Nonlinear Absorption of Intense Microwave Pulses

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Free-electron lasers in the electron cyclotron frequency range have been proposed as an efficient means of heating and driving current in tokamak plasmas. The proposed free-electron laser will have short pulses ($\approx 5\times 10^{-8}$ sec) with a peak power of $(4-8)\times 10^{9}$ W, and electric fields within the plasma in the range $10^{5}-10^{6}$ V/cm. At these high intensities nonlinear effects compete with thermal effects in the absorption of microwave power. Analytic and numerical calculations of the nonlinear absorption are presented.

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The efficient production of microwave power by pulsed free-electron–laser (FEL) technology has recently been demonstrated.[1] Such FEL's operating in the electron cyclotron frequency range are an attractive option for both heating and driving current in tokamak plasmas.[2] The Lawrence Livermore National Laboratory in collaboration with the Massachusetts Institute of Technology has proposed the Microwave Tokamak Experiment (MTX) for which the Alcator-C tokamak would be brought to Lawrence Livermore National Laboratory for pulsed FEL heating experiments. In this Letter we study the direct wave-particle absorption of the intense pulses of microwave radiation that would be used in heating reactor-relevant plasmas. Another issue for intense pulsed heating is the parametric coupling to other modes. Such considerations are beyond the scope of this Letter, but are examined in Ref. 2.

The microwave beam power will have a Gaussian profile with an elliptical cross section. The full height of the Gaussian is h, while the full width is w. It will be incident from the low-field side nearly perpendicular to the magnetic field, $\mathbf{B}$. The rf electric field, $\tilde{E}$, may be estimated from the Poynting vector and peak power. For the MTX experiment, $\tilde{E}$ is about 1700 statvolts/cm or 0.5×10^{6} V/cm.

We consider a model problem in which an electron in a uniform magnetic field interacts with a wave propagating nearly perpendicular to $\mathbf{B}$. We make the following assumptions: $\tilde{E}/B_0 \ll 1$ (for MTX, $\tilde{E}/B_0 \approx 0.01$), $k_\parallel \approx 0$ so that $\langle v_\parallel \rangle \approx 0$, and $\epsilon/m_e c^2 \ll 1$ (where ϵ is the electron's kinetic energy) so that $\Omega_{ce}(\epsilon) \approx \Omega_0(1-\epsilon/m_e c^2)$. Only the first of these assumptions is necessary. When $\tilde{E}/B_0 \ll 1$, the energy gain of resonant electrons is small compared to $m_e c^2$, and we need keep only one cyclotron harmonic in the analysis. With use of canonical variables, the Hamiltonian may be put in the form[3]

$$H = \tfrac{1}{2}(P_\phi - P_r)^2 + \alpha P_\phi^q \cos(l\phi),$$

where we have chosen units such that $m_e = c = \Omega_0 = 1$. To lowest order in the wave amplitude, P_ϕ is the relativistic generalization of the magnetic moment, $P_\phi \approx \frac{1}{2}\gamma^2 v_\perp^2/\Omega_0$; $\gamma = (1 - v^2/c^2)^{-1/2}$ is the relativistic factor; $P_r = 1/\gamma_0 - \omega/l$ [where $\gamma_0 = (1+p_z^2)^{1/2}$] is a measure of the frequency mismatch between the gyro motion and the wave; and l is the resonant cyclotron harmonic number. The small-argument expansion for the Bessel function has been used in the writing of this Hamiltonian. Its validity follows directly from the perpendicular index of refraction, $N_\perp = k_\perp c/\omega$, being generally of order 1 for electron cyclotron waves, together with the weakly relativistic approximation, $\epsilon/m_e c^2 \ll 1$.

The ordinary mode near the first harmonic ($\tilde{\mathbf{E}} \parallel \mathbf{B}_0$) and the extraordinary mode near the second harmonic ($\tilde{\mathbf{E}} \perp \mathbf{k}$ and $\tilde{\mathbf{E}} \perp \mathbf{B}_0$) give the best linear absorption when launched from the low-field side of the plasma. For the O mode at the first harmonic[3] $\alpha_1^{(O)} = (N_\perp/\sqrt{2})(v_\parallel/c)(\tilde{E}_\parallel/B_0)$ and $q_1^{(O)} = \frac{1}{2}$, while for the X mode at the second harmonic[3] $\alpha_2^{(X)} = (N_\perp/2)(\tilde{E}_\perp/B_0)$ and $q_2^{(X)} = 1$.

Since the Hamiltonian is independent of time, it is a constant of motion. Hence, the electron orbits are given by curves of constant H in (P_ϕ, ϕ) phase space. When $\alpha \ll P_r^{2-q}$, there is a small set of closed orbits about an elliptic fixed point at $P_\phi \approx P_r$, and $\phi = \pi$. The characteristic width of a trapped orbit in this low-wave-amplitude limit is $\delta P \approx (l\alpha P_r^q)^{1/2}$. The characteristic frequency of the motion about the fixed point is $\omega_T^{(1)} \approx \Omega_0 \delta P$. In the opposite limit, $\alpha \gg P_r^{2-q}$, the region of closed orbits about the elliptic fixed point in the neighborhood of $P_\phi = P_r$ extends to $P_\phi = 0$. The orbit size is $\Delta P \approx \alpha^{1/(2-q)}$ and the frequency is $\omega_T^{(2)} \approx \Omega_0 \Delta P$.

In the FEL experiment, an electron will stream across the microwave beam in a time $w/v_\parallel \sim 10^{-9}$ sec much greater than the cyclotron period ($\sim 10^{-11}$ sec). The amplitude of $\tilde{\mathbf{E}}$, and hence α, will vary on the $w/v_\parallel$ time scale. Nonlinear physics will occur when $\omega_T w/v_\parallel \gtrsim 1$. For the MTX experiment (and possible future experiments in Compact Ignition Tokamak and Engineering Test Reactor) we estimate that $\omega_T w/v_\parallel \gtrsim 10$, so that nonlinear effects will limit the opacity of the plasma, and determine the region in phase space where the energy is deposited. When the microwave beam has a square profile, the change in the magnetic moment of a typical electron is given by ΔP (or δP) when α is greater than (less than) P_r^{2-q}. In the MTX experiment the microwave beam will have a Gaussian profile. The wave amplitude $\tilde{E}$, and hence α, will vary slowly compared to the period of the motion of electrons trapped or nearly trapped in the wave. As a consequence of the separation of time scales, there will be an adiabatic invariant, $\mathcal{J}$, associated with the rapid oscillation of these electrons, and the rapid motion will be restricted to a two-dimensional manifold within the four-dimensional (P_ϕ,ϕ,P_z,z) phase space.[4] It follows from our Hamiltonian that $\dot{P}_z/\dot{P}_\phi \sim \lambda/w \ll 1$, where $\lambda = 2\pi c/\omega$ is the wavelength of the incident FEL beam. Hence, the two-dimensional manifold containing the rapid motion is well approximated by the (P_ϕ,ϕ) plane and

$$\mathcal{J} = \oint P_\phi d\phi + O((\omega_T w/v_\parallel)^{-1}, (\lambda/w)^2).$$

If an electron is not trapped, $\mathcal{J}$ is the area under a curve of constant H; for trapped electrons $\mathcal{J}$ is the area enclosed by one of the closed contours surrounding the elliptic fixed point (see Fig. 1). In the limit $\alpha \to 0$, $\mathcal{J}$ is proportional to the magnetic moment, P_ϕ. Hence, the $\mathcal{J}$ must be broken if there is to be a net change in P_ϕ for an electron that passes through the Gaussian beam.

The hyperbolic fixed point at $\phi = 0$ in Fig. 1 provides a mechanism for breaking the adiabatic invariant of those electrons that become trapped in the wave. This leads to electron heating in the nonlinear regime as follows. As electrons stream into the beam, α increases and a region of trapped orbits appears about the elliptic fixed point. Because we use canonical variables the phase-space flow is incompressible. Hence, as the trapped region expands, phase volume flows from the region of open orbits below the separatrix, through the hyperbolic fixed point, and into the trapped region. If the wave amplitude is sufficiently large, all of the phase volume below the separatrix is pulled into the trapped region [see Fig. 1(b)]. As the electrons stream out of the beam, α decreases and the trapped region collapses. The area enclosed by the separatrix is equal to twice the decrease in the area below the separatrix.[3] Hence, the phase volume is expelled through the hyperbolic fixed point into both the regions above and below the separatrix. Those electrons that are pulled into the trapped region from below the separatrix and expelled into the region above the separatrix will have a net increase in their magnetic moment.

To quantify the heating, we define an opacity, $\tau_{NL} = P'_a/P_0$, where P'_a is the power absorbed by the particles when wave attenuation is ignored, and P_0 is the incident power. This is consistent with the definition of the opacity, τ_L, of linear wave theory, and is relatively easy to estimate. The relation between the transmission coefficient and τ_{NL} is somewhat different from the linear analog. However, the wave absorption is determined by τ_{NL}, and there is nearly complete absorption when $\tau_{NL} > 1$. We consider a plane stratified slab, and retain gradients in B_0 in the direction parallel to the propagation of the beam. The microwave power is then absorbed in a narrow layer about the surface on which $\omega = l\Omega_0$. We may estimate P'_a as the product of the flux of electrons through the microwave beam, Γ, and the mean energy gain of the electrons, $\langle\Delta\epsilon\rangle$, where $\Gamma \approx n_e v_e h d$, n_e is the electron number density, v_e is the electron thermal velocity, and d is thickness of the absorption layer.

We calculate $\langle\Delta\epsilon\rangle$ for the linear absorption regime and two nonlinear regimes. In each regime the opacity may be expressed as the product of the linear opacity, τ_L,

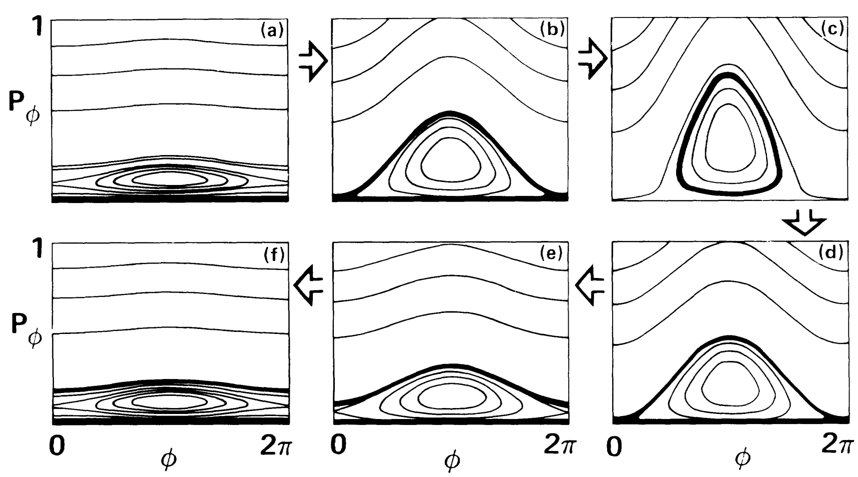

FIG. 1. (a)-(f) A sequence of snapshots of phase space moving across the microwave beam illustrating the heating mechanism in the strongly nonlinear regime. The thin lines are surfaces of constant H. The particles are indicated by the heavy lines. They are first pulled through the hyperbolic fixed point from below [(a)-(c)]. Half of the particles are expelled above the separatrix [(d)-(f)].

and a function of two dimensionless parameters, $p_1 \equiv \Delta P(m_e c^2/T_e)$ and $p_2 \equiv (m_e c^2/T_e)(2\pi/\omega\tau_c)$, where the linear correlation time τ_c is given by $w/v_\parallel$. The parameter p_1 is the ratio of the nonlinear resonance width to the thermal resonance width, while p_2 is the ratio of the resonance width arising from a finite linear correlation time to the thermal resonance width.

In the linear regime the change in energy of a resonant particle, $\Delta\epsilon_L$, is of order

$$\frac{\Delta\epsilon_L}{T_e} \approx \frac{m_e c^2}{T_e}\dot{P}_\phi \tau_c \approx \alpha\Omega_0\tau_c\left(\frac{T_e}{m_e c^2}\right)^{q-1} \sim \frac{p_1^{2-q}}{p_2}.$$

Some resonance electrons increase their energy by $\Delta\epsilon_L$, while the energy of others is reduced. In addition, only a fraction p_2 of the electrons are in resonance with the wave even within the linear absorption layer. Averaging over the electron distribution function yields

$$\langle\Delta\epsilon_L\rangle/T_e \approx p_2(\Delta\epsilon_L/T_e)^2 \sim p_1^{4-2q}/p_2.$$

Hence, P_a' is given by

$$P_a^{(L)} \sim n_e v_e T_e h d_L p_1^{4-2q}/p_2$$
$$\sim n_e v_e T_e h d_L (m_e c^2/T_e)^{4-2q}\alpha^2/p_2,$$

where $d_L \approx 4(T_e/m_e c^2)R$ is the width of the linear absorption layer, and R is the magnetic field scale length. The incident power is

$$P_0 \approx (\tilde{E}^2/8\pi)Nchw \sim \alpha^2 B_0^2 chw(m_e c^2/T_e)^{2-2q}/N,$$

where the second estimate holds for both the first-harmonic O mode and the second-harmonic X mode. Hence, we obtain the well-known scaling of linear opacity,

$$\tau_L = \frac{P_a^{(L)}}{P_0} = A\frac{T_e}{m_e c^2}\frac{R}{\lambda_0}\frac{\omega_{pe}^2}{\Omega_{ce}^2}N.$$

In this Letter, we seek the scaling of τ with parameters; the dimensionless constant A must be set equal to π^2 to match linear absorption theory. In nonlinear absorption regimes, A will have different values which we determine through numerical simulation.

Nonlinear effects first become important when $\omega_T^{(1)}\tau_{tr} \gtrsim 1$, or $p_1 > p_2^{1/(1-q/2)}$. Resonant electrons now perform several oscillations about the elliptic fixed point while passing through the microwave beam: α will first increase, and then decrease with time in crossing the beam. While α is increasing, electrons are pulled through the hyperbolic fixed point into the trapped region. When α decreases, the electrons are forced back out. This results in a mixing of the phase space within δP of P_r. The fraction of electrons that participate in these oscillations is of order $\delta P(m_e c^2/T_e)$. Hence, the mean energy gain is

$$\langle\Delta\epsilon_{NL1}\rangle/T_e \sim (m_e c^2/T_e)^3\delta P^3 \sim p_1^{3-3q/2},$$

the power absorbed is $P_a^{(NL1)} \sim n_e v_e T_e h d_L p_1^{3-3q/2}$, and the opacity is given by

$$\tau_{NL1} \sim \tau_L p_2/p_1^{1-q/2}.$$

The strongly nonlinear regime of wave amplitude corresponds to $p_1 > \max(1,p_2)$. The width of the trapped region in phase space is large compared to the electron temperature. Hence, essentially all of the electrons that pass through the microwave beam on flux surfaces in the absorption region are pulled into the trapped region of phase space as they move toward the center of the beam. They are pushed back into the passing region as they leave the microwave beam, but half of them will be expelled above the separatrix, as shown in Fig. 1. When these electrons leave the microwave beam they will have gained an amount of energy $2P_r m_e c^2$. Since electrons on flux surfaces for which $P_r \lesssim \Delta P$ participate in this process, $\langle\Delta\epsilon_{NL2}\rangle/T_e \sim p_1$. Note that the depth of the absorption region is increased by a factor of p_1 relative to the linear absorption depth, d_L. Hence, the absorbed power is $P_a^{(NL2)} \sim n_e v_e T_e h d_L p_1^2$, and the opacity is given by

$$\tau_{NL2} \sim \tau_L p_2/p_1^{2-2q}.$$

We have compared results of the analytic theory with two numerical codes. First, a Monte Carlo code is used to evaluate the energy gain of an ensemble of electrons chosen from a Maxwellian distribution that pass through an intense, Gaussian-shaped rf electric field profile. The Monte Carlo code integrates the equations of motion for the electrons in a specified rf field.[5] The results of a series of runs studying O-mode absorption near the first

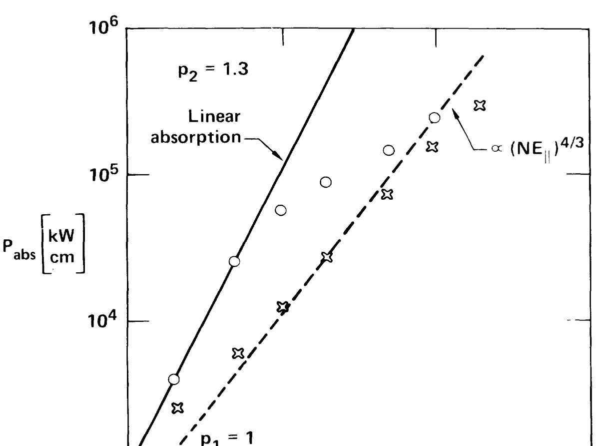

FIG. 2. Power absorbed from Monte Carlo code for $k_\parallel = \partial B_0/\partial s = 0$ corresponding to a minor radius of $r=0$ (crosses), and for $k_\parallel = 1.4 + 0.5(s - s_0)$ cm^{-1} with $\partial B_0/\partial s = 0.003\,663$ T/cm corresponding to $r = 3$ cm in MTX (circles). We use $T_e = 1$ keV, $B_0 = 5$ T, and $n_e = 1 \times 10^{14}$ cm^3. The width along $\mathbf{B}_0$ is $w = 3.7$ cm, yielding $p_2 = 1.3$.

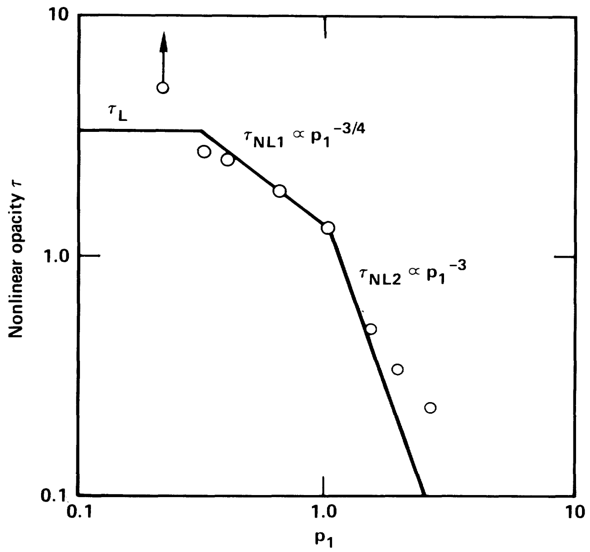

FIG. 3. Nonlinear opacity observed in ZOHAR simulations. The solid lines are the predictions of the theory developed here as corrected for the finite width of the plasma slab. τ is defined as the negative of the natural logarithm of the transmitted power divided by the incident power.

harmonic are summarized in Fig. 2, which shows the absorbed power plotted against $\tilde{E}_{\parallel}$. Linear theory predicts absorbed power increasing as $\tilde{E}_{\parallel}^2$. The data points (crosses) for a uniform B_0 and $k_{\parallel}=0$ show reasonable agreement with the prediction of the strongly nonlinear regime, $P_a^{(NL2)} \sim \tilde{E}_{\parallel}^{4/3}$, where the multiplicative factor, $A=1.6$, has been chosen in plotting the dashed line.

Parallel gradients in $k_{\parallel}$ and B_0 broaden the resonance with the rf fields. This can increase p_2 to give improved absorption. This effect follows from the time dependence of $\tilde{E}$ as seen by an electron,

$$\tilde{E}(\mathbf{x}(t),t) \sim \exp[-i(\omega - l\Omega_{ce} - k_{\parallel}v_{\parallel})t].$$

We assume that $k_{\parallel}(t) \approx k_{\parallel 0} + k'_{\parallel}v_{\parallel}t$, and $\Omega_{ce}(t) \approx (\Omega_0/\gamma)(1+v_{\parallel}t/L_B)$. When the stationary-phase approximation is used to compute the (linear) change in an electron's energy as it passes through resonance, the resonance condition is approximately satisfied for a time

$$\tau_c \sim [(\rho_{\parallel}/L_B) + N'_{\parallel}\rho_{\parallel}(v_{\parallel}/c)]^{-1/2}/l\,\Omega_{ce},$$

where $\rho_{\parallel} \equiv v_{\parallel}/\Omega_{ce}$, and $N'_{\parallel} \equiv k'_{\parallel}c/\omega$. When this time is less than $w/v_{\parallel}$ it should be used in evaluating p_2. This leads to an increase in p_2, and hence an increase in the opacity as illustrated by the circles in Fig. 2.

The second code we have used to simulate the absorption is the two-dimensional, relativistic particle code ZOHAR.[6] This code provides a self-consistent solution of Maxwell's equations and the particle orbit equations. Hence, these simulations include the effects of wave attenuation. Here the O mode was incident on a plasma slab in which the equilibrium magnetic field, $B_0(x)\hat{\mathbf{y}}$, was taken to be in the y direction, and had a linear variation with x. The boundary conditions were periodic in y and open in x (outgoing boundary conditions on the particles and radiation). The predictions of our theory have been compared with these simulations, noting that the width of the plasma slab in the simulations is comparable to the width of linear absorption layer. This reduces the opacity in all absorption regimes (by about a factor of 3 here), and changes the scaling of the opacity in the strongly nonlinear regime to $\tau'_{NL2} \sim \tau_L p_2/p_1^3$. Figure 3 shows a comparison between the opacities observed in the simulations and the predictions of our theory. The parameter p_2 has been held fixed at $p_2=0.4$, while p_1 has been varied from 0.3 to 3.

In this Letter we have studied the effect of electron trapping on the absorption of intense microwave pulses using a plane stratified slab model. Electron trapping always reduces the opacity in this model relative to the linear opacity. However, our study indicates that the nonlinear opacity will still be sufficient in both the MTX and Compact Ignition Tokamak experiments to guarantee essentially complete single-pass absorption. Hence, pulsed FEL's should provide an efficient source of power for heating of electrons in plasma fusion experiments.

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