$E2/M1$ Ratio of $\gamma N \to \Delta(1232)$ Transitions in a Relativistic Three-Quark Model

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The $E2/M1$ ratio of electromagnetic $N(939) \to \Delta(1232)$ transitions is calculated in a relativistic three-quark model for the nucleon and $\Delta(1232)$. Relativistic kinematics in the three-quark N and Δ wave functions yields an $E2/M1$ contribution of about -0.2% *without* any tensor forces in the dynamics.

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The ratio of electric quadrupole to magnetic dipole amplitudes in $\gamma N \leftrightarrow \Delta(1232)$ transitions, $R = \langle\Delta|E2|N\rangle / \langle\Delta|M1|N\rangle$, is known as a sensitive test of quark models; $R=0$ for spherically symmetric nonrelativistic N and Δ states.[1] The tensor component of the hyperfine interaction[2] between quarks yields $R=-0.3\%$ to -0.4% obtained from the current operator and $R=-0.7\%$ from the charge density,[3] which is known to be more reliable. In the Skyrme model,[4] R is predicted to be relatively large (about -5%).

A recent reanalysis[5] of pion photoproduction data with Olsson's unitary *Ansatz*[6] for the resonant amplitude gives an $E2/M1$ ratio $R=(-1.5\pm0.2)\%$, and again revised to substantially smaller values -0.3% to -0.6%; another[7] based on Noelle's unitary K-matrix *Ansatz*,[8] which does not rely on Olsson's Δ-dominance assumption, yields $E2/M1$ values between $(-0.50\pm0.14)\%$ and $(-1.3\pm0.1)\%$, while Grushin *et al.*[9] report $R=(-1.83\pm0.22)\%$. The one-pion-exchange potential between quarks at long distances contributes[3] about -0.4% to R.

In this Letter, we calculate the $E2/M1$ ratio in a relativistic model,[10] in which the nucleon and $\Delta(1232)$ are taken to be composite systems of three-valence quarks with constituent mass $m_u=m_d=m_q\simeq330$ MeV/c^2. The model does not include the component of the hyperfine interaction or the one-pion-exchange potential. Nonetheless, we show that nonspherical components required in the N and Δ wave functions by relativistic invariance alone yield a small, but negligible, contribution to the $E2/M1$ ratio between -0.1% and -0.2% for an acceptable range of parameters, viz., $300\le m_q\le350$ MeV/c^2 and internal momentum scale (harmonic-oscillator constant) $280\le\alpha\le360$ MeV/c^2.

We follow Ref. 10 and use Dirac's light-cone formalism, which provides a consistent relativistic many-body theory in momentum space in terms of a Fock-state basis defined at equal light-cone time $x^+=t+z$, rather than the conventional equal-time wave function of the instant form. The three-quark nucleon (N) and $\Delta(1232)$ wave functions (with internal quantum numbers σ_i) are taken to be simple relativistic generalizations of the nonrelativistic constituent quark model assuming that the N and Δ states are dominated by a three-valence-quark configuration with constituent quark mass $m_u=m_d=m_q\simeq330$ MeV/c^2. In view of the relativistic motion of these light quarks, the totally symmetric N and Δ momentum distributions are taken as a common relativistic Gaussian[11]

$$\phi(x,\mathbf{k}_\perp)=N\exp(-M_3^2/6\alpha^2)$$
$$=N\exp\left[-\frac{1}{6\alpha^2}\sum_{j=1}^{3}(\mathbf{k}_{\perp j}^2+m_q^2)/x_j\right], \qquad (1)$$

where M_3^2 is the covariant three-body mass squared. The size parameter α is determined by the proton quark core charge radius, magnetic moments, and its axial form factor.[10]

To determine the relativistic spin-wave functions of the nucleon and $\Delta(1232)$, we need some approximation to deal with the problem of the angular momentum $\mathbf{J}^2$ in light-cone dynamics, where only J_z is well defined. For the small $E2$ transition amplitude from N to Δ, we want to avoid contributions from spurious spin-$\frac{3}{2}$ components in the nucleon and spin $\frac{1}{2}$ in the Δ wave function. To this end, we construct the spin-$\frac{1}{2}$ and -$\frac{3}{2}$ wave functions in terms of the conventional Clebsch-Gordan prescription for free quarks, but with the total energy of the free quarks taken to be the N or Δ mass, respectively, i.e., $\sum_{j=1}^{3}p_j^\mu \to P_{N/\Delta}^\mu$. For free spin-$\frac{1}{2}$ constituents the one-particle instant (equal time) and light-cone (equal x^+) states are related by the so-called Melosh transformation.[12] Using this relation, we obtain the following model for the symmetric and Lorentz-invariant light-cone wave functions

$$\psi_\uparrow^{N/\Delta}(x,\mathbf{k}_\perp,\lambda)$$
$$=\phi(x,\mathbf{k}_\perp)\chi_\uparrow^{N/\Delta}(x,\mathbf{k}_\perp,\lambda)\left[\prod_{j=1}^{3}x_j\right]^{-1/2}, \qquad (2)$$

where

$$\chi_\uparrow^N(\hat{1},\hat{2},\hat{3})=J_\uparrow^N(13,2)+J_\uparrow^N(23,1),\qquad \chi_\uparrow^\Delta(\hat{1},\hat{2},\hat{3})=J_\uparrow^\Delta(12,3)+J_\uparrow^\Delta(23,1)+J_\uparrow^\Delta(31,2), \tag{3}$$

with

$$J_\uparrow^N(12,3)=\bar{u}_{\lambda_2}(m_N+\gamma\cdot P)\gamma_5 v_{\lambda_1}\bar{u}_{\lambda_3}u_\uparrow^N,\qquad J_\uparrow^\Delta(12,3)=\bar{u}_{\lambda_2}\left[\gamma^\mu+i\sigma^{\mu\nu}\frac{P_\nu}{m_\Delta}\right]v_{\lambda_1}\bar{u}_{\lambda_3}u_{\mu\uparrow}^\Delta. \tag{4}$$

Their normalizations are given by

$$2\int dx\,d^2k_\perp\,|\phi(x,\mathbf{k}_\perp)|^2\sum_{\lambda's}|\chi^{N/\Delta}(\hat{1},\hat{2},\hat{3})|^2=1. \tag{5}$$

The u_λ in Eq. (4) are the standard light-cone spinors,[13] and $\hat{1},\hat{2},\hat{3}$ denote collective momentum-helicity indices $(x_j,\mathbf{k}_{\perp j},\lambda_j)$ for $j=1,2,3$. Further details on the wave functions are given elsewhere.[10,14] The N and Δ states (2) are written in the so-called *uds* basis, where only the u (and d) quark antisymmetrization is carried out explicitly. The wave functions in Eqs. (2)–(4) are obviously symmetric under the exchange of the first two quarks and symmetrized in the third quark.

The electromagnetic $N\to\Delta$ amplitude is defined[15] in terms of the transition form factors for the charge G_c^*, magnetic dipole G_M^*, and quadrupole G_E^*, respectively. In Fig. 1 the diagrams are shown that contribute to the electromagnetic $N\to\Delta$ transition current

$$\Gamma_\lambda^+=\langle\Delta(p+q,\lambda)|\frac{j^+(0)}{p^+}|N(p,\lambda-1)\rangle=\sum\int dx\,d^2k_\perp\psi_\lambda^{\dagger\Delta}(x,\mathbf{k}'_\perp,\lambda')\frac{\bar{u}_m}{(k_m^+)^{1/2}}\gamma^+Q_m\frac{u_m}{(k_m^+)^{1/2}}\psi_{\lambda-1}^N(x,\mathbf{k}_\perp,\lambda), \tag{6}$$

where Q_m is the charge of the struck quark with the final momentum $\mathbf{k}'_{\perp m}=\mathbf{k}_{\perp m}+(1-x_m)\mathbf{q}_\perp$, while the spectator quarks have final momentum $\mathbf{k}'_{\perp i}=\mathbf{k}_{\perp i}-x_i\mathbf{q}_\perp$ for $i\neq m$. If the light-cone coordinates are chosen[16] as $p^\mu=(p^+,M^2/p^+,\mathbf{0}_\perp)$ for the nucleon moving along the z axis and $q^\mu=(0,2pq/p^+,\mathbf{q}_\perp)$ for the photon, the helicity matrix elements of the current $j^+=j^0+j^3$ have the simple form

$$\Gamma_{3/2}^+=\frac{3q_L}{2\sqrt{2}m_N}(G_M^*+G_E^*),$$
$$\Gamma_{1/2}^+=\frac{3q_L}{2\sqrt{2}m_N}(G_M^*-3G_E^*), \tag{7}$$

where $q_L=q_1-iq_2$ is used for the transverse momentum transfer. Hence[17]

$$G_M^*(0)=\frac{m_N}{\sqrt{6}}\frac{\partial}{\partial q_L}(\sqrt{3}\Gamma_{3/2}^++\Gamma_{1/2}^+)\Big|_{q=0}, \tag{8}$$

$$G_E^*(0)=\frac{m_N}{\sqrt{6}}\frac{\partial}{\partial q_L}\left[\frac{1}{\sqrt{3}}\Gamma_{3/2}^+-\Gamma_{1/2}^+\right]\Bigg|_{q=0}, \tag{9}$$

and the $E2/M1$ ratio becomes

$$R=G_E^*(0)/G_M^*(0). \tag{10}$$

We have performed our calculations for the ranges of parameters suggested by earlier work on the electromagnetic and weak properties of the nucleon in the relativistic quark model,[10] viz., $\alpha\in[280,360]$ MeV and $m_q\in[300,350]$ MeV. Our $E2/M1$ ratio depends sensitively on the quark mass m_q and the momentum scale α. We show in Table I our results. They show that relativistic effects from nonspherical components in the three-quark nucleon and $\Delta(1232)$ wave functions lead to mixed orbital configurations which in nonrelativistic quark models originate only from (tensor) interactions. The small Dirac components responsible for this are required by relativistic invariance and lead to small, but nonnegligible, $E2/M1$ contributions of the same sign as those from the color hyperfine and pion tensor potentials.

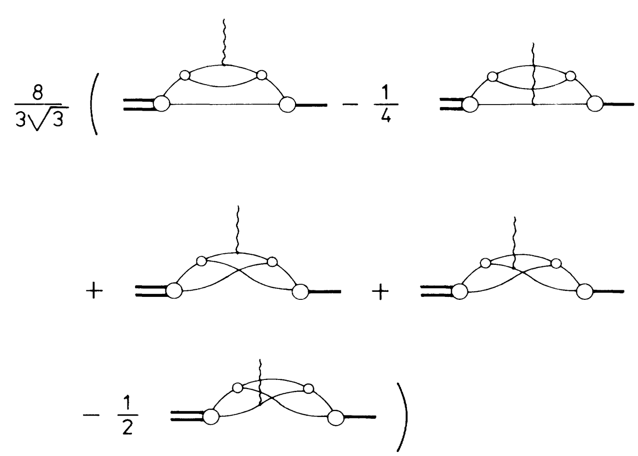

FIG. 1. Calculation of electromagnetic current matrix element for three-valence-quark state contribution.

TABLE I. $E2/M1$ ratio for $N^\gamma\to\Delta$ in percent.

	M_q (MeV/c^2)			
α (MeV)	250	300	350	400
280	−0.156	−0.132	−0.113	−0.096
320	−0.190	−0.162	−0.139	−0.120
360	−0.225	−0.193	−0.166	0.144

In conclusion, then, our results suggest that relativistic kinematics in hadron wave functions can play an important role in some properties of the hadron spectrum.

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