First-Order Phase Transition in Compact Lattice QED with Light Fermions

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(Received 15 May 1987)

We present a numerical study of lattice (compact) QED with dynamical fermions using hybrid stochastic differential equations. Working with an 8^4 lattice and fermions of mass 0.10 (lattice units), we have found an unexpected first-order transition separating the strong- and weak-coupling regions.

PACS numbers: 11.15.Ha, 12.20.Ds

Many years ago Gell-Mann and Low[1] and Landau[2] remarked that in spite of its success, weak-coupling QED is a logically incomplete theory. The reason is that zero coupling is an infrared-stable fixed point rather than an ultraviolet-stable fixed point as in QCD. This means that the running coupling constant of QED increases with the cutoff (Λ) making the weak-coupling expansion inconsistent at very short distances or high energies. In addition, Landau[2] also gave arguments suggesting that the renormalized coupling constant may vanish when λ goes to infinity (the "zero-charge" situation).

A resolution to these problems could be the existence of a nontrivial zero of the theory's β function at a nonzero coupling.[3] The search for this new fixed point of QED must necessarily involve nonperturbative techniques.

A nonperturbative study of QED is not expected to improve the physical results obtained from weak-coupling "phenomenological" QED because at high energies where perturbation theory is expected to fail other interactions will be important.[2] However, from the point of view of QED as an abstract field theory, it is clear that the search for a nontrivial zero of the β function is an interesting challenge.

There have been basically two approaches to the problem: (1) a study of the Schwinger-Dyson equation in the ladder approximation[4,5] and (2) the lattice technique.

With use of the first technique, a new ultraviolet fixed point of the β function was found. The new strong-coupling phase generates fermion masses dynamically. A nontrivial continuum theory can be recovered by varying the coupling constant ($\alpha = e^2/4\pi$) as a function of the cutoff. In Refs. 4 and 5, it has been shown that the expected scaling law should be

$$m = C\exp[-\pi/(\alpha/\alpha_c - 1)^{1/2}], \tag{1}$$

where α_c is the critical coupling and C is an unknown constant.

The lattice formulation of QED allows nonperturbative calculations in a more systematic way. In the pure gauge case compact QED has a rich structure that has been analyzed by numerical and renormalization-group techniques.[6] In this limit (heavy fermions) a phase transition separates a strong-coupling regime where there is confinement, as in QCD, from a massless phase in weak coupling. When dynamical fermions are included, the numerical calculations are more difficult. In a quenched approximation[7] to the noncompact version of lattice QED, it was shown that the lattice approach gives results in qualitative agreement with Refs. 4 and 5. On symmetric 8^4 and 10^4 lattices, a phase transition was found with $\langle\bar{\psi}\psi\rangle$ vanishing (nonvanishing) on its weak-(strong) coupling side. For $\beta = 1/e^2$ below the critical point β_c, $\langle\bar{\psi}\psi\rangle$ behaved as $(\beta_c - \beta)^{0.60\pm0.10}$ as expected from mean-field theory. However, studies on asymmetric lattices, 2×8^3, 4×8^3, and 6×8^3, revealed that the critical coupling was independent of the lattice asymmetry (temperature). Thus, no evidence was found for a transition temperature which scales with Eq. (1). This result could be a limitation of the quenched approximation and the trivial (free-field) nature of pure noncompact Abelian gauge fields. It might, however, support the "collapse" picture of the chiral condensate, i.e., the size of the condensate wave function is $O(\Lambda^{-1})$, and the critical temperature is $O(\Lambda)$ and thus *independent* of β. Miransky, in particular, has proposed that chiral symmetry breaking in nonasymptotically free theories is analogous to the collapse of the single-particle electron wave function bound to a point charge $Z > Z_c \sim 137$. He stressed that through nonperturbative renormalization QED would possess a nontrivial continuum limit at criticality. See Ref. 4 for further details. These arguments deserve clarification and they motivated the present study which puts light fermions into the compact theory to make the dynamics more interesting.

A study of this type was done earlier.[8] There it was found that the two-phase structure of the quenched theory survives the introduction of fermions. The strong-coupling phase is characterized by spontaneous breakdown of chiral symmetry. A phase transition occurs with a critical coupling of order 1, and at weak coupling the vacuum is chirally symmetric. These results were obtained with relatively large fermion bare masses ($m = 0.25$ and 0.20 in lattice units) and with limited statistics. In Ref. 8 it was noticed that the relaxation time to achieve equilibrium starting with hot and cold configurations was rather high in the vicinity of the transition.

In this Letter we present a high-statistics study of lattice *compact* QED with light staggered fermions. We worked on an 8^4 lattice with fermion bare masses $m=0.10$ and 0.25. The main result of the Letter is that contrary to the naive expectations described above, a first-order transition appears to $m=0.10$, introducing problems in the study of the continuum limit of compact lattice QED with massless fermions when the standard Wilson action is used for the gauge-field sector.

The action is given by

$$S=\beta\sum_p \mathrm{Re}U_p+\tfrac{1}{2}\sum_{x,\mu}\eta_\mu(x)\bar{\psi}(x)[U_\mu(x)\psi(x+\mu)-U_\mu^*(x-\mu)\psi(x-\mu)]+m\sum_x\bar{\psi}(x)\psi(x), \quad (2)$$

where x, p, and μ, denote sites, plaquettes and directions of a four-dimensional hybercubic lattice. The U(1) link gauge variable is denoted by $U_\mu(x)$ while the fermionic fields on the sites are $\bar{\psi},\psi$ (staggered fermions). The rest of the notation is standard.

To numerically generate configurations with probability $\exp(-S)$ we use the hybrid method.[9] This is a stochastic differential equation method in which molecular-dynamics equations are iterated, but the velocities are refreshed periodically (i.e., from time to time the Langevin equation is used). The molecular-dynamics equations are the following:

$$L=\tfrac{1}{2}\sum_{x,\mu}\dot{\theta}_\mu^2(x)+\sum_{x,y}\dot{\phi}_x^*[A^\dagger(U)A(U)]_{xy}\dot{\phi}_y-\omega^2\sum_x\phi_x^*\phi_x-\beta\sum_p\mathrm{Re}U_p, \quad (3)$$

where $U_\mu(x)=\exp(i\theta)\mu^{(x)}$, $A(U)$ is the lattice Dirac operator, ϕ are "pseudofermion" fields, and ω^2 is a constant. For details see Ref. 9.

Every 25 iterations we refresh the velocities putting the system in contact with a heat bath at coupling β. To avoid systematic errors and instabilities in the algorithm, we use a small value of the time step $dt=0.01$ for $m=0.10$, and $dt=0.02$ for $m=0.25$. To analyze the phase diagram of the theory, we measure the chiral condensate and the mean value of the pure-gauge field term S_0 in the action of Eq. (2).

In Fig. 1 we show $\langle\bar{\psi}\psi\rangle$ as a function of β for masses $m=0.25$ and 0.10. We see an abrupt change of behavior in a narrow interval suggesting the possibility of a discontinuity in this observable. The algorithm was iterated 10000–50000 times at each point.

The critical couplings are $\beta_c=0.894$ for $m=0.10$, and $\beta_c=0.919$ for $m=0.25$. This represents a shift of about 10% with respect to the pure gauge results where the critical β is near[6] 1.0. We also did a mean-field study of lattice QED with fermions (for details see Ref. 8). In this case, the shift in the critical β between the pure gauge theory and the theory with zero-mass fermions is approximately 7% in good agreement with the numerical result.

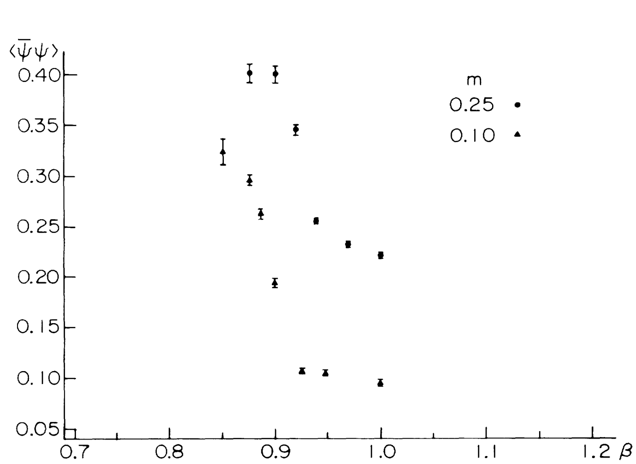

FIG. 1. $\langle\bar{\psi}\psi\rangle$ vs β with masses 0.10 and 0.25.

To clarify whether we are observing a first- or higher-order transition, we show in Fig. 2(a) the results of the computer time evolution of the gauge-field term in the action, S_0, at $m=0.25$. We needed over 40000 iterations of the algorithm with a time step of $dt=0.02$ to find convergence of the two signals, a disordered start and an ordered one, showing the existence of only quasimetastability at this value of the mass. In Fig. 2(b), we show $\langle\bar{\psi}\psi\rangle$ as a quasimetastability at this value of the mass. In Fig. 2(b), we show $\langle\bar{\psi}\psi\rangle$ as a function of iteration number. These results are in good agreement with the conclusions of Ref. 8 at this value of the mass.

However, by reduction of the mass, the order of the transition apparently changes. In Fig. 3(a), the results for $m=0.10$ are presented. Now after 100000 iterations ($dt=0.01$) the two-state signal survives suggesting the existence of a first-order transition. Comparison of the mass $=0.25$ and 0.10 runs suggests that as mass approaches zero, a clean signal for metastability will be produced. Further simulations at $m=0.050$ are in progress to confirm this point.

In Fig. 3(b), we plot $\langle\bar{\psi}\psi\rangle$ versus computer time. Although the signal is quite noisy, the existence of two states is still fairly compelling.

In conclusion, the phase diagram of lattice compact QED with the Wilson gauge action and staggered fermions resembles Fig. 4. In the pure gauge case, there is a weak first-order transition[10] that presumably disappears very quickly when the dynamical fermions are turned on. For intermediate values of the mass, the transition is replaced by a smooth crossover phenomenon, while for very small masses it is again of first order.

What about the influence of finite-size effects (or finite temperature) on the transition? Simulations on 6^4 and

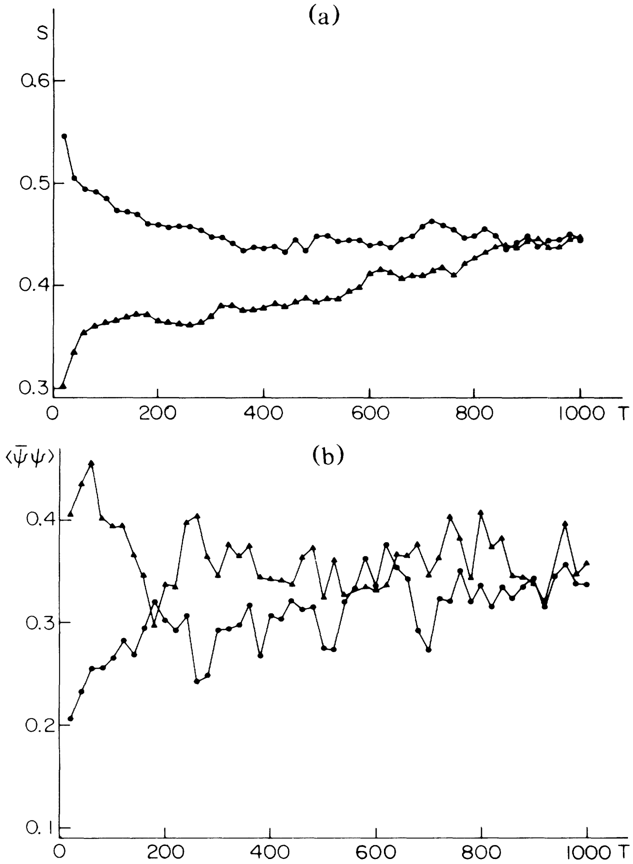

FIG. 2. (a) Mean value of the action S_0 as a function of computer time at $m=0.25$ and $\beta=0.919$. (b) $\langle\bar{\psi}\psi\rangle$ vs computer time with the same parameters as in (a).

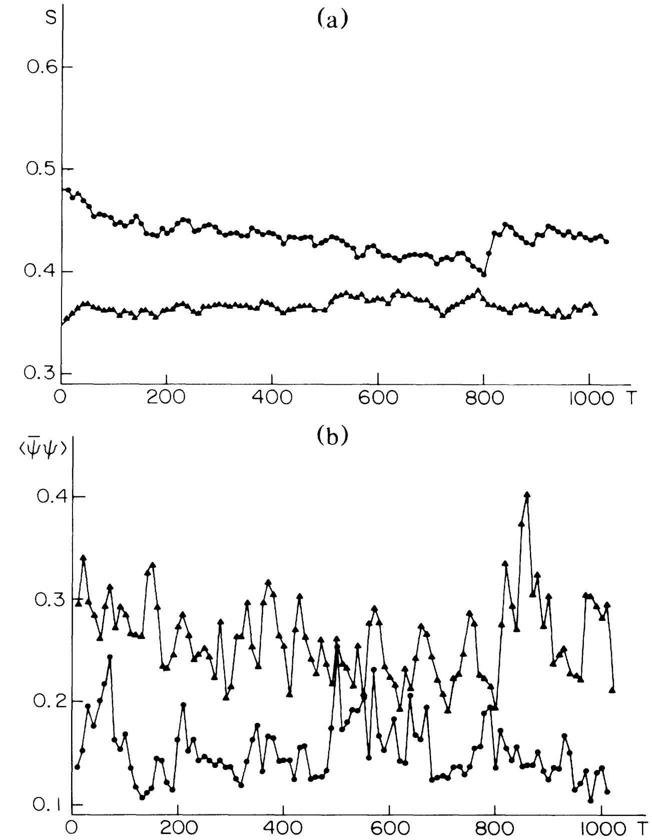

FIG. 3. (a) Mean value of the action S_0 as a function of computer time at $m=0.10$ and $\beta=0.894$. (b) Same as (a) but plotting $\langle\bar{\psi}\psi\rangle$.

10^4 lattices at $m=0.10$ are in progress, and results on the 6^4 lattice strongly suggest that the transition is a zero temperature, bulk phenomenon since β_c, as measured on the 6^4 lattice, is shifted only slightly (0.025 units toward stronger coupling) relative to the 8^4 data shown here. On the 6^4 lattice $\langle\bar{\psi}\psi\rangle$ jumps from 0.338 ± 0.011 at $\beta=0.8675$, to 0.144 ± 0.005 at $\beta=0.875$, and S_0 jumps from 0.484 ± 0.001 to 0.381 ± 0.002.

It is clear that the existence of a first-order transition for lattice QED with light fermions is an undesirable result—the theory does not have an interesting continuum limit. Other approaches that will be used in the future to search for a physically interesting strong-coupling phase of QED are (1) the noncompact formulation and (2) the introduction of additional parameters in the compact formulation.[11] This later method has proved able to change the order of the transition in the pure gauge case.[12] Of course, we would like to develop an intuitive physical picture of the existence of this first-order transition. Perhaps the number N_f of continuum flavors is important here as in QCD at high temperature. We have run simulations on 6^4 lattices with $N_f=1$, 4, and 16, and have found very clear metastability in each case. In addition, it would be interesting to understand the interplay

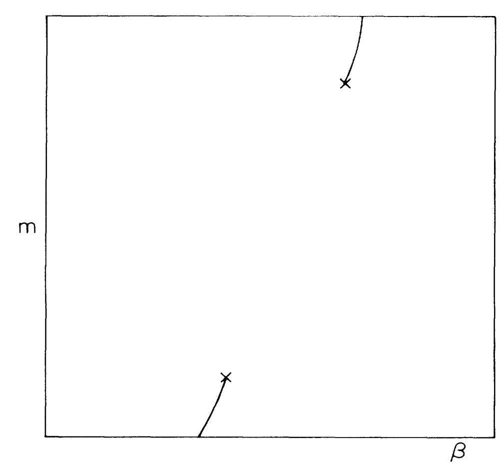

FIG. 4. Probable phase diagram of QED_{3+1} in the plane (m,β).

of the dynamical vortices[13] which drive the transition in the pure gauge theory and the massless fermions. Since the size of the condensate wave function is presumably $O(\Lambda^{-1})$, it could couple strongly to the cores of the ultraviolet singular vortex loops of the pure U(1) gauge model. Since the noncompact formulation of lattice QED does not have vortices, it may be that the chiral transition is continuous here even in the unquenched theory. Preliminary simulations do not rule out this possibility. If this is true, we must then understand the limitations of Landau's zero-charge scenario and the fate of fermion vacuum polarization at criticality.

In future studies we shall implement Fourier acceleration techniques to evade long correlation times in the computer simulations.[14]

This work was done as part of the Materials Research Program, Grant No. DMR83-16981. It was also supported in part by the National Science Foundation Grants No. DMR-84-15063 and No. PHY83-01948. The computer simulations were done on a FPS-264 which is supported in part by National Science Foundation Grant No. PHY83-04872. Program development was done on the Cray X-MP/48 of the National Center for Supercomputing Applications and the Cray X-MP/22 of the National Magnetic Fusion Energy Computer Center.

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