First Observation of a Near-Harmonic Vibrational Nucleus

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Evidence is presented for five closely spaced states in ^{118}Cd near $E_{ex} \simeq 2$ MeV, which are interpreted as a near-harmonic three-phonon quintuplet. Candidates for even higher-lying multiphonon states are also found. The experimental spectrum is compared with an anharmonic vibrator [or U(5) spectrum] and an interacting-boson-approximation calculation incorporating a two-particle, four-hole intruder configuration.

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The geometrical concepts of the spherical vibrator[1] and axially deformed rotor[2] are two benchmarks of nuclear-structure physics. The excitations of the former show equally spaced, degenerate phonon multiplets, whereas the structure for deformed nuclei is rotational in character. Although evidence abounds for deformed nuclei, the existence of spherical nuclei with nearly harmonic vibrational structure has come to be viewed with some degree of skepticism due to the lack of experimental support.[3] While many even-even nuclei near closed shells have an $(E_{4^+})_1/(E_{2^+})_1$ ratio of ≈ 2, the expected, closely spaced 0^+, 2^+, and 4^+ triplet of two-phonon states is seldom observed.[4] Evidence for three- and more-phonon levels is virtually nonexistent. The best known candidate for a vibrational character may be ^{102}Ru, but even here, the levels of the three-phonon quintuplet are rather widely spaced and the interconnecting $E2$ transitions strongly violate the $\Delta N_{ph} = \pm 1$ selection rule. The Cd nuclei with $Z=48$ have also traditionally been considered as possible examples of vibrational structure. However, their low-lying levels display extra degrees of freedom. In fact, in ^{112,114}Cd, there are five states[5] at the energy of the two-phonon triplet. Recently, these extra states have been successfully interpreted[6] in terms of the coexistence and mixing of normal vibrational states with two-particle, four-hole ($2p$-$4h$) intruder states which arise from the excitation of a pair of protons across the $Z=50$ shell gap. The result is a V-shaped energy systematics for the intruding configuration with a minimum near midshell and a rise thereafter.[6]

The testing of these ideas was a primary motivation for the detailed study[7] of intruder states in two neutron-rich isotopes of Cd ($A=118,120$), where it was indeed found that the intruding configuration is higher in energy than in ^{112,114}Cd. Similar results have been obtained[8] for a series of Pb nuclei. The interesting point for the present purpose is that, in ^{118}Cd, the lowest intruder level lies well above the two-phonon triplet of states. Thus, its mixing with the normal states is reduced, thereby providing a unique opportunity to isolate the latter and to test their description in terms of a nearly unperturbed vibrational structure.

The purpose of this Letter is, therefore, to report on the first observation of a set of closely spaced levels which seem to represent a complete, nearly harmonic, three-phonon multiplet and to discuss the vibrational structure of ^{118}Cd.

The Cd nuclei were studied from the decay of Ag nuclei produced at the isotope separator TRISTAN at Brookhaven National Laboratory by fission of enriched ^{235}U in a modified Febiad-type[9] ion source. The radioactive beam of fission fragments is ionized, extracted, mass separated, and electrostatically focused onto a movable aluminized-Mylar tape system at a suitable counting station. The measurements included γ-ray singles, γ-γ coincidences, β singles, β-γ coincidences, γ-γ angular correlations, and γ multispectrum scaling. The resulting coincidence relations were used to construct extensive level schemes; the angular correlations were used to assign level spins and the singles γ-ray data to extract transition branching ratios. Full details will be published separately: The main emphasis here is on the five closely spaced states found near 2 MeV in ^{118}Cd. Especially important for these were the angular correlation measurements which were made by a multidetector technique[10] with four Ge detectors positioned to provide six different pair combinations at angles of 90°, 105°, 120°, 135°, 150°, and 165°. Examples of the correlations analyzed by the arctanδ method[11] are shown in Fig. 1. For the

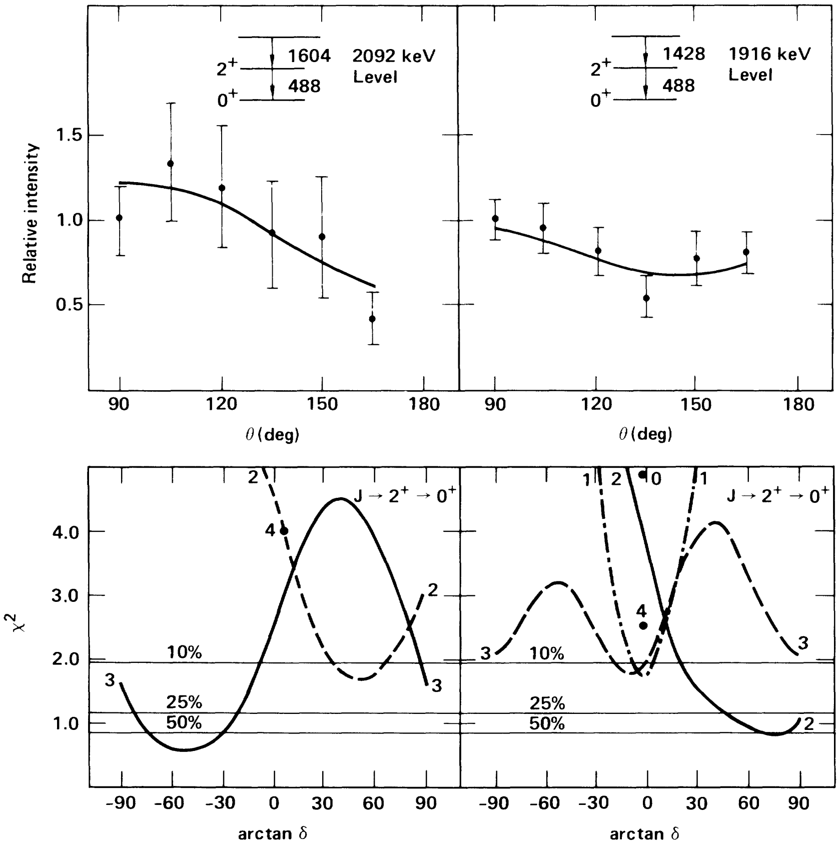

FIG. 1. Top: Two angular correlations in ^{118}Cd (normalized to unity at 90°). Solid line: Theoretical correlation for the adopted spin sequence and mixing ratio. Bottom: χ^2 values for fits with different spin sequences and δ values. (Horizontal lines give various confidence limits.) δ is the ratio $(L+1)/L$, where L is the highest multipole in a given $J_1 \rightarrow J_f$ transition. Single points are shown at arctan$\delta=0$, where $L+1$ is greater than 2. The point for 0^+ is off scale for the 1604-488-0 cascade.

1428-488-0 cascade the χ^2 minimum occurs for $J=2$ at arctan$\delta=75°$, corresponding to a transition with 79% $E2$ and 21% $M1$ mixed multipolarity and a 2^+ assignment for the 1916-keV level. The 1604-488-0 cascade is seen to have a minimum in χ^2 for $J=3$ at arctanδ $=-53°$, yielding a transition multipolarity with 57% $E2$ and 43% $M1$, and a 3+ assignment for the level at 2092 keV. Similarly, the 2074-keV level can be assigned as 0^+. The correlation data for the 1929-keV level allow $J=3,4$. If $J=4$, the χ^2 minimum is at arctan$\delta=33°$ (39% $E2$). If $J=3$, the data give pure $E2$ multipolarity. The measured correlation of the 1936-keV level does not allow any effective spin discrimination but, when it is combined with the γ-multiscaling data, J^π can be restricted to 5^+, 6^+, and, less likely, 4^+.

In summary, we have found a quintuplet of levels in ^{118}Cd near 2 MeV, three of which have definitive assignments of 0^+, 2^+, and 3^+. The spins of the remaining two levels at 1929 and 1936 keV are less certain but consistent with the assignments 4^+ and 6^+, respectively, which we shall assume for the remainder of our discussion. The level scheme from this study for ^{118}Cd is shown in Fig. 2. The most remarkable feature is the observed clustering of levels, a 2^+ state at 488 keV, a closely spaced triplet of 4^+, 2^+, and 0^+ levels at 1165, 1270, and 1286 keV, respectively, a rather isolated intruder 0^+ state at 1615 keV, and a set of five closely spaced levels near 2 MeV.

In addition to the close spacing of the latter group, there is a strong preference for decay to the two-phonon triplet levels rather than to the one-phonon 2^+ state. For example, the $E2$ branching ratios for the 1916-keV 2^+ and 1929-keV 4^+ levels are

$$B(E2;2^+ \rightarrow 2^+_{2\mathrm{ph}})/B(E2;2^+_{1\mathrm{ph}})=33,$$

$$B(E2;4^+ \rightarrow 2^+_{2\mathrm{ph}})/B(E2;4^+ \rightarrow 2^+_{1\mathrm{ph}})=100.$$

A broader view of this dominance is shown on the right-hand side of Fig. 2 by means of the average $E2$ branching ratios for the triplet and quintuplet levels. The description of these levels as rather pure two-and-three-phonon multiplets is thus strongly supported. A more detailed test of the vibrational wave functions is the relative $E2$ branching ratios of allowed transitions. Both the

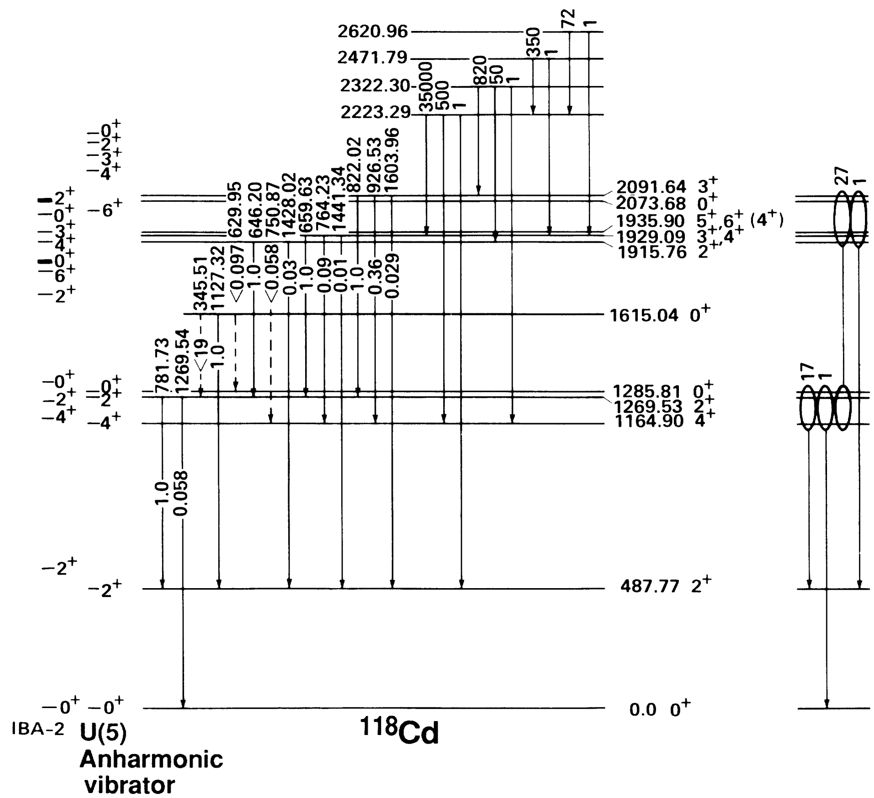

FIG. 2. Partial level scheme for ^{118}Cd: Transition arrows and relative $B(E2)$ branching ratios are indicated on the vertical lines for levels with more than one observed decay route. Upper limits are shown for (dashed) unobserved transitions. A few of the unusual decay routes of higher lying states are also included. At the left are comparisons with IBA-2 calculations (Ref. 12) and with the identical predictions of the U(5) limit (Ref. 13) of IBA-1 and an anharmonic vibrator (Ref. 14). The thicker lines for the IBA-2 calculations are the intruder levels. To the right are the average of the $B(E2)$ ratios showing the preferred depopulation of the three- and two-phonon states by $\Delta N_{\rm ph}=1$ transitions.

1929- and 2092-keV levels decay by two γ rays to members of the two-phonon triplet. For the latter, the empirical branching ratio of 2.78 is in good agreement with the pure vibrator[15] value $B(E2;3_1^+\rightarrow 2_2^+)/B(E2;3_1^+\rightarrow 4_1^+)=2.5$ but for the 1929-keV level, the empirical ratio $B(E2;4_2^+\rightarrow 2_2^+)/B(E2;4_2^+\rightarrow 4_1^+)$ is 10.6 in disagreement with the predicted value of 1.1.

Since the multiplet levels in ^{118}Cd are closely spaced but not rigorously degenerate, it is interesting to compare the observed level structure with an anharmonic-vibrator model. The eigenvalue equation of the U(5) interacting-boson-approximation (IBA) Hamiltonian is[13]

$$E=\epsilon n_d+\tfrac{1}{2}\alpha n_d(n_d-1)+\beta(n_d-v)(n_d+v+3)+\gamma[L(L+1)-6n_d], \qquad (1)$$

where ϵ, α, β, and γ are parameters, and n_d and v are quantum numbers giving the total number of d bosons and the number of d bosons *not* pairwise coupled to zero angular momentum, respectively. This equation permits much flexibility and highly anharmonic level sequences can be produced. Nevertheless, although the U(5) limit encompasses a rich variety of possibilities, one can write down universal expressions for the $n_d=3$ (or three-phonon quintuplet) level in terms of those for the $n_d=1$ and 2 (one- and two-phonon) states, that are parameter *independent*. These are given in Table I. It is also possible to introduce anharmonicities in the geometric vibrational model. In Ref. 14, the Hamiltonian $H=H_o+H_c+H_q$ was studied where H_o is the harmonic vibrator Hamiltonian, and H_c and H_q represent the most general set of cubic and quartic anharmonic terms, which are treated in lowest-order perturbation theory. Again, two-phonon anharmonicities can be used to calculate energies of the three-phonon quintuplet. It is useful to emphasize the interesting result, not widely known, that the anharmonic-vibrator predictions are *precisely* the same as for the U(5) symmetry (Table I). A comparison of these U(5) and anharmonic-vibrator predictions with the data is shown in Fig. 2. The predictions are in qualitative but clearly not quantitative agreement with the observed excitation structure, indicating that the normal states of ^{118}Cd cannot be quite so simply described.

Detailed calculations in the configuration-mixing approach[16] in the IBA-2 have also been performed[12] for

TABLE I. Energies of the three-phonon quintuplet in the U(5) limit of the IBA and also in the anharmonic vibrator of Ref. 14.

n_d	L^π	Energies
3	6_1^+	$3E(4_1^+)-3E(2_1^+)$
3	4_2^+	$-3E(2_1^+)+\frac{10}{7}E(4_1^+)+\frac{11}{7}E(2_2^+)$
3	3_1^+	$-3E(2_1^+)+\frac{6}{7}E(4_1^+)+\frac{15}{7}E(2_2^+)$
3	2_3^+	$-3E(2_1^+)+\frac{36}{35}E(4_1^+)+\frac{4}{7}E(2_2^+)+\frac{7}{5}E(0_2^+)$
3	0_3^+	$3E(2_2^+)-3E(2_1^+)$

^{118}Cd. Since $^{118}_{48}\mathrm{Cd}_{70}$ has 12 valence-neutron holes, $N_\nu=6$. Similarly, for the normal states, $N_\pi=1$, while for the two-particle, four-hole intruder levels, $N_\pi=3$. Thus, the Hamiltonian is

$$H=H_{N_\pi=1}+H_{N_\pi=3}+H_{\mathrm{mix}},$$

where the first two terms are standard IBA-2 Hamiltonians for different N_π values. The resulting energies are also shown in Fig. 2. The 2_1^+ state and two-phonon triplet levels are in good agreement with experiment. The third calculated state at 1.831 MeV has a large $N_\pi=3$ component and is identified as the intruder. It is within ≈ 200 keV of the observed energy. Also in agreement with experiment is the predicted splitting of the quintuplet levels into two groups, the lower consisting of 6^+, 4^+, 2^+, and the higher one containing 0^+, 3^+.

Finally, there is one other surprising feature of the empirical ^{118}Cd level scheme. Above the quintuplet, there are several levels whose relative decay favors population of quintuplet levels by orders of magnitude. For example, on the assumption of pure $E2$ multipolarities, Fig. 2 shows that the 2223- and 2322-keV levels populate levels of the quintuplet by low-energy γ rays whose relative $B(E2)$ values dominate the higher-energy decay transitions by factors of 35000 and 820. This gives rise to the speculation that these two levels may even contain amplitudes for a four-phonon structure. In addition, there are levels at still higher excitation energies which, in turn, show preferential decay to these levels. These low-energy decay routes are very unusual and defy any standard interpretation. (Note that, if they are not $E2$ but rather $M1$ with an E_γ^3 rather than E_γ^5 energy dependence, their dominance is numerically smaller but no less puzzling.)

In conclusion, the identification of the set of five states in ^{118}Cd, with a centroid at 1989 keV, with appropriate spins, and with their preferential decay routes, suggests that they comprise a nearly harmonic three-phonon excitation. Possible evidence for higher phonon excitations was also presented. These observations provide strong evidence, three decades after the original suggestion, that, well-developed nuclear vibrational structure can indeed exist and point to ^{118}Cd as the first well-defined example of such behavior.

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