Quantum Transport in an Electron-Wave Guide

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We have fabricated high-mobility, one-dimensional wires in GaAs-AlGaAs heterostructures and measured the resistance as a function of magnetic field and temperature. Because of the size of the devices and the high mobility, only a few channels carry the current at 35 mK with minimal scattering. Fluctuations in the resistance as a function of magnetic field due to quantum interference are observed for $0 < \omega_c \tau < 300$, where ω_c is the cyclotron frequency and τ is the scattering time, superimposed upon Shubnikov–de Haas oscillations and the Hall effect.

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It is now technologically feasible to fabricate devices in GaAs-AlGaAs heterostructures which are comparable in width to the electron wavelength, and yet possess high mobility.[1-3] Such devices provide a unique opportunity to investigate the role of quantum interference (QI) and the quantized Hall effect in transport, because phase coherence is maintained on the length scale of a device, and because the role of current-carrying edge states is exaggerated. The following describes our measurements of the temperature dependence of the magnetoresistance of wires fabricated in modulation-doped GaAs-AlGaAs which vary in length from 700 nm to 4.7 μm, and are estimated to be 220–75 nm wide. The carrier density determined from the Hall effect is $n \approx (2.0\text{–}2.6)\times 10^{11}$ cm^{-2} corresponding to a Fermi wavelength of $\lambda_F = (2\pi/n)^{1/2} \approx 50\text{–}56$ nm, while the mobility μ at 4 K is larger than 3.3×10^5 cm^2/V·s, so that the elastic mean free path $L_e = h\mu/e\lambda_F$ is greater than 1.6 μm. Thus, the width of the sample is comparable to an electron wavelength, while the distance between voltage probes is less than L_e. Because of quantization in the transverse direction, the current is carried by only a few (<8) channels (or transverse subbands) at the Fermi energy with minimal scattering. Consequently, we have measured the electrical resistance associated with a few channels in an electron-wave guide.

Fluctuations in the resistance are observed as a function of H for $0 \le \omega_c \tau \le 300$, where ω_c is the cyclotron frequency and τ is the scattering time, concomitantly with Shubnikov–de-Haas oscillations and the quantum Hall effect. The fluctuations are correlated and the amplitude is so large at 35 mK that negative dynamic resistance is observed. The fluctuations change to a lower frequency of oscillation and smaller amplitude in the extreme magnetic quantum limit where only the $N=0$ Landau level is filled. We tentatively propose that the fluctuations in the resistance of the wire are due to the Aharonov-Bohm effect, and that the change in the typical frequency is indicative of change with magnetic field in the width of the distribution of electron trajectories across the waveguide. Recent experiments have demonstrated QI in magnetotransport of disordered metals,[4] and of silicon inversion layers,[5] but have been exclusively concerned with the quasi one-dimensional (Q1D) regime $L_e \ll L$, $W < L_\phi$, where L is the sample length, W is the width, and L_ϕ is the phase coherence length, for magnetic fields such that $\omega_c \tau \le 1$. As a result, $\Delta g/g \ll 1$, where Δg represents the rms conductance fluctuations associated with interference and g is the conductance of the device. Here we present results of measurements in which $\Delta g/g \approx 1$ in the regime $L < L_e \approx L_\phi$ for magnetic fields in the range $0 \le \omega_c \tau \le 300$.

The devices were fabricated[1,2] by electron-beam lithography and low-voltage (75–150 V) reactive ion etching on modulation-doped GaAs-$Al_{0.33}Ga_{0.67}As$ heterostructures prepared by molecular-beam epitaxy. Electron-beam lithography was used to pattern an etch mask which protects the underlying AlGaAs from a subsequent partial etch of the AlGaAs layer in a Freon-helium-oxygen gas mixture. The anisotropic partial etch, which removes the doped layer, stops in the AlGaAs spacer layer and so laterally confines the electron gas to the region immediately beneath the etch mask. The remaining doped AlGaAs is partially depleted[1,2] by pinning of the Fermi level in the band gap at the AlGaAs surface and by the presence of traps in the layer. The inset to Fig. 1(a) shows a typical device with a linewidth of 500 nm which widens to 1.0 μm for the interconnection to the outlying lead frame.

We measured the four-terminal resistances of three devices with the geometry shown in Fig. 1(a) using an ac resistance bridge operating at 16 Hz. The resistance was linear in the excitation voltage up to 50 μV, but under typical operating conditions the excitation across any segment was only about 1 μV. The resistance scaled within 3% to the length at $T = 4$ K, and the magnetoresistance was reproducible within 0.1%. We found that wires which are lithographically $<400\text{–}450$ nm wide do not conduct at low temperature ($T < 4$ K). Consequently, we anticipate that wires which are lithographically 500 nm wide have a homogeneous, conducting width $W \approx 100$ nm. Furthermore, we have observed

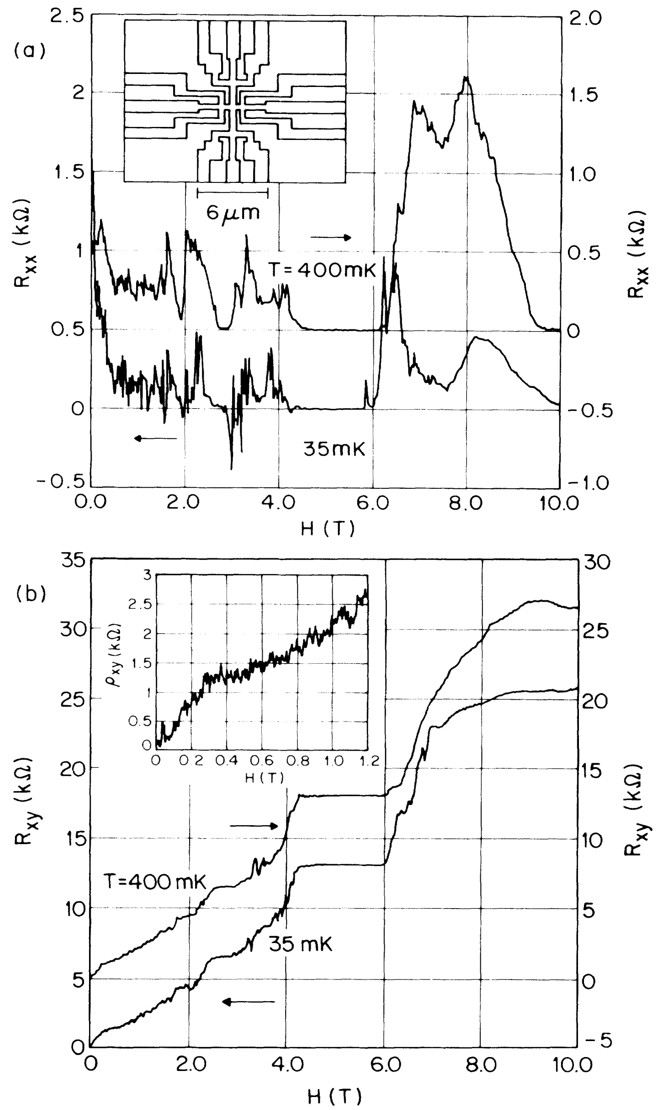

FIG. 1. The magnetoresistance of the 900-nm segment as a function of temperature. (a),(b) R_{xx} and R_{xy}, respectively. Inset in (a): The cross pattern of a typical device. The center-to-center distances between the branches are clockwise from the top: 1.70 μm, 700 nm, 2.00 μm, and 900 nm. Inset in (b): The low-field Hall effect.

the hc/e Aharonov-Bohm effect for a 1-μm-diam annulus[4] fabricated by the same techniques in the same material, and found the width of the 5.8-mT fundamental in the Fourier spectra of the magnetoresistance to be approximately 50 T^{-1} indicating that $W \approx 100$ nm.

The magnetoresistances, R_{xx} and R_{xy}, found in a 900-nm-long segment are shown in Figs. 1(a) and (b), respectively. The zero-resistance states observed in R_{xx} and the Hall resistance plateaus in R_{xy} correspond to the Landau indices $N=0$, 1, and 2 (i.e., filling factors of $i=1$, 2, 3, 4, and 6). There is an abrupt change in the slope of the Hall effect near 0.3 T [see Fig. 1(b), inset]. From the low-field ($H<0.4$ T) Hall effect we estimate the carrier density in the wire to be $(2.0\pm0.2)\times10^{11}$ cm^{-2}, in contrast with a density of $(3.9\pm0.1)\times10^{11}$ cm^{-2} measured in the 2D lead frame and in a 100×650 $\mu\mathrm{m}^2$ Hall bridge fabricated from the same wafer. The Hall plateau resistances ($i=1$ and 2) are quantized in units of $h/e^2 i$ within 1%, but the positions of the centers of the Hall plateaus as well as the zero-resistance states in R_{xx} are shifted to higher fields than anticipated for a low-field density of $n=2.0\times10^{11}$ cm^{-2}. The positions of the $i=1$, 2, and 4 plateaus and the slope of R_{xy} above $H=0.5$ T correspond to a density of $(2.6\pm0.2)\times10^{11}$ cm^{-2}.

The change in the density in the wire from the 2D value may be due to the partial depletion of the doped layer from the AlGaAs sidewalls. Corresponding to the change in density, there is a shift in the lowest subband energy E_0 associated with confinement perpendicular to the interface. In the extreme magnetic quantum limit, $n=2.6\times10^{11}$ cm^{-2} and $E_F-E_{0'}=\pi\hbar^2 n/m^*=9.2$ meV, whereas $E_F-E_0=14$ meV in the 2D lead frame. If our interpretation of the low-field Hall effect is correct, the inferred increase in carrier density in the wire with increasing magnetic field may be indicative of confinement. According to Kaplan and Warren,[6] when a magnetic field is applied perpendicular to the interface, the magnetic potential, $\frac{1}{2}m^*\omega_c^2(x-X)^2$ where $X=k_y l_c^2$, k_y is a wave vector quantized in units of $2\pi/L$, $l_c=(\hbar c/eH)^{1/2}$, further confines an electron and the density increases. We estimated the width using the carrier density determined from the low-field Hall effect in conjunction with an infinite square-well model for the confining potential.[3] For a square well of width W along x with $H=0$,

$$(\hbar^2/2m^*)[(m\pi/W)^2+k_y^2]=E_F-E_{0'}$$

with m an integer, and

$$n_{2D}=\frac{n_{1D}}{W}=\frac{2}{W\pi}\sum_m k_m=2.0\times10^{11}\ \mathrm{cm}^{-2},$$

where

$$k_m=\left\{\frac{2m^*}{\hbar^2}\left[E_F-E_{0'}-\frac{\hbar^2}{2m^*}\left(m\frac{\pi}{W}\right)^2\right]\right\}^{1/2},$$

so that $W=75$ nm and $m=3$, i.e., there are three conducting channels and so only $N\geq2$ will be observed in the magnetoresistance.

We have not determined the width unequivocally, however. If the data are digitally filtered to enhance low-frequency components, we find oscillations in the resistance which may be associated with Landau-level indices up to $N=7$ (with an amplitude $>7\%$ of the background resistance). If the square well is adjusted to accommodate eight levels the width would be 220 nm. Although the width where the wires cross must be wider than 75 nm and may be as wide as 220 nm, we cannot unambiguously associate the observation of Landau levels $N>2$ with these regions. For a width of 220 nm, the mobility of the wire is 3.3×10^5 $cm^2/V\cdot s$ and $L_e=1.6$ μm which is comparable to $(3.0\pm0.2)\times10^5$ $cm^2/V\cdot s$, the mobility observed in a 2D Hall bridge. There is no degradation in mobility presumably because variations in the width of the conducting channel are small compared to λ_F.[7]

Fluctuations in the magnetoresistance indicative of QI are observed in both R_{xx} and R_{xy} over the entire field range, except near the extreme magnetic quantum limit (i.e., $i=1$), with a reduced amplitude at the Hall-resistance plateaus and at the corresponding zero-resistance states for R_{xx}. Figure 2(a) shows an example of the low-field fluctuations at 35 mK, 400 mK, and 4.2 K found in the 700-nm long segment. The rms amplitude at 35 mK is so large that the dynamic resistance becomes negative. In the neighborhood of 0.5 T, the typical spacing of the fluctuations decreases from 180 to 10 mT when the temperature is lowered from 400 to 35 mK. Figure 2(b) contrasts the fluctuations observed in the resistance near 0.5 and 7.0 T for the 900-nm segment at 35 mK. The typical frequency decreases with increasing magnetic field. At 7.0 T the typical frequency is approximately 4 times lower than at 0.5 T.

Sample-specific, aperiodic fluctuations arise in the magnetoresistance of very small, disordered, Q1D conductors for $\omega_c\tau<1$ because of the Aharonov-Bohm effect and the lack of self-averaging.[8] Interference gives rise to oscillatory terms in the conductance which depend on the relative phases of transmission amplitudes associated with different trajectories through the wire. An applied magnetic field penetrates the wire and changes the relative phase by an amount $2\pi\Phi_{pp'}/\Phi_0$, where Φ_i is the flux through an area enclosed by trajectories p and p', and $\Phi_0=hc/e$. Thus, the conductance oscillations are periodic in Φ_0 and depend only on the change in magnetic field, not on the absolute field. For a Q1D wire the large variety of trajectories enclose different amounts of flux, and consequently, variation of the magnetic field causes phase-dependent contributions to the resistance to vary incoherently. Generally, variation of the magnetic field does not give rise to a single periodicity, but the phase-dependent contributions do not self-average either. Their variation gives aperiodic magnetoresistance fluctuations[4,5] which are characterized by a magnetic field correlation range $H_c=2.4\Phi_0/LW$ beyond which the correlation function vanishes.[5] Furthermore, the rms amplitude is predicted to be universal[9] (for $\omega_c\tau\leq1$) with a value of $0.729e^2/h$ (for 1D) independent of the scale of the device.

In our devices, we observe that both the amplitude and the typical spacing of fluctuations in the resistance are functions of field in the regime $\omega_c\tau>1$. In the inset in Fig. 2(b) the autocorrelation functions corresponding to the magnetoresistance near 0.5 T (solid line) and 7.0 T

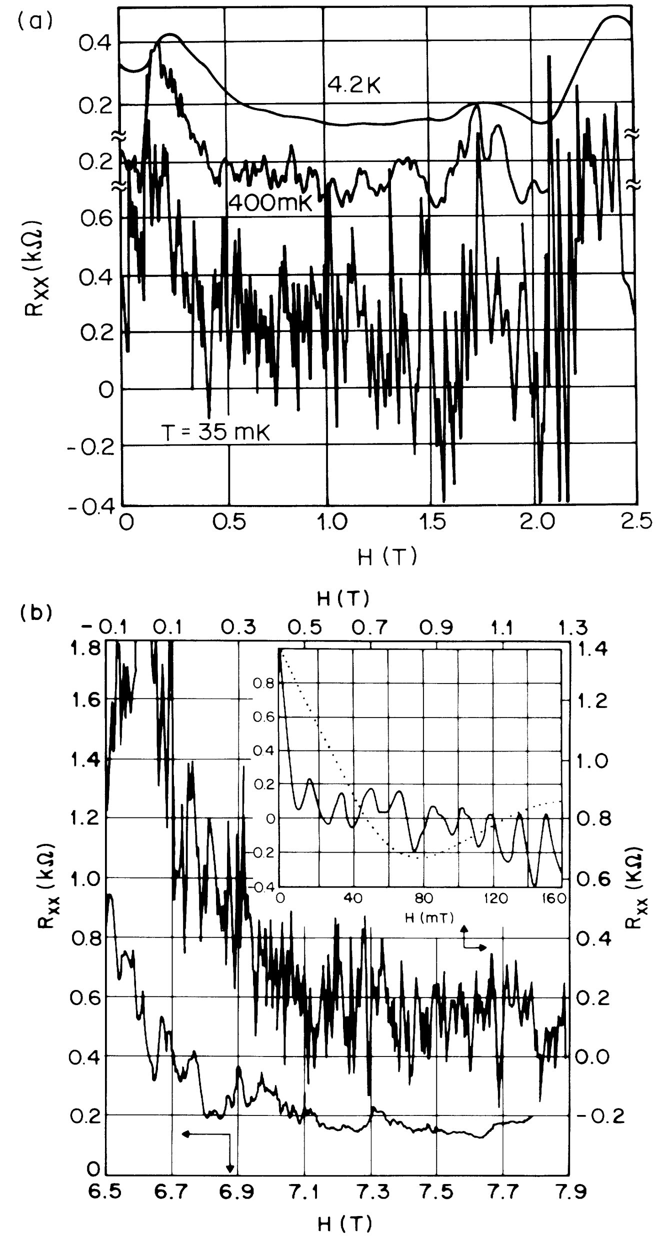

FIG. 2. (a) The magnetoresistance of the 700-nm segment at 35 mK, 400 mK, and 4.2 K. (b) The magnetoresistance of the 900-nm segment at 35 mK in the vicinity of both 0.5 and 7.0 T. Inset: The autocorrelation function associated with the trace taken near 0.5 T (solid line) and 7.0 T (dotted line).

(dotted line) are shown. The autocorrelation coefficient near 0.5 T monotonically decays to zero over 10 mT, and oscillates with a 16-mT frequency thereafter, while near 7.0 T the fluctuations have a typical spacing of 39 mT (estimated from the full width of the autocorrelation function at 0.5). Although the origin of the fluctuations is still the subject of conjecture, we propose that the fluctuations in the magnetoresistance develop from changes in the relative phase between two trajectories running the length of the waveguide (between the 1-μm outlying lead frame), separated on average by the distance between maxima in the probability amplitude across the wire. The trajectories presumably close near the 1-μm connections to the waveguide where we expect the probability amplitude to be more homogenous. Because there is less than one state within $\pm k_B T$ of the Fermi energy at 35 mK for a wire 6 μm long and 75 nm wide, and because $L_e \approx L$, only a few channels, predominantly a single channel (specifically the highest-energy subband which gives the largest contribution to the density of states), carry the current with minimal scattering.[10] For 0.5 T, if the wave function associated with this single channel resembles

$$\Psi = \frac{1}{(\pi^{1/2} l_c/2)^{1/2}} \left[1 - 2\left(\frac{x-X}{l_c}\right)^2\right] e^{-(x-X)^2/2l_c^2} e^{ik_y y}$$

(corresponding to $m=3$), then the maxima in the probability density are separated by $\Delta x = \sqrt{2.5} l_c = 56$ nm. The separation of maxima will be smaller for edge states ($X \approx W/2$), where the magnetic potential is dominated by the square well, and consequently $\Delta x \leq 56$ nm. However, in the extreme magnetic quantum limit,

$$\Psi = (\pi^{1/2} l_c)^{-1/2} e^{-(x-X)^2/2l_c^2} e^{ik_y y},$$

which has a maximum only at $x = X$. Because many paths near X will contribute with similar amplitude, the variance of the wave function will determine the typical frequency: i.e., $\langle x^2 - \langle x \rangle^2 \rangle = 2l_c^2$, or $(\langle \Delta x^2 \rangle)^{1/2} = 14$ nm at 7.0 T. Accordingly, as estimate for the period of oscillation is $\Delta H = \Phi_0 / L\Delta x \geq 12$ mT at 0.5 T, and $\Delta H = \Phi_0 / L 2(\langle \Delta x^2 \rangle)^{1/2} \geq 25$ mT near 7.0 T, where we have used $L = 6.0$ μm, the length of the waveguide running between the 1-μm interconnections to the lead frame, in correspondence with the data.

Finally, it is apparent from the data obtained on the shorter segments at 400 mK, and on all segments at 35 mK, that the rms conductance can exceed e^2/h, e.g., $\Delta G_{xx} \approx 5e^2/h$ deduced from $G_{xx} = R_{xx}/[R_{xx}^2 + (L/W) \times R_{xy}^2]$ in the 700-nm segment at 35 mK. The rms amplitude for $\omega_c \tau > 1$ is not e^2/h independent of the sample size if the device is smaller than both L_ϕ and L_e at zero field.

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