## Not-So-Resonant, Resonant Absorption

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When an intense electromagnetic wave is incident obliquely on a sharply bounded overdense plasma, strong energy absorption can be accounted for by the electrons that are dragged into the vacuum and sent back into the plasma with velocities  $v \approx v_{osc}$ . This mechanism is more efficient than usual resonant absorption for  $v_{osc}/\omega > L$ , with L being the density gradient length. In the very high-intensity CO<sub>2</sub>-laser-target interaction, this mechanism may account for most of the energy absorption.

PACS numbers: 52.50.Jm, 52.25.Wz, 52.75.Di

When a very intense laser field is incident obliquely on a metallic surface or a sharply bounded overdense plasma, a large absorption rate can be accounted for by electrons that are pulled into the vacuum and sent back into the plasma with  $v \simeq v_{\rm osc}$ , with  $v_{\rm osc} = eE/m_e\omega$  being the quiver velocity. This mechanism is effective when the overdense plasma density is well over the critical density and when a strong density gradient or discontinuity is present; the presence of an underdense plasma is not necessary. Those conditions are met in the laser-grating accelerator concept,<sup>1</sup> where an intense laser field incident on a grating is used to accelerate electrons. For this purpose, high-power  $CO_2$  lasers are now available<sup>2</sup> that can deliver up to 100 mJ in a time scale of 1 ps: A short enough time scale is necessary if we want to maximize the electric field without damaging the grating surface.<sup>3</sup> I find that the present mechanism has a strong limiting effect on the magnitude that the electric field normal to a grating can reach without significant absorption of the wave energy. In the laser inertial fusion context in the very-high intensity regime (i.e.,  $I\lambda^2 \gtrsim 10^{16}$  W  $\mu m^2/cm^2$ ), a strong density steepening with an overdense region where the density may be as large as  $40n_0$  ( $n_0$  being the critical density) arises in the region around critical density.<sup>4</sup> Under those conditions, the present mecha-

$$E(x_l=0) = E_{\text{ext}}(t_l) + \Delta E_l = 0 \text{ or } \Delta E_l = -E_{\text{ext}}(t_l),$$

since we have a null field at x = 0, and  $x_i = 0$  at  $t = t_i$ .

From the equation of motion,  $dv_l/dt = -eE/m_e$ , one can integrate to obtain the exact velocity and position,

$$v_l = v_{\rm osc}(\cos\omega t - \cos\omega t_l) + \omega v_{\rm osc}(t - t_l)\sin\omega t_l \quad , \tag{4}$$

$$x_l = (v_{\rm osc}/\omega)(\sin\omega t - \sin\omega t_l) - v_{\rm osc}(t - t_l)\cos\omega t_l + \frac{1}{2}\omega v_{\rm osc}(t - t_l)^2\sin\omega t_l,$$
(5)

if  $W_{abs}$  is written as

with  $v_{\rm osc} = eE_0/m\omega$ .

An evaluation of the absorbed energy per cycle can be found by solving numerically for the velocity of reentry; i.e., in Eq. (5) I solve for  $t_l$  at any given t when  $x_l=0$ and substitute it into Eq. (4) to obtain  $v_{l0}=v_l(x_l=0)$ . The absorbed energy over one cycle is given by

$$W_{\rm abs} \simeq \int_{\pi/\omega}^{5\pi/2\omega} (\frac{1}{2} m_e v_{l0}^2) (n v_{l,0} dt)$$

nism may become more efficient than resonant absorption<sup>5,6</sup> to absorb energy. Also, some laser fusion schemes involving a magnetic field<sup>7</sup> may become more attractive since, with this new mechanism, the energy absorption takes place directly at the focal spot (the dragged electron goes back to the overdense plasma in about one cycle).

To describe the energy absorption mechanism, let us consider a one-dimensional capacitor model where we have at  $x \ge 0$  a perfect conductor which can freely emit electrons. For x < 0 I assume a vacuum region where a uniform electric field  $E_{\text{ext}} = E_0 \sin \omega t$  is present in the x direction. As the field builds up for t > 0, electrons will be pulled out in order to maintain a zero field on the conductor surface at x = 0. The *l*th particle that is emitted at time  $t = t_l$  will see a field

$$E(x_l) = E_{\text{ext}} + \Delta E_l, \tag{1}$$

where

$$\Delta E_{l} = -4\pi e \int_{x_{1}(t)}^{x_{l}(t)} n \, dx \tag{2}$$

comes from Poisson's equation with *n* being the electron density and  $x_1(t)$  and  $x_l(t)$  are the positions of the first (left-most) particle and *l*th particle, respectively. Since the particles cannot overtake one another,  $\Delta E_l$  is constant in time and can be determined initially by

(3)

The density *n* can be found by differentiating Eqs. (5) and (2) with respect to  $t_l$ , and I obtain  $n=2n_0/(\omega t -\omega t_l)^2$ , where  $n_0$  is the critical density defined as  $4\pi n_0 e^2/m_e = \omega^2$ . The integral is done numerically, and

$$W_{\rm abs} = \eta \, \frac{1}{2} \, N m_e v_{\rm osc}^2, \tag{6}$$

where  $N = E_0/4\pi e$  is the maximum number of electrons drawn into the vacuum per cycle, we find  $\eta = 1.57$ . To find the absorbed power, I divide  $W_{abs}$ , which can also be written as  $W_{abs} = \eta (v_{osc}/\omega) E_0^2/8\pi$ , by the time period  $2\pi/\omega$ ; I obtain

$$I_{\rm abs} = (\eta/2\pi) v_{\rm osc} E_0^2 / 8\pi.$$
 (7)

To compare with "classical" resonant absorption, <sup>5,6</sup> one may note (using Poisson's equation) that the minimum length necessary to shield the electrostatic field for a plasma at critical density is  $v_{osc}/\omega$ ; therefore, within that distance the electric field has to be monotonic since the plasma is nonneutral. One can see that if  $v_{osc}/\omega > L$ , where  $L = (\partial \ln n/\partial x)^{-1}$ , the usual resonant absorption as given by Refs. 5 and 6 cannot take place. From a different point of view, the maximum value of the field  $E_p$  in resonant absorption is given by  $v_p = (2v_{osc}L\omega)^{1/2}$ , where  $v_p = eE_p/m\omega$ ; therefore, for  $v_{osc}/\omega > 2L$ , the maximum field in the resonant absorption mechanism becomes less than the pump field itself.

From an energetic point of view, the maximum energy stored in the resonant field is  $W_r = (E_p^2/8\pi)l$ , where the length *l* at wave breaking is approximately  $l = v_p/\omega$ . If all this energy is lost, it will take a time<sup>6</sup>  $t_p = (8L/v_{osc}\omega)^{1/2}$  to rebuild. Thus the highest absorption rate is  $I_{abs} = W_r/t_p \simeq (E_0^2/8\pi)L\omega$ . For  $v_{osc}/\omega \gtrsim L$ , the resonant absorption mechanism will absorb less energy than the mechanism already described in the text.

I have performed simulations with a  $1\frac{2}{2}$ -dimensional (i.e., x,  $v_x$ ,  $v_y$ ,  $v_z$ ) electrostatic particle code,<sup>8</sup> and I use an ion to electron mass ratio of 1836. I use nearly the same setup as in the theoretical model in which I have on the left-hand side a vacuum region, while on the righthand side, I define a sharp boundary overdense plasma. I implement throughout the system a capacitor field  $E_{\text{ext}} = E_0 \sin \omega t$ . As time goes on, the electric field will be shielded inside the overdense plasma. In Fig. 1 the electron phase space (i.e.,  $v_x$  vs x) is shown at different times. We see that, as the field is turned on, the electrons are dragged into the vacuum; at later times as the field reverses, the particles reverse their direction and are pushed back into the overdense plasma. At  $t = 2\pi/\omega$ , two-thirds of the particles have reentered the plasma. At a later time, more of those particles go back to the plasma as new particles are being dragged out of the plasma. Since the field is null inside, one can see that the kinetic energy given to the particles that reenter the plasma is lost. To evaluate this lost energy, the particles are absorbed as they reach the right-hand-side boundary and their energy is added.

Several runs have been made for  $\omega/\omega_p = 0.1, 0.2$ , over few (approximately six) initial cycles, and I varied  $v_T/v_{osc}$  from 0.05 to 0.2;  $\omega_p$  and  $v_T$  are the electron plasma frequency and the electron thermal velocity (I start with a Maxwellian distribution function) of the overdense plasma, respectively. I find that  $\eta$  is a function of



FIG. 1 Electron phase space at  $t = (a) \pi/2\omega$ , (b)  $\pi/\omega$ , (c)  $3\pi/2\omega$ , (d)  $2\pi/\omega$ , and (e)  $5\pi/2\omega$ .

 $v_T/v_{osc}$  and a good fit is  $\eta = 1.75(1 + 2v_T/v_{osc})$ ; I find little variation with respect to  $\omega/\omega_p$  in the range under consideration. On that time scale, the ions essentially do not move. In the small-absorption regime, however, enhancement in the absorption is expected as time goes on if an underdense plasma forms and expands; as the density gradient length L increases beyond the excursion length, the "classical" resonant absorption will become the main absorption mechanism.

In the case where we have a *p*-polarized electromagnetic wave incident with an angle  $\theta$  from the normal, its absorption through the present mechanism can be evaluated when no resonance effects are present (as in the case with no corona). Since the incident power is given by  $I_{in} = c (E_L^2/8\pi) \cos\theta$ ,  $E_L$  being the laser field, the ratio f of the absorbed power, given by Eq. (7), to the incident one is

$$f = (\eta/2\pi) \left( v_{\rm osc}^3 / v_L^2 c \cos\theta \right), \tag{8}$$

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FIG. 2. (a) f vs I: Solid curves are from Eqs. (8) and (9) with  $\eta = 1.57$  (lower one) and  $\eta = 1.75(1 + 2v_T/v_{osc})$  (upper one);  $v_T$  corresponds to a cold-electron temperature of 250 eV; dashed curves are from relativistic Eq. (10) instead of Eq. (8); experimental results are reproduced from Ref. 9. (b) Hot electron temperature vs I: Solid curve is from our model; dashed curve is from Ref. 10 with  $T_{cold} = 1$  keV; experimental results are reproduced from Ref. 11, filled circle is from Ref. 12.

where  $v_L = eE_L/m\omega$  and  $v_{osc}$  is the quiver velocity due to the electric field  $E_0$  normal to the interface. In the case of a perfect conductor (f=0) or an overdense plasma with  $\omega_p \gg \omega$ , the laws of reflection and refraction give  $E_0 = 2E_L \sin \theta$ . However, at very high intensity, other effects that will saturate the value of f have to be taken into account. The most important one is the pump depletion itself: If a fraction f of the intensity is absorbed, the electric field of the reflected wave is  $E_L(1-f)^{1/2}$ . A good approximation for  $E_0$  is

$$E_0 = E_L [1 + (1 - f)^{1/2}] \sin\theta, \tag{9}$$

which gives the proper limit value for  $f \ll 1$  and  $f \simeq 1$ .

As I still increase the pump, i.e.,  $v_L \gtrsim c$ , relativistic corrections also become important. Equation (6) can be written as  $W_{abs} = \eta N K_e$ , with  $K_e$  being the electron kinetic energy now given by  $K_e = (\gamma - 1)m_ec^2$ . Because the electron velocity scales as  $v_{osc}/\gamma$ , we obtain  $\gamma = (1 + v_{osc}^2/c^2)^{1/2}$ ; one can therefore obtain, in a way similar to Eq. (8),

$$f = (\eta/\pi) c v_{\rm osc} [(1 + v_{\rm osc}^2/c^2)^{1/2} - 1] / (v_L^2 \cos\theta).$$
(10)

I have checked the value of  $W_{abs}$  against  $1\frac{2}{2}$ -dimensional electrostatic particle simulations and I find that  $\eta$  varies very little over a wide range of  $v_{osc}/c$ ; for instance, I obtain  $\eta = 1.66$  for  $v_{osc}/c = 2$  [one should remember that  $v_{osc}$  is only an expression of  $E_0$  as defined below Eq. (5)], and  $v_T/v_{osc} = 0.05$ .

I have assumed that the  $\mathbf{v} \times \mathbf{B}$  term in the Lorentz force can be minimized and neglected if the pump is split into two equal components at equal and opposite angles of incidence, i.e., at  $\pm \theta$ : There will be no net component of the Poynting vector along the surface; and on it the maximum normal electric field positions, where the electrons are dragged out, will correspond to magnetic field nodes.

The absorption coefficient f according to Eqs. (8)– (10) has been plotted in Fig. 2(a) with respect to intensity for the special case where  $\theta = 45^{\circ}$ . This also corresponds to the case of a uniform irradiation with respect to  $\theta$ , with no correlation between pumps from different directions, since the root-mean-square value of  $\sin\theta$  and  $\cos\theta$  would also be  $2^{-1/2}$ . As it is compared with experimental data obtained from Ref. 9, in the case where an intense CO<sub>2</sub> laser is incident on a target, the present mechanism (even with no resonant effect) is observed to account for most of the energy absorption for  $I\lambda^2 \gtrsim 4 \times 10^{16}$  W  $\mu$ m<sup>2</sup>/cm<sup>2</sup>. In that particular regime, even in the long-pulse case, a strong density steepening is observed with a density gradient length equivalent or smaller than the excursion length.<sup>4,9</sup>

In Fig. 2(b) comparisons are made with the hotelectron temperature, as given by the kinetic energy of the electrons in the quiver field  $E_0$  given by Eq. (9), and experimental data from Refs. 11 and 12, where a good agreement is also observed. I have also plotted the hotelectron scaling law due to usual resonant absorption,<sup>10</sup> which is known to break down in that regime.

In the laser-grating acceleration concept, the present mechanism also plays an important role in the damping of a surface wave,<sup>1</sup> if the electric field is strong enough to extract electrons (field emission) from a metallic surface<sup>13</sup> (i.e.,  $E \gtrsim 5 \times 10^5$  V/cm for a static field). For a surface wave that has the fields that decay as  $\exp(-\kappa z)$  from the surface, its energy per unit of surface is  $W_{\text{surf}} = \frac{1}{2} (E_0^2/8\pi)/\kappa$ . Using (7), one finds that the

damping rate of the field is

$$v = \frac{1}{2} I_{\text{abs}} / W_{\text{surf}} = (\eta / 2\pi) v_{\text{osc}} \kappa.$$

The accelerating or longitudinal field is maximized for  $\kappa \simeq \omega/c$ ; so  $v \simeq (\eta/2\pi)(v_{\rm osc}/c)\omega$ . By keeping the electric field such as  $v_{\rm osc}/c \sim 1\%$ , this should keep the average absorption rate  $v/\omega$  below 0.25%, in the case where the ions are not involved and remain bound to the crystalline structure. By extrapolating experimental results from Ref. 3, damage thresholds for polished copper should be approximately 300 mJ/cm<sup>2</sup> for a 1-ps pulse. At  $v_{\rm osc}/c = 1\%$ , one also achieves an electric field of  $3 \times 10^7$  V/cm, which is 200 times larger than the maximum electric field found in present-day accelerators.

The author would like to thank N. Burnett, P. Corkum, N. Sherman, J. Young, and W. L. Kruer for many helpful discussions.

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