Squeezing of Spontaneous Emission in a Laser

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It is predicted that a laser's phase-diffusion rate (its Schawlow-Townes linewidth) may be reduced by as much as one-half when the laser is coupled out to "squeezed vacuum" as opposed to ordinary vacuum. The effect is important because it is directly related to spontaneous emission in a squeezed vacuum. It shows that a part of spontaneous emission is due to amplified zero-point noise and that, therefore, in a squeezed field the random phases of the spontaneously emitted photons are no longer uniformly distributed.

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To what extent can spontaneous emission be understood as stimulated emission, stimulated by the vacuum fluctuations? This question received a good deal of theoretical attention several years ago.[1] The conclusion most widely held was that the issue was one of interpretation: The role played by vacuum fluctuations in spontaneous emission seemed to depend on the particular choice of ordering for the radiation field operators, in perturbation-theoretical treatments. Nevertheless, a very persuasive argument was given[2] for a completely symmetric ordering which led to Hermitean operators at every stage of the calculation, and thus to "interpretable" quantities. In this approach the effects of vacuum fluctuations could be understood as arising from the presence of a vacuum field corresponding to an extra half photon per mode (with a totally random phase). Spontaneous emission in a given mode, with n real photons present, appears as a 1 in a factor $n+1$ (that is, as if there was an one extra photon in the field). The result above suggests that stimulated radiation due to vacuum fluctuations could account for *one-half* of spontaneous emission in a given mode, i.e., one half of the extra photon.[3]

Could this stimulated part of spontaneous emission be observable? One distinctive feature of stimulated emission is that it is in phase with the stimulating field. Vacuum fluctuations have no particular phase, nor do the zero-point fluctuations of, for instance, a coherent state (i.e., the fluctuations of the operator $a-\langle a\rangle$). But we know now, both theoretically[4] and experimentally,[5] that it is possible to "squeeze" the vacuum, to produce a state with an anisotropic distribution of phase fluctuations. The possibility immediately arises of studying spontaneous emission in such a field.[6] In particular, one would expect that if there really is a part of spontaneous emission that may be identified with amplified zero-point noise, it should be sensitive to the squeezing of this noise.

The laser is a natural system to investigate these phenomena, for it is based on stimulated emission and (ideally) limited by spontaneous emission. The so-called Schawlow-Townes linewidth is usually attributed to the emission of one spontaneous photon, with random phase, every $1/\gamma$ sec (where γ is the cavity loss rate).[7] If n_0 is the average number of photons in the laser cavity, the average squared phase change induced by the spontaneous photon is $\delta\phi^2=1/2n_0$ giving a phase diffusion rate $D=\Delta\phi^2/\Delta t=\gamma/2n_0$. But if one could influence the phases of the spontaneously emitted photons, this phase diffusion rate would be modified.

In an ordinary laser, vacuum is entering the cavity through the out-coupling mirror. If the foregoing interpretation of spontaneous emission is correct, some of the spontaneously emitted photons result from amplifying the fluctuations of this vacuum (along with those of the intracavity field). Hence the injection of a "squeezed vacuum" should partially bias the phases of the spontaneously emitted photons and modify the laser phase-diffusion rate.

This is precisely the result that this Letter establishes. As shown below, the ultimate linewidth of a laser coupled to a squeezed vacuum (on the other side of the output mirror) may be reduced by as much as a factor of $\frac{1}{2}$ (in the limit of negligible absorption losses and infinite squeezing).

The physical system to be considered is most simply illustrated by the ring laser of Fig. 1, with only one running-wave mode above threshold. The dashed line indicates the mode of the external field which couples to the lasing mode. (The ring arrangement is not really necessary; one could as well couple to a standing-wave cavity by using a polarization rotator and reflection polarizers.)

The conventional laser theory[8-10] needs to be modified to allow for the fact that the external field leaking into the cavity is not ordinary vacuum. As Yamamoto, Machida, and Nilsson[11] suggest, one may write a quantum Langevin equation for the intracavity field as follows:

$$da/dt=-\tfrac{1}{2}[\gamma-\alpha(N_2-N_1)]a+G_{at}(t)+F_a(t)+F_t(t). \quad (1)$$

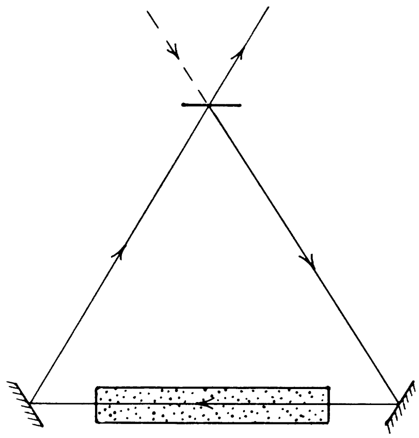

FIG. 1. Experimental arrangement considered: A single-mode ring laser coupled to the outside world by a partially transmitting mirror. The dashed line indicates the mode of the external field which couples to the intracavity field.

Here N_2 and N_1 are the operators for the populations of the upper and lower lasing levels, respectively, and G_{at}, F_a, and F_t are Langevin-force operators, associated respectively with the atomic dipole (after adiabatic elimination of the same; see Lax and Louisell[9]), the absorption (and other essentially irreversible) losses for the radiation field, and the transmission losses. The constant α is a linear gain coefficient ("gain rate per atom").

All the Langevin operators in (1) commute with one another at all times. The correlation function for G_{at} is[9]

$$\langle G_{at}^\dagger(t)G_{at}(t')\rangle = \alpha N_2\delta(t-t'), \tag{2a}$$

$$\langle G_{at}(t)G_{at}^\dagger(t')\rangle = \alpha N_1\delta(t-t'), \tag{2b}$$

while $\langle G_{at}(t)\rangle = \langle G_{at}(t)G_{at}(t')\rangle = 0$. The operators F_a and F_t may be written in terms of boson annihilation operators $c(\omega)$ and $b(\omega)$ as follows[12]:

$$F_a = \left(\frac{\gamma_a}{2\pi}\right)^{1/2}\int d\omega\, e^{-i(\omega-\omega_0)t}c(\omega), \tag{3a}$$

$$F_t = \left(\frac{\gamma_t}{2\pi}\right)^{1/2}\int d\omega\, e^{-i(\omega-\omega_0)t}b(\omega), \tag{3b}$$

where ω_0 is the nominal frequency of the laser field, and γ_a and γ_t are the cavity loss rates associated with the absorption and transmission losses, respectively.

The field described by the operators $c(\omega)$ is taken to be a pure vacuum (more properly, a thermal field, but the number of thermal photons at optical frequencies at room temperature is small enough to be negligible). The field described by the $b(\omega)$, on the other hand, is the outside field (dashed line in Fig. 1) being transmitted into the cavity through the out coupling mirror, and it may be a squeeze field.

Assume, indeed, that the b modes are uniformly squeezed vacuum (with a bandwidth larger than the cavity γ) so that their state is given by[13]

$$|\psi_b\rangle = \prod_{\epsilon>0}\exp[-re^{2i\theta}b(\omega_0+\epsilon)b(\omega_0-\epsilon)+re^{-2i\theta}b^\dagger(\omega_0+\epsilon)b^\dagger(\omega_0-\epsilon)]\,|0\rangle, \tag{4}$$

where r is a real, positive "squeeze parameter." Then the correlation functions for the Langevin operator F_t are

$$\langle F_t^\dagger(t)F_t(t')\rangle = \gamma_t\sinh^2 r\,\delta(t-t'), \tag{5a}$$

$$\langle F_t(t)F_t^\dagger(t')\rangle = \gamma_t(\sinh^2 r+1)\delta(t-t'), \tag{5b}$$

$$\langle F_t(t)F_t(t')\rangle = \gamma_t(\sinh r\cosh r)e^{-2i\theta}\delta(t-t'), \tag{5c}$$

$$\langle F_t^\dagger(t)F_t^\dagger(t')\rangle = \gamma_t(\sinh r\cosh r)e^{2i\theta}\delta(t-t'). \tag{5d}$$

Equations (5a) and (5b) are the same that would hold for a thermal field with an average number of photons $\bar{n}=\sinh^2 r$. [In particular, F_a is taken to satisfy (5a) and (5b) with $r=0$ ($\bar{n}=0$).] But Eqs. (5c) and (5d) are unique to the squeezed-state input and exhibit its phase-sensitive nature: For a field with random uniformly distributed phase fluctuations (such as F_a) the right-hand sides of Eqs. (5c) and (5d) would be zero.

The Langevin equations for the field a [Eq. (1)] and the populations N_1 and N_2 (see Ref. 9) may now be solved near the steady state by a quasilinearization procedure (as in, e.g., Ref. 8, Chap. 20) to obtain the phase diffusion rate. The result is

$$D = \langle(\Delta\phi)^2\rangle/\Delta t = (4n_0)^{-1}[\alpha(N_{20}+N_{10})+\gamma_a+\gamma_t+2\gamma_t\sinh^2 r-2\gamma_t\sinh r\cosh r\cos(2\phi-2\theta)] \tag{6}$$

(the subscript 0 denotes steady-state average values).

Together, γ_a and γ_t makeup the total loss rate γ of the cavity, and in steady state $\alpha(N_{20}-N_{10})=\gamma_a+\gamma_t$, so that, for large inversion ($N_{20}\gg N_{10}$) and with no squeezing ($r=0$) the second and third terms of (6) add up to equal the first one and give a total diffusion rate of $D\simeq\gamma/4n_0+\gamma/4n_0=\gamma/2n_0$. But now suppose that the squeezed light is shined upon the cavity. Choosing the phase of the squeezed field $\theta=\phi$ or $\theta=\phi+\pi$ yields

$$D \simeq (4n_0)^{-1}[\gamma+\gamma_a+\gamma_t e^{-2r}]. \tag{7}$$

If $\gamma_a \ll \gamma_t$, then $\gamma \simeq \gamma_t$, and the possibility of reducing the laser linewidth D by essentially a factor of $\frac{1}{2}$ is apparent from Eq. (7).

The same result may be established in a somewhat more formal manner by making use of the master equation for the intracavity field density operator and the associated Fokker-Planck equation. Gardiner and Collett[12] have shown how the loss-related part of the master equation is written with a squeezed input. For the state (4)

$$(\partial\rho/\partial t)_t = \tfrac{1}{2}\gamma_t(1+\sinh^2 r)(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) + \tfrac{1}{2}\gamma_t \sinh^2 r(2a^\dagger \rho a - aa^\dagger\rho - \rho a a^\dagger) - \tfrac{1}{2}\gamma_t \sinh r \cosh r[e^{-2i\theta}(a^\dagger\rho a^\dagger - a^\dagger a^\dagger \rho - \rho a^\dagger a^\dagger) + \text{H.c.}]. \tag{8}$$

(The subscript t is used to denote the part of $\partial\rho/\partial t$ due to transmission.) The corresponding part of the Fokker-Planck equation for the function $P(\alpha)$ may be seen to contain the phase diffusion term

$$(\gamma_t/4n)(\partial^2/\partial\phi^2)\{[\sinh^2 r - \sinh r\cosh r\cos(2\phi-2\theta)]P(n,\theta)\} \tag{9}$$

(where the coherent-state amplitude α has been written as $\alpha=n^{1/2}e^{-i\phi}$). Note that none of these terms is present when the squeezed vacuum is replaced by ordinary vacuum ($r \to 0$); the vacuum fluctuations (which are entering the cavity all the time, through the out coupling mirror) do not explicitly show up in the Fokker-Planck equation. This is consistent with the fact that the P representation is appropriate for normally ordered operators, and when normal ordering is used vacuum fluctuations tend to drop from the picture.[1]

When squeezing is present, however, the diffusion coefficient in Eq. (9) may add to, or subtract from, the usual phase diffusion coefficient $a(N_{20}-N_{10})/4n_0$, to yield the same result expressed by Eq. (6). The interpretation of this result has already been suggested: The squeezed state has less noise in its phase quadrature than the vacuum. This causes the spontaneously emitted photons to have phases preferentially near θ or $\theta+\pi$. When $\phi=\theta$ or $\theta+\pi$, this slows down the laser phase diffusion process. Note that the actual rate of spontaneous emission is still equal to γ to a very good approximation, since the steady-state condition saturated gain=loss is not modified much by the relatively small number of photons ($\sinh^2 r$) entering now through the output mirror.

Why is only a reduction of one-half achievable? It may be seen from Eq. (1) that in the absence of the gain medium the field in the cavity would become as squeezed as the outside field; for infinite squeezing, it would have a perfectly well-defined phase (up to an additive π). If spontaneous emission were only amplified zero-point noise, no phase diffusion should result. The residual phase diffusion stems from the addition, at a rate γ, of about half a photon's worth of phase-insensitive noise —the amplifier's "added noise," in the language of Caves's paper[14]—which may be said to constitute the other half of spontaneous emission; this is contained in the operator G_{at} in Eq. (1).

From an experimental point of view, the main signature of the effect described here might be its dependence on the phase θ of the squeezed field (see Fig. 2). This might suffice to distinguish it from, e.g., injection locking. In practice, the possibility of injection locking taking place in this system must be considered, because it is likely to occur if the external field is not pure "squeezed vacuum," but has a small coherent component (in other words, if $\langle F_t \rangle \neq 0$). Even when $\langle F_t \rangle = 0$, a careful analysis reveals the existence of a phase-locking term[15] with a characteristic locking time $\tau \simeq 2n_0 e^{-2r}/\gamma_t$. In the presence of phase locking however, the change in the phase diffusion rate could still be observed by looking at the evolution of the system in times shorter than the

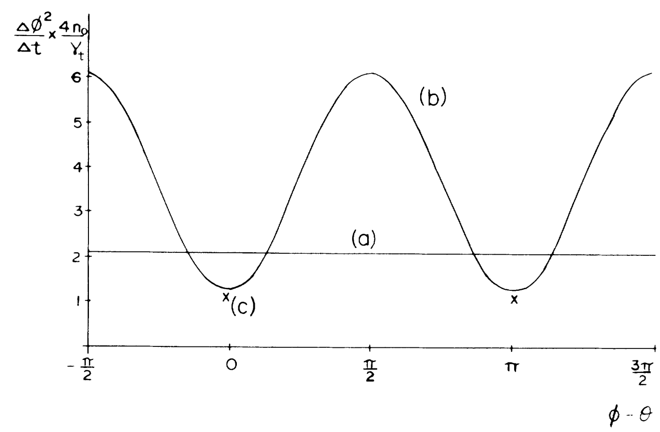

FIG. 2. Laser linewidth (phase-diffusion rate) $D=\Delta\phi^2/\Delta t$, plotted in units of $\gamma_t/4n_0$, for a hypothetical case with $\gamma_a = 0.05\gamma_t$, as a function of the phase difference between the squeezed external field and the intracavity field. The line a shows the usual Schawlow-Townes linewidth for zero squeezing (laser coupled to ordinary vacuum). Curve b is drawn for the case when the external field is 80% squeezed ($e^{-2r}=0.2$). The cross c shows the theoretically possible reduction of the linewidth for infinite squeezing and zero absorption losses. The minima of curve b correspond to a 38% reduction below the Schawlow-Townes linewidth.

locking time τ: In normal injection locking the rate of diffusion for times shorter than τ is still essentially the Schawlow-Townes linewidth, whereas here it could be almost twice as small.

Alternatively, one might consider driving the squeezing device with the laser field itself, so that θ in Eqs. (6) or (9) is actually determined by the instantaneous value of ϕ. The detailed theory of such an arrangement remains to be worked out, but it seems clear that no locking of the phase should take place under these conditions.

Two concluding remarks: The condition $\gamma_a \ll \gamma_t$ is essential to achieve the linewidth reductions discussed here. Transmission losses are controllable and can be made phase sensitive [see Eqs. (5c) and (5d)], while absorption losses are typically phase insensitive and represent therefore the true ultimate limit to any reduction in the system's fluctuations[16] (unless one is interested in measurement times smaller than γ_a^{-1}, that is).

The effect predicted here is different from the one discussed in Ref. 6 (inhibition of dipole decay). Indeed, for the purpose of this paper the atomic dipole may be assumed to decay at its usual rate, since only one plane-wave mode of the field is squeezed (Ref. 6 then shows the change in the decay rate to be negligible). Yet, precisely because of this approximately one-dimensional situation (only the spontaneous emission in the lasing mode contributes to the laser linewidth), this effect might be more readily observable than the inhibition of dipole decay of Ref. 6.

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