Reconstructing the Nuclear Profile in Gauge Space

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We introduce an expression for the deformed nuclear density in gauge space in terms of the pairing deformation parameter β_P. The Fourier components of this quantity as functions of the ordinary space variables can be used to define macroscopic form factors for multiparticle transfer processes in superfluid nuclei. Experimental ΔN-particle transfer cross sections for $\Delta N = \pm 2, \pm 4$, etc. analyzed within this framework yield the relative magnitudes of the Fourier amplitudes. This procedure therefore allows for the reconstruction of density contours in the abstract space.

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A major example of success in the description of nuclear phenomena is provided by the macroscopic approach to nuclear vibrational and rotational motion.[1] The specific probes to investigate these phenomena are the inelastic processes in nuclear collisions which are induced by the nuclear and Coulomb fields. Through the systematic analysis of a large variety of reactions it has been possible to acquire a clear picture of the deformation degrees of freedom which can be used to characterize excitations of nuclei either close to or away from completely filled shells. The former undergo deviations from their spherical shape which can be described in terms of harmonic dynamical variables. For the latter, on the other hand, the strength of the residual interactions is such that a phase transition to a permanently deformed system takes place.

It has been known for a long time that nuclei exhibit also an interesting affinity with neighbors in the mass table whose numbers of particles differ by a pair of identical nucleons.[1] The experimental data are here interpreted as evidence of a collective motion which takes place in an abstract space where the number of particles is not strictly conserved. In this so-called gauge space, the number of nucleons is the conjugate coordinate to a gauge variable (angle) ϕ. A correspondence with the familiar vibrational and rotational modes in ordinary space can be established.[2,3] Nuclei in the normal phase perform oscillations in particle number with respect to an equilibrium position which is symmetric in gauge space. Strong residual pairing interactions eventually break that symmetry, leading to superfluid nuclei which are permanently deformed in gauge space.

The concepts summarized in the last paragraph find their natural use in the analysis of two-particle transfer reactions. These processes, however, have been traditionally studied on microscopic terms. This is understandable. Surface modes and the rotation of a deformed object are easily visualized in the space of our everyday experience. Pair vibrations and rotations, on the other hand, tax our imagination. Only recently, and in the context of heavy-ion–induced reactions, has the formal analogy between inelastic surface modes and pairing vibrations been exploited to introduce macroscopic pair-transfer form factors.[4] With the assumption of a scaling of the nuclear volume as a function of particle number, a simple parametrization of the pair transition density as

$$\delta\rho(r) = \beta_P(\partial\rho/\partial A) \simeq (\beta_P R/3A_0)(\partial\rho/\partial r)$$

leads to form factors which are essentially proportional to those used for inelastic excitations. We note that within this approach the two-particle transfer process is globally described as the excitation of pair collective modes by a generalized one-body field. In this respect it can be viewed as an alternative to the description of the process in terms of the transfer of individual nucleons (either "simultaneous" or sequential).

In this note we indicate how these ideas can be extended to the case of superfluid nuclei. The major hurdle will be, as in the case of vibrations, to establish a plausible connection between the ordinary spatial variables and the angle ϕ characterizing motion in the abstract space. This link is essential to construct the radial form factors which are needed to calculate pair-transfer cross sections. The proposed solution opens, as we shall see, the way to use multiparticle transfer processes to map the surface of superfluid nuclei in gauge space.

We start by noting that the spontaneous breaking of symmetry in the superfluid phase justifies the introduction of an intrinsic frame. We recall the analogous situation in the case of ordinary rotations, where the members of a band share a common intrinsic state $|\Phi(\omega_0,\beta)\rangle$ characterized by an orientation angle ω_0 and the intrinsic deformation parameter β. The relevant couplings for direct excitation from the ground state are specified through the radial functions

$$\delta\rho_\lambda(r)=\int\rho(r,\theta)Y_{\lambda 0}(\theta)d\theta.$$

In this expression the spherical harmonic $Y_{\lambda 0}$ is the appropiate kernel to project from the deformed density in the intrinsic frame, $\rho(r,\theta)$, the piece that connects with the state of angular momentum λ.

In the pair-transfer case the angular momentum role is taken over by the number of particles N. The states with different N can be viewed as members of a family that share a common intrinsic state $|\Phi(\phi_0,\beta_P)\rangle$, where ϕ_0 specifies the orientation of the deformed system in gauge space. The information to construct the couplings between states with different numbers of particles is contained in a generalized density $\rho(r,\phi)$ which plays a role analogous to the deformed density $\rho(r,\theta)$ in the case of standard rotations. The moment decomposition of the function $\rho(r,\phi)$ that we need here is

$$\delta\rho_k(r)=\int\rho(r,\phi)e^{ik\phi}d\phi,$$

i.e., its Fourier expansion. Note that the spherical harmonic is replaced in this expression by the exponential $e^{ik\phi}$, as follows from the fact that gauge transformations are equivalent to ordinary rotations in two dimensions. From these moments the deformed density in gauge space can be reconstructed by the inverse transformation

$$\rho(r,\phi)=(1/2\pi)\sum_k\delta\rho_k(r)e^{-ik\phi}.$$

As we have pointed out, in the superfluid phase the intrinsic state is statically deformed. Thus, contours of the density in gauge space are no longer circles, as was the case for normal systems. The situation can be interpreted as follows. As we move about the ϕ-spanned plane, the nucleus presents to us different facets. These reflect the admixture of particles which is present in number-nonconserving states such as the BCS. With the parameter β_P as a measure of this deformation, we adopt as a simple *Ansatz* the following expression:

$$A(\phi)=A_0+2\beta_P\cos 2\phi,$$

where we have chosen the orientation of the symmetry axis to be at $\phi_0=0$. With the aforementioned scaling assumption this can also be turned into a radial dependence

$$R(\phi)=R_0[1+(2\beta_P/3A_0)\cos 2\phi].$$

The distribution of A around A_0 which follows from this expression has a second moment which is directly measured by β_P. The distribution itself, however, deviates significantly from an expected bell-shaped curve. This behavior is well known from the case of coordinates in a classical oscillator and it is also analogous to the distribution of radii in a quadrupole-deformed object. Note also that for small-amplitude vibrations (i.e., dynamical deviations from the circular shape) the $\Delta N=\pm 2$ term reproduces the result obtained for normal systems. A sketch of the function $A(\phi)$ in the intrinsic frame is shown in Fig. 1. We finally observe that within a microscopic approach such as the BCS formalism the macroscopic pairing deformation parameter β_P can be put in correspondence with the traditional gap parameter Δ according to $\beta_P\leftrightarrow\Delta/G$, G being the strength of the pairing interaction.

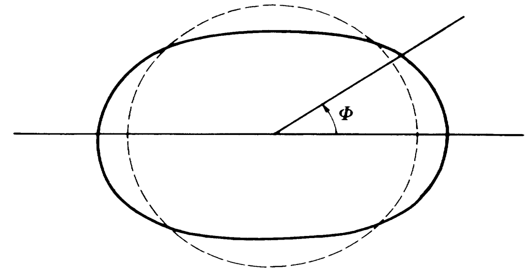

FIG. 1. Sketch of the function $A(\phi)$ in the intrinsic frame in gauge space. The dashed curve corresponds to the case of a normal system with fixed number of particles $A_0=125$, while the solid line corresponds to the superfluid case with pairing deformation parameter $\beta_P=10$.

The dependence of the nuclear radius on the gauge angle can be used to introduce a macroscopic expression for the density $\rho(r,\phi)$, namely

$$\rho(r,\phi)=\frac{\rho_0}{1+\exp\{[r-R(\phi)]/a\}}.$$

This function can then be Fourier expanded to construct the transition densities for $\Delta N=0, \pm 2, \pm 4$, etc. In Fig. 2 we show the radial dependence of these components for two typical values of β_P. The $\Delta N=0$ term corresponds to the usual one-body nuclear density. The projection for $\Delta N=\pm 2$ (corresponding to the particle-particle transition density) has the surface-peaked behavior expected from the first derivative of the density for small deformations. The $\Delta N=\pm 4$ radial dependence displays the characteristic surface node which is also present for the angular momentum $\lambda=4$ couplings in ordinary rotations. The qualitative aspects of the curves are similar for both values of β_P. As we compare absolute values, however, we notice the increasing importance of the four-nucleon direct coupling relative to the two-nucleon term for the larger deformation.

The form factors for particle transfer are sensitive only to the values of these densities outside the nuclear surface. For the examples displayed in Fig. 2, this corresponds to the tail region, i.e., $r\geq 8$ fm. At these large distances the ratios between the different moments become independent of r. A measure of the relative impor-

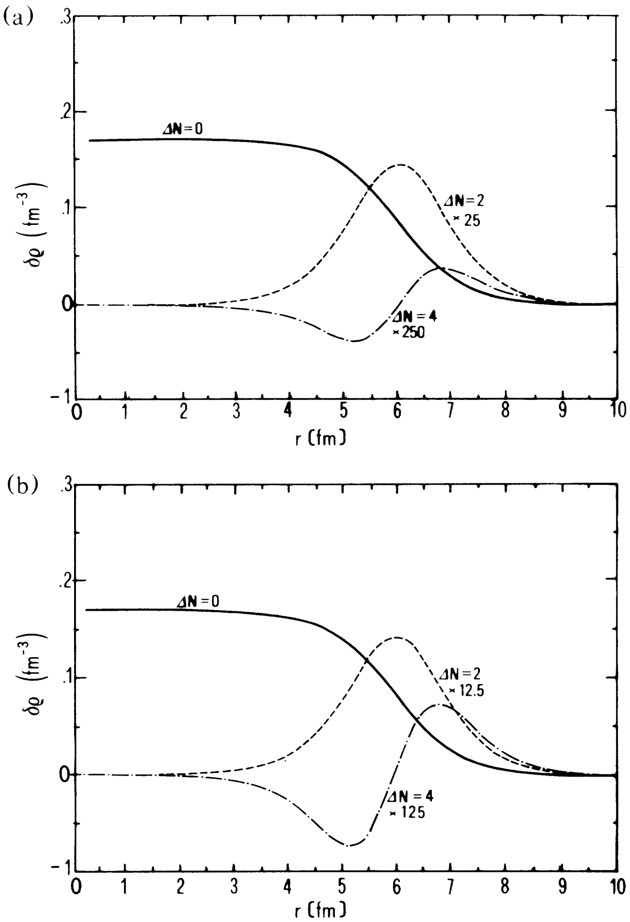

FIG. 2. Radial dependence of the transition densities for $\Delta N=0$, $\Delta N=2$, and $\Delta N=4$ obtained by Fourier expansion of the macroscopic deformed density in gauge space. The results corresponding to the values (a) $\beta_P=5$ and (b) $\beta_P=10$.

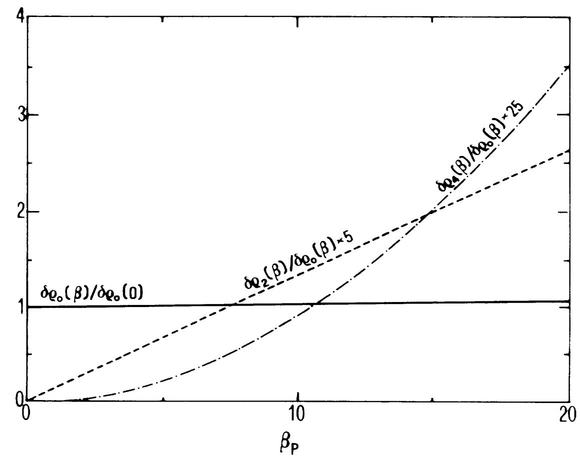

FIG. 3. Transition densities for $\Delta N=0$, $\Delta N=2$, and $\Delta N=4$ as a function of the pairing deformation parameter β_P. The quantities are evaluated at a common point in the exponential-tail region ($r=10$ fm), and are normalized to $\delta\rho_0(\beta_P=0)$ for $\Delta N=0$ and to $\delta\rho_0(\beta)$ for $\Delta N=2,4$.

tance of the two- and four-particle direct transfer couplings can be obtained from Fig. 3. Here the asymptotic ratios $\delta\rho_2/\delta\rho_0$ and $\delta\rho_4/\delta\rho_0$ are shown as a function of the deformation parameter for the range $0\leq\beta_P\leq 20$. This covers the values of β_P which are expected to be representative for real nuclei. The normalized to $\delta\rho_0$ was included to insure the conservation of the volume. The range of β_P is, however, such that for all values the effective deformation $(\beta_P/3A_0)$ is small. This can also be appreciated from the fact that the ratio $\delta\rho_2/\delta\rho_0$ keeps a constant slope throughout.

An interesting aspect of the present approach is the prediction of direct four–particle transfer. Such a transfer process has been experimentally observed, for example in the Sn+Sn reaction.[5] It is known that in the inelastic excitation of the 4^+ member of an ordinary rotational band the direct $\Delta\lambda=4$ transition competes with the two-step transition through the 2^+ state. Similarly, in our case, the $\Delta N=\pm 4$ transfer process can effectively compete with the sequential transfer of two pairs. An idea of the relative importance of the two processes can be obtained by comparing the amplitudes $a_{\rm dir}$ and $a_{\rm seq}$ associated with $\Delta N=\pm 4$ transfer processes. These amplitudes can be estimated to be (aside from adiabatic cutoff effects)

$$|a_{\rm dir}|\sim\tau F_4(r_0),\qquad |a_{\rm seq}|\sim\tfrac{1}{2}\tau^2F_2(r_0)^2,$$

where $F_2(r_0)$ and $F_4(r_0)$ are the form factors for $\Delta N=\pm 2$ and $\Delta N=\pm 4$ direct transfer evaluated at the distance r_0 of closest approach and τ an effective collision time. Since the ion-ion potential $U(r)$ and the coupling form factors are proportional to the corresponding transition densities, expanding up to second order in β_P, one can take

$$\frac{F_4(r_0)}{U(r_0)}\simeq\frac{1}{2}\left[\frac{F_2(r_0)}{U(r_0)}\right]^2,$$

so that

$$\frac{|a_{\rm seq}|}{|a_{\rm dir}|}\sim\tau U(r_0).$$

For $\tau/\hbar\simeq 0.1$ MeV^{-1} and $U(r_0)\simeq 1$ MeV, the direct cross section would therefore be about 2 orders of magnitude larger than the sequential one. An even further increase of $\Delta N=\pm 4$ transfer would require the occurrence of a term proportional to $\cos 4\phi$ in the expression of $R(\phi)$. This situation has its counterpart in the inelastic excitation of a rotor, where a major presence of $\Delta\lambda=4$ processes is often taken as the signature of hexadecapole deformations of the nuclear surface. The previous arguments centered on the magnitude of the transition matrix elements. The experimental conditions can in addition be adjusted to emphasize the weak coupling situation (below the barrier, for instance) where the direct transitions eventually overcome the two-step processes.

Through the analysis of low-energy data it should be possible to extract the magnitude of the different direct transfer terms. The previous considerations then open the actual possibility of reconstructing by inverse Fourier transform the profile of the nucleus in gauge space. A similar idea has been successfully applied in electron scattering experiments, which identified the different moments of the transition densities in space-deformed systems.[6] In the case of the gauge space the empirically extracted values of β_P could then be used to test microscopic BCS calculations.

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