Is There Koba-Nielsen-Olesen Scaling at Fermilab Tevatron Collider Energies (1600–2000 GeV)?

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It is shown that the parton branching distribution P_{mn} of m quarks and n gluons does not obey exact Koba-Nielsen-Olesen scaling. When the quark evolution is neglected the probability distribution becomes wider as energy increases in agreement with experimental data. In this model it is predicted that the widening of the probability distribution will stop at Fermilab Tevatron Collider energies (1600–2000 GeV). Also given are theoretical predictions for the multiplicities and moments for Tevatron Collider energies.

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This past year has witnessed an impressive renewal of interest in multiparticle production and particularly in Koba-Nielsen-Olesen (KNO) scaling and its violations.[1] The most recent experimental data[2] indicate that the problem of understanding the shape of the hadronic multiplicity distributions still represents an outstanding problem in strong-interaction physics. The first theoretical contribution to this problem came in 1972 when Koba, Nielsen, and Olesen[3] predicted, on the basis of validity of Feynman scaling for the many-particle inclusive cross section, a scaling law for the probability distribution, namely that

$$\bar{n}\sigma_n/\sum \sigma_n = \psi(z = n/\bar{n}), \qquad (1)$$

where $\bar{n}$ is the mean multiplicity, σ_n is the semi-inclusive n-particle cross section, and $\psi(z)$ is an energy-independent function. KNO scaling also implies that the moments defined as

$$C_q = \langle n^q \rangle / \bar{n}^q \qquad (2)$$

are energy independent. This scaling law was found to be approximately valid up to CERN Intersecting Storage Ring energies,[4] but at CERN collider energies dramatic scaling violations have been observed.[5] Scaling violations of the probability distribution and moments observed by the Alner *et al.* (UA5 group) can be seen in Fig. 1.

In this Letter, I derive the probability distribution, multiplicities, and moments in the parton branching model. I show that the probability distribution P_{mn} of m quarks and n gluons does not obey a KNO scaling law.[6] The violation of the scaling is due to the fact that gluons can produce quarks by converting into quark- and antiquark pairs and quarks can produce gluons by bremsstrahlung. In the limit when the quark evolution is neglected we obtain the *exact analytic* expression for the probability distribution, multiplicities, and moments. I assume that hadron-hadron collision takes place in three steps. In step one, partons from the hadrons collide. Their collisions are assumed to be $2 \rightarrow 2$ processes. There are a total of m_0 quarks and n_0 gluons involved in the collision (since m_0 and n_0 are the average initial numbers of quarks and gluons involved in the collision, m_0 and n_0 need not be integers). After the collision, in step two, these quarks and gluons branch and lose their energy. Finally, in step three, they hadronize. Here we consider steps one and two, and, as usual, I assume that

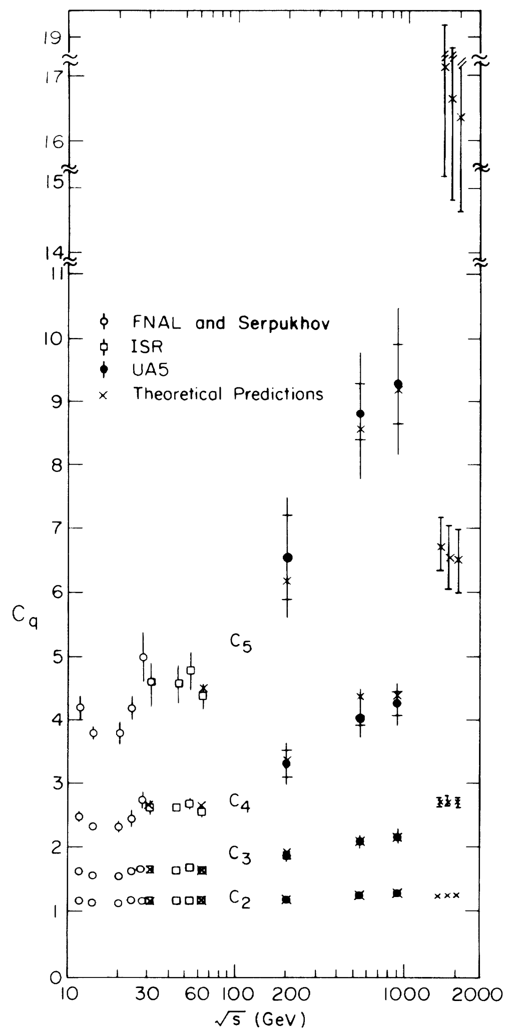

FIG. 1. Theoretical predictions for the moments C_2, C_3, C_4, and C_5 plotted as functions of energy and compared with the experimental data in the energy range 10 GeV $\leq \sqrt{s} \leq$ 900 GeV. Theoretical prediction for the moments C_q for the Fermilab Tevatron Collider energies is included.

the hadronization process does not alter the main features of the hard process. As a result of the fact that as the energy increases, the activity of gluons inside hadrons increases, and the contribution from gluons to the cross section and multiplicities increases with energy,[7] I assume that n_0 increases slowly with energy while m_0 decreases with energy. Therefore at some asymptotic energies only gluon-gluon collisions will contribute to the multiplicities and quarks will just pass through each other. The probability distribution at these energies will be pure gluon branching distribution.[8] This implies that the widening of the distribution and multiplicity moments have upper bounds.

In our parton branching model we consider the following branching processes: $g \to gg$, $q \to qg$, and $g \to q\bar{q}$ with probabilities A, $\tilde{A}$, and B, respectively. In the leading logarithmic approximation these probabilities can be obtained by integration of splitting functions $P_{g \to g} = 2N_c/x$, $P_{q \to g} = (N_c^2 - 1)/N_c x$, and $P_{g \to q} = N_f/2$ over the fractional momenta x. The probability distribution P_{mn} of m quarks and n gluons satisfies the following evolution equation[9]:

$$\partial P_{mn}/\partial t = -AnP_{mn} + A(n-1)P_{m,n-1} - \tilde{A}mP_{mn} + \tilde{A}mP_{m,n-1} - BnP_{mn} + B(n+1)P_{m-2,n-1}, \tag{3}$$

where t is the natural evolution parameter,

$$t = [6/(11N_c - 2N_f)] \ln[\ln(Q^2/\mu^2)/\ln(Q_0^2/\mu^2)]. \tag{4}$$

Q is the initial parton energy, Q_0 is the hadronization energy, and μ is a typical QCD energy scale. If we assume that P_{mn} is a smooth function of m and n, and nP_{mn} (mP_{mn}) varies slowly between n and $n+1$ (m and $m+1$), Eq. (3) becomes a differential equation for the probability $P(m,n,t)$

$$\frac{\partial P(m,n,t)}{\partial t} = -(A-B)P(m,n,t) - [(A-B)n + \tilde{A}m]\frac{\partial P(m,n,t)}{\partial n} - 2Bn\frac{\partial P(m,n,t)}{\partial m} + \ldots . \tag{5}$$

Assuming n_0 initial gluons and m_0 initial quarks, we obtain a *new* nonscaling law for the probability distribution,[6]

$$(\bar{m} - 2B/\lambda^+ \bar{n})(\bar{n} + \tilde{A}/\lambda^+ \bar{m})P(m,n,t) = \psi\left(\frac{m - 2B/\lambda^+ n}{\bar{m} - 2B/\lambda^+ \bar{n}}, \frac{n + \tilde{A}/\lambda^+ m}{\bar{n} + \tilde{A}/\lambda^+ \bar{m}}\right), \tag{6}$$

where

$$\lambda^+ = \tfrac{1}{2}(A-B)\{1 + [1 + 8\tilde{A}B/(A-B)^2]^{1/2}\}.$$

At high energies $\bar{m}/\bar{n} \sim 2B/A \ll 1$ and we can neglect quark evolution ($\bar{m} = m_0 = \text{const}$). The evolution equation (3) then becomes

$$\partial P_n(t)/\partial t = -AnP_n + A(n-1)P_{n-1} - BnP_n + B(n+1)P_{n+1} + \tilde{A}m_0 P_{n-1} - \tilde{A}m_0 P_n. \tag{7}$$

Assuming n_0 initial gluons and m_0 initial quarks, we get the following probability distribution[9]:

$$P_n(t) = \left[1 + \frac{A}{A-B}(e^{(A-B)t} - 1)\right]^{n - n_0 - k} (e^{(A-B)t} - 1)^{n_0 + n} \frac{B^{n_0} A^n}{(A-B)^{n_0+n}} \times \frac{(n + n_0 + k - 1)!}{n!(n_0 + k - 1)!}\, {}_2F_1(-n, -n_0; -n_0 - k + 1, u), \tag{8}$$

where $k = \tilde{A}m_0/A$ and $u = 1 - (A-B)^2 e^{(A-B)t}/AB(e^{(A-B)t} - 1)^2$. It has been shown that this distribution approaches KNO scaling in the large-n and -$\bar{n}$ limit *only* in the case when n_0 and m_0 are energy independent.[6] By rewriting Eq. (3) in terms of the generating function defined as $G(y,t) = \sum y^n P_n(t)$ we can solve this equation with the initial condition $G(y,0) = y^{n_0}$. The solution is

$$G(y,t) = \left[1 - \frac{A}{A-B}(e^{(A-B)t} - 1)(y-1)\right]^k \left[1 + \frac{e^{(A-B)t}(y-1)}{1 - [A/(A+B)](e^{(A-B)t} - 1)(y-1)}\right]^{n_0}. \tag{9}$$

Clearly, the multiplicity $\bar{n}$ and moments C_q can be obtained by taking the qth derivative of the generating function and setting $y = 0$. We get

$$\bar{n} = k\frac{A}{A-B}(e^{(A-B)t} - 1) + n_0 e^{(A-B)t}, \quad C_2 = 1 + \frac{\beta\alpha^2 - n_0(1-\alpha)^2}{\delta^2} + \frac{1}{\bar{n}}\left[1 - \frac{2n_0\alpha\beta}{\delta^2}\right],$$

$$C_3 = 1 + \frac{3[\beta\alpha^2 - n_0(1-\alpha)^2]}{\delta^2} + \frac{2[\alpha^3\beta + n_0(1-\alpha)^3]}{\delta^3} + \frac{1}{\bar{n}}\left[3 + \frac{\beta\alpha(3\alpha - 6n_0) - 3n_0(1-\alpha)^2}{\delta^2} + \frac{6n_0\alpha\beta(1-2\alpha)}{\delta^3}\right]$$
$$+ \frac{1}{\bar{n}^2}\left[1 - \frac{3\alpha n_0\beta(2 + \alpha 2k)}{\delta^2} + \frac{6\alpha^2\beta n_0[n_0\alpha + (1-\alpha)\beta]}{\delta^3}\right] + \frac{1}{\bar{n}^3}\left[\frac{-3\alpha^2 k n_0\beta}{\delta^2} + \frac{2n_0\alpha^3\beta(\beta^2 - n_0^2)}{\delta^3}\right], \tag{10}$$

where $\alpha = A/(A-B)$, $\beta = k + n_0$, and $\delta = k\alpha + n_0$. The analytic expressions for C_4 and C_5 will be displayed elsewhere.[10]

Assuming that the average initial number of gluons n_0 increases slowly with energy while the average initial number of quarks m_0 decreases, I fit the data with the multiplicity moments C_2, C_3, C_4, C_4, and C_5 in the energy range 30.4 GeV $\le \sqrt{s} \le$ 900 GeV (Table I). Since $B \ll A$ at high energies, the parameter α decreases with energy approaching 1 at $\sqrt{s} = 900$ GeV. The fact that the initial number of quarks is large at low energy ($\sqrt{s} \simeq 30$ GeV) indicates that our approximation of neglecting the quark evolution is not good at these energies. We need to consider the probability distribution P_{mn} which is a solution of the evolution equation (3). However, for the high enough energies ($\sqrt{s} \simeq 200$ GeV, $k=3$, $m_0 = 6$) we can safely neglect quark evolution and our distribution describes the data remarkably well. The values for $\bar{n}$ in Table I are taken from the experimental fit[2] ($\bar{n} \simeq 2.7 - 0.03 \ln s + 0.167 \ln^2 s$). In order to satisfy the assumption that k decreases with energy while n_0 increases with energy, we find that the best fits for the moments C_2 are obtained with $k \simeq 11.4 - 1.51 \ln\sqrt{s}$ and $n_0 \simeq -0.007 + 0.295 \ln\sqrt{s}$. The values for the moments C_3, C_4, and C_5 are calculated with these values of k and n_0. Extrapolating the energy dependence of parameters k and n_0 (Fig. 2), I predict that at $\sqrt{s} \simeq 1700 \pm 100$ GeV (the solid line in Fig. 2 corresponds to $\sqrt{s} \simeq 1600$ GeV, while the dashed line corresponds to $\sqrt{s} \simeq 1800$ GeV) the widening of the distribution will stop (i.e., $m_0 = 0$ at this energy).

This implies the following upper bounds on moments

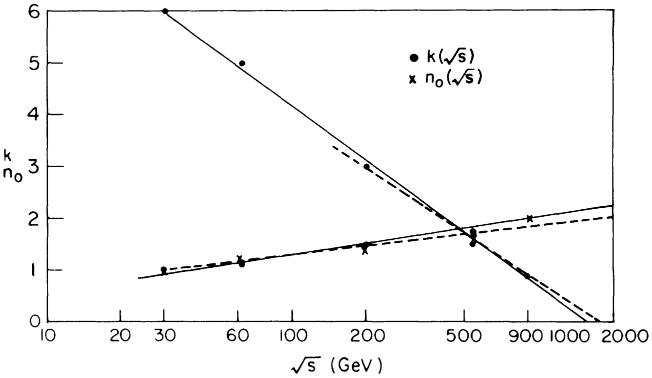

FIG. 2. Theoretical parameters k and n_0 plotted as functions of energy with the assumption that the initial number of gluons increases with energy while the initial number of quarks decreases with energy. Extrapolating values for k and n_0 to higher energies, we find that k approaches 0 (m_0 approaches 0) at $\sqrt{s} \simeq 1700 \pm 100$ GeV. This implies that the probability distribution reaches its maximum width. It also sets the upper limits on the moments C_q.

C_q[11]:

$$C_{2\max} \le 1.46 \pm 0.02, \quad C_{3\max} \le 2.85 \pm 0.13, \qquad (11)$$
$$C_{4\max} \le 6.79 \pm 0.52, \quad C_{5\max} \le 17.22 \pm 1.95.$$

As energy increases from $\sqrt{s} \simeq 1700 \pm 100$ GeV, the contribution to the multiplicities is only coming from gluons. The probability distribution is the pure gluon branching distribution [see Eq. (12)]. The initial number of gluons n_0 will continue to increase, resulting in the

TABLE I. Theoretical predictions for the moments C_2, C_3, C_4, and C_5 for the energy range 30.4 GeV $\le \sqrt{s} \le$ 900 GeV compared with their experimental values given in the brackets. Parameters k and n_0 are fitted with $k \simeq 11.4 - 1.51 \ln\sqrt{s}$ and $n_0 \simeq -0.007 + 0.295 \ln\sqrt{s}$. Theoretical predictions for the multiplicities and moments for Fermilab Tevatron Collider energies ($\sqrt{s} \simeq 1600$, 1800, and 2000 GeV) indicate the upper bounds for the moments C_q. The error bars in the predictions for the moments C_q at Fermilab Tevatron Collider energies are due to the uncertainty in extrapolation of the energy dependence of the parameters k and n_0.

$\sqrt{s}$ (GeV)	k	n_0	α	$\bar{n}$	C_2	C_3	C_4	C_5
30.4	6	1	1.1	10.3	1.22	1.68	2.72	4.60
				(10.07 ± 0.1)	(1.21 ± 0.01)	(1.70 ± 0.02)	(2.64 ± 0.1)	(4.6 ± 0.3)
62.6	5	1.2	1.09	13.9	1.21	1.68	2.71	4.49
				(13.6 ± 0.1)	(1.20 ± 0.01)	(1.65 ± 0.02)	(2.60 ± 0.08)	(4.4 ± 0.2)
200	3	1.3	1.05	21.1	1.26	1.89	3.33	6.13
				(21.6 ± 0.5)	(1.26 ± 0.03)	(1.91 ± 0.12)	(3.3 ± 0.03)	(6.6 ± 0.9)
540	1.55	1.6	1.03	28.8	1.32	2.19	4.23	8.50
				(28.3 ± 0.2)	(1.31 ± 0.03)	(2.12 ± 0.11)	(4.05 ± 0.32)	(8.8 ± 1.0)
900	0.9	2	1	33.2	1.33	2.24	4.46	9.28
				(35.1 ± 0.6)	(1.34 ± 0.03)	(2.22 ± 0.13)	(4.3 ± 0.4)	(9.3 ± 1.1)
1600	0	2.06 ± 0.11	1	38.6	1.46 ± 0.02	2.85 ± 0.13	6.79 ± 0.52	17.22 ± 1.95
1800	0	2.09 ± 0.11	1	39.78	1.45 ± 0.02	2.82 ± 0.13	6.67 ± 0.52	16.77 ± 1.93
2000	0	2.11 ± 0.11	1	40.84	1.45 ± 0.02	2.81 ± 0.12	6.59 ± 0.49	16.44 ± 1.79

narrowing of the probability distribution which is in agreement with other QCD-based approaches to the KNO scaling problem in hadron-hadron collisions.[12] I give predictions for the multiplicities and moments C_q for the Fermilab Tevatron Collider energies ($\sqrt{s} \simeq 1600$, 1800, and 2000 GeV in Table I and Fig. 1) indicating the slow narrowing of the probability distribution in this energy range.

The parton branching distribution given by Eq. (8) is very similar to the very popular negative binomial distribution. Namely, in the limit when $\alpha=1$ and $\delta=k$ (or n_0) the probability distribution is the negative binomial (or simple gluon branching distribution). In this special limit the probability distribution, multiplicities, and moments are given by

$$P_n^\delta = \frac{(n-\frac{1}{2}\pm\delta\mp\frac{1}{2})!}{n!(\delta-1)!}\left[\frac{\delta}{\bar{n}}\right]^{\delta}\left[1\pm\frac{\delta}{\bar{n}}\right]^{\mp n-\delta}, \quad \bar{n}=\delta e^{At}-\frac{\delta}{2}\mp\frac{\delta}{2}, \quad C_2=1+\frac{1}{\delta}\pm\frac{1}{\bar{n}},$$
$$C_3=1+\frac{3}{\delta}+\frac{2}{\delta^2}\pm\frac{1}{\bar{n}}\left[3+\frac{3}{\delta}\right]+\frac{1}{\bar{n}^2}, \quad C_4=1+\frac{6}{\delta}+\frac{11}{\delta^2}+\frac{6}{\delta^3}\pm\frac{1}{\bar{n}}\left[6+\frac{18}{\delta}+\frac{12}{\delta^2}\right]+\frac{1}{\bar{n}^2}\left[7+\frac{7}{\delta}\right]\pm\frac{1}{\bar{n}^3}, \tag{12}$$
$$C_5=1+\frac{10}{\delta}+\frac{35}{\delta^2}+\frac{50}{\delta^3}\pm\frac{1}{\bar{n}}\left[10+\frac{60}{\delta}+\frac{110}{\delta^2}-\frac{24}{\delta^4}\frac{60}{\delta^3}\right]+\frac{1}{\bar{n}^2}\left[25+\frac{75}{\delta}+\frac{190}{\delta^2}\right]\pm\frac{1}{\bar{n}^3}\left[15+\frac{15}{\delta}\right]+\frac{1}{\bar{n}^4}\left[1+\frac{50}{\delta}\right].$$

The negative-binomial distribution has been proposed in many stochastic models.[13] This distribution has been widely used by Alner *et al.* to fit the experimental data[14] even though in all stochastic models there is *no* physical understanding of the behavior of the parameter k (it decreases drastically with energy from 20 at $\sqrt{s}=10$ GeV to 3 at $\sqrt{s}=900$ GeV). In our model, k is related to the average number of initial quarks and for the Collider energies (200 GeV $\le \sqrt{s} \le$ 900 GeV)[15] it decreases from 3 to 0.9 while the average number of initial gluons n_0 increases from 1.3 to 2 in the same energy range. We also predict the values for the parameters k and n_0 for the Tevatron Collider energies.

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[1]For review see P. Carruthers and C. C. Shih, Los Alamos Report No. LA-UR-87-1655, Phys. Rep. (to be published), and the references therein.

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[15]As noted before, for Fermilab and Intersecting Storage Ring energies we cannot neglect quark evolution. We need to consider the probability distribution which is a solution of Eq. (3).