. Channel labels (c) are in spectroscopic notation, with (d) meaning the d-t arrangement and (n) the n-α arrangement. Eigenenergies E_λ are center-of-mass values in megaelectronvolts relative to the d-t threshold. Entries in the body of the table are values of reduced-width amplitudes $\gamma_{c\lambda}$, also center-of-mass, in units MeV$^{1/2}$.

$c(J=\frac{3}{2})$	a_c (fm)	B_c	λ / E_λ	1 / 0.0837559	2 / 6.4713043	3 / 13.7357067	4 / 47.475246
$^4S(d)$	5.1	−0.37		1.1760678	0.0693397	−0.4955438	1.1052421
$^4D(d)$	5.1	−2.00		0.1688724	−0.2729805	1.9910681	1.9847048
$^2D(d)$	5.1	−2.00		−0.0484797	0.8862475	0.0958513	0.2422464
$^2D(n)$	3.0	−0.59		0.3768218	−0.1562737	0.9994494	−3.8556539

the nuclear surface on which the R matrix is specified, and the second term corresponds to a bounded normalization outside the nuclear surface in the channel region. This second term can be obtained by analytical continuation of the bound-state relation from the physical sheet to the unphysical sheet on which E_0 lies.[3] With this normalization, the definition of partial widths as

$$\Gamma_c = |\rho_{0c}|^2 \tag{6}$$

is consistent[4] with the definition in terms of the transition probability rate that one usually finds in time-dependent treatments. A consequence of this definition, however, is that the partial widths do not, in general, sum to the total width $\Gamma = -2\mathrm{Im}E_0$.

The complete R-matrix analysis of reactions in the ^{5}He system[5] at excitation energies below 21.5 MeV required four channels and four levels for $J^{\pi}=\frac{3}{2}^{+}$. The parameters of this four-level description are given in Table I. The first level is mainly associated with the low-energy resonance under consideration here, the next two with higher-energy ^2D and ^4D resonances in the d-t channel, and the fourth serves primarily as a background term, especially in the n-α channel.

A two-level fit to low-energy cross-section data in the reaction ^{3}H$(d,n)^4$He alone was reported in conjunction with the latest measurements.[6] For completeness, the parameters of that two-level, two-channel (D waves in the d-t channel are neglected) fit are repeated in Table II. The first level is again the resonance level, and the second level serves primarily as a background.

TABLE II. R-matrix parameters from the two-level fit (see Ref. 6), with labeling and units as in Table I. The eigenenergy E_2 was held fixed at 10 MeV.

$c(J=\frac{3}{2})$	a_c (fm)	B_c	λ / E_λ	1 / 0.021626	2 / 10.000
$^4S(d)$	5.0	−0.27864		0.95838	0.48304
$^2D(n)$	3.0	−0.5570		0.27781	1.51753

Since the cuts due to channel thresholds are automatically built into the theory, the relatively simple analytic structure of the R matrix (meromorphic, with poles only on the real energy axis) leads to a more complicated structure of the multisheeted S matrix. Applying the procedure and definitions contained in Eqs. (1)–(3) to the R-matrix parameters of Tables I and II, we searched for poles of S in the vicinity of the $d+t$ threshold on all three unphysical sheets of the two-(arrangement) channel Riemann energy surface. Following Eden and Taylor,[2] we use the notation $U_{(i)}$ and $U_{(i,j)}$ for i or $j=n,d$ to denote the unphysical sheet on which k_i and k_j change signs when one moves from the physical sheet P at the same energy. Poles were found on sheets $U_{(n)}$ and $U_{(n,d)}$, which correspond to sheets II and III in the convention of Oehme.[7] Partial widths were determined from the residues of the poles with use of Eqs. (4)–(6). Values of the Coulomb function $\mathcal{O}_c$ at complex energies were obtained from the elegant numerical subroutine of Thompson and Barnett.[8]

The results for both sets of R-matrix parameters are given in Tables III and IV. In addition to the resonance energy, total width, and partial widths, we have also tabulated a quantity called

$$\text{"strength"} = \rho_0^\dagger \rho_0/\Gamma = \textstyle\sum_c \Gamma_c/\Gamma, \tag{7}$$

which is a measure of the magnitude of the residue compared to the displacement of the pole from the real axis, and also of the consistency of the partial widths with the

TABLE III. $J^{\pi}=\frac{3}{2}^{+}$ resonance parameters from S-matrix poles of the four-level R matrix of Table I. The center-of-mass resonance energy E_r and widths Γ are in kiloelectronvolts. Only the S-wave partial width Γ_d is given, since the D-wave partial widths are <0.05 keV. As explained in the text, the partial widths sum to the product of the strength times Γ.

Sheet	E_r	Γ	Γ_d	Γ_n	Strength
$U_{(n,d)}$	46.97	74.20	25.10	39.83	0.875
$U_{(n)}$	81.57	7.28	2861.6	68.77	402.5

TABLE IV. $J^\pi = \frac{3}{2}^+$ resonance parameters from S-matrix poles of the two-level R matrix of Table II. The center-of-mass resonance energy E_r and widths Γ are in kiloelectronvolts. As explained in the text, the partial widths sum to product of the strength times Γ.

Sheet	E_r	Γ	Γ_d	Γ_n	Strength
$U_{(n,d)}$	48.10	74.16	24.51	43.57	0.918
$U_{(n)}$	78.94	16.52	2542.5	65.95	157.9

total. One expects the strength to be about 1 for isolated, narrow resonances that lie closest to the physical sheet. In Ref. 6, following the conventions of Humblet,[9] we used a slightly different definition of partial widths, and normalized them to sum to the total width. With the more meaningful normalization convention used here, that artificially imposed property has been discarded.

The R-matrix parameters of Tables I and II give very similar pole structures, with a "conventional" resonance (strength$\simeq 1$) on the $U_{(n,d)}$ sheet and an "unconventional" one (strength$\gg 1$) on the $U_{(n)}$ sheet. In both the two-level and four-level cases, the first level is primarily ($>99\%$) responsible for the two S-matrix poles found on different unphysical sheets. Since the results are so similar, we shall refer only to those from the four-level case in the subsequent discussion.

The conventional pole on $U_{(n,d)}$ (the unphysical sheet closest to P above the d-t threshold) is mainly responsible for the peak in the n-α total cross section, as is seen from the calculated curve labeled $(1-\mathrm{Re}S_{nn}) \sim \frac{1}{2}\sigma_T$ in Fig. 1. The position and width associated with this pole, $E_r = 47.0$ keV and $\Gamma = 74.2$ keV, are somewhat smaller than (but well within the errors of) the recommended[1] values of $E_r = 60 \pm 130$ keV and $\Gamma = 100 \pm 50$ keV. Converting our value of E_r to the corresponding laboratory neutron energy, 22.124 MeV, we find good agreement with the values $E_r = 22.133 \pm 0.010$ MeV and $\Gamma = 76 \pm 12$ keV that Haesner *et al.*[10] extract from a Breit-Wigner fit to their n-α total-cross-section measurements. Our n-α partial width, $\Gamma_n = 39.8$ keV, also agrees well with their value of 37 ± 5 keV. However, since our d-t partial width, $\Gamma_d = 25.1$ keV, is not constrained to add to the total width, we find a different ordering of partial widths ($\Gamma_n > \Gamma_d$) from that implied by their Breit-Wigner parameters if Γ_d is taken to be $\Gamma - \Gamma_n = 39$ keV.

The properties of the pole on the $U_{(n)}$ sheet are more intriguing. Despite its apparently small width, $\Gamma = 7.3$ keV, the pole is on a sheet far enough removed from P that it causes no narrow resonance phenomena, as is reflected in the sizes of the partial widths. The significance of the small Γ is rather the following: Associated with a pole on $U_{(n)}$ is a *zero* of the diagonal S-matrix element S_{nn} located on P at the same energy, as

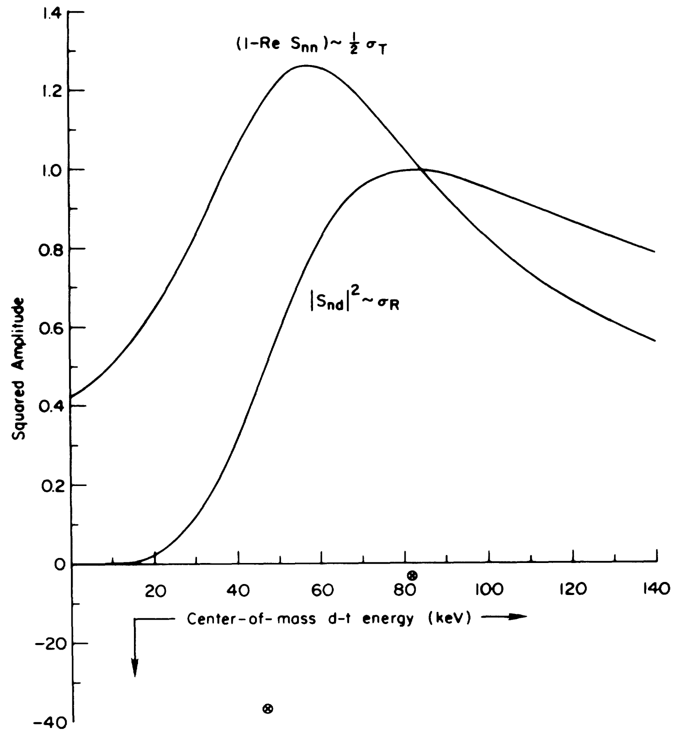

FIG. 1. $J^\pi = \frac{3}{2}^+$ S-matrix amplitudes calculated as functions of center-of-mass d-t channel energy from the R-matrix parameters of Table I. The curve labeled $(1-\mathrm{Re}S_{nn})$ is proportional to one-half the n-α total cross section σ_T, and the one labeled $|S_{nd}|^2$ is proportional to the σ_R, the cross section for the reaction $^3H(d,n)^4He$. The pole positions are shown as direct product symbols in the lower half of the complex energy plane at the bottom of the figure, the lower symbol (the conventional resonance) being on the $U_{(n,d)}$ sheet, and the upper symbol (the shadow pole) on the $U_{(n)}$ sheet.

was pointed out by Kato[11] for the case of two channels, and can be seen in the more general analytic continuation relations of Eden and Taylor.[2] Because Γ is small, the zero of S_{nn} in this case lies close to the real axis of the physical sheet P. Through unitary constraints, the zero forces the off-diagonal S-matrix element S_{nd} to approach its maximum value of unity when E reaches $E_r \approx 82$ keV, as can be seen in Fig. 1 from the calculated curve labeled $|S_{nd}|^2 \sim \sigma_R$ [the cross section for the reaction $^3H(d,n)$]. Figure 1 also shows that the second pole induces effects that are clearly visible in the displacement and broadening of the peak of the cross section for the reaction $^3H(d,n)$ relative to that of the n-α total cross section, explaining why the widths are not the same when the resonance is observed in the n-α channel and in the reaction $^3H(d,n)^4He$.

Even more interesting from a theoretical standpoint is that the pole on the $U_{(n)}$ sheet appears to be the first experimental evidence for what Eden and Taylor[2] termed a shadow pole. The presence of a shadow pole on $U_{(n)}$ alone (and not on $U_{(d)}$) implies that only a pole in S_{nn} would persist in the absence of coupling between the n-α and d-t channels. Thus, the pole structure we have found suggests that the $J^\pi = \frac{3}{2}^+$ resonance in ^{5}He is fundamentally an n-α resonance on the $U_{(n,d)}$ sheet. In the presence of coupling between the d-t and n-α channels, a second pole (the shadow pole) develops close to the real axis on the $U_{(n)}$ sheet that, through its associated zero on the physical sheet, contributes to the large reaction cross section that has made this resonance so important in fusion energy applications.

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