Hadronic Spectrum of the Quark Plasma

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We have carried out a numerical simulation in SU(3) lattice gauge theory with four flavors of low-mass staggered fermions at zero baryon-number density in the vicinity of the high-temperature phase transition on 6×10^3 and $6\times 10^2\times 20$ lattices. Static screening masses (static susceptibilities) were measured in several color-singlet channels with quark valence $q\bar{q}$ and qqq. Clear evidence for a hadronic screening spectrum is found, suggesting the presence of hadronic plasma modes. The spectral multiplets, extrapolated to zero quark mass, are consistent with a restoration of the $SU(N)\otimes SU(N)$ chiral symmetry.

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At high temperatures and zero baryon-number density, hadronic matter is expected to undergo a phase change to a new form of matter called the quark plasma. Recently various laboratories have mounted efforts to produce quark matter in high-energy heavy-ion collisions.[1] It is therefore increasingly important to develop a solid theoretical understanding of the expected structure and properties of this novel phase, not merely for its intrinsic theoretical interest, but to suggest and criticize possible experimental signals for plasma formation.

A popular model of the plasma is based upon a naive interpretation of the consequences of asymptotic freedom that obtains at extremely high temperatures. According to this "deconfinement folklore," the quark plasma is to be regarded as a gas of weakly interacting quarks and gluons. However, it is known that long-range, nonperturbative effects disrupt this simple picture even at the highest temperatures.[2,3] Knowing the long-range structure of this plasma is tantamount to knowing its large-scale composition. A great deal depends upon these degrees of freedom: the equation of state, the rate of entropy production upon a phase change, plasma transport properties, multiplicities and flavors of low-energy particles, and production rates of low-energy lepton pairs, to name a few items.

In an effort to develop a more rigorous nonperturbative picture of the structure of the plasma, we have carried out a numerical simulation of quantum chromodynamics on small lattices [6×10^3 and $6\times 10^2\times 20$] at temperatures near the phase transition. Fermions are incorporated in the staggered scheme[4] with a modification of the Illinois hybrid microcanonical algorithm.[5] We measured several hadron propagators at large spacelike separation. Although our methods are well known in studies of the zero-temperature hadron mass spectrum,[6] to our knowledge, ours is the first deliberate application of these methods to the quark plasma. The inspiration for the present work comes from conjectures concerning the confining properties of the quark plasma and similar work on glueball analog modes of the pure gluon plasma.[3,7]

With N flavors of massless quarks, QCD is symmetric under the chiral $SU(N)\otimes SU(N)\otimes U(1)\otimes U(1)$ group. The $U(1)\otimes U(1)$ chiral symmetry is explicitly broken by the gauge anomaly, and the $SU(N)\otimes SU(N)$ symmetry is spontaneously broken at zero temperature, resulting in the appearance of a massless Goldstone boson. A modified version of the Goldstone theorem applies at finite temperature as well, with the Goldstone boson appearing as a zero-frequency excitation at zero wave number. It has been known for some time that there must be a high-temperature phase transition in QCD that restores some part of the chiral symmetry. The restoration of the $SU(N)\otimes SU(N)$ symmetry is signaled by a vanishing of the order parameter $\langle\bar{q}q\rangle$. This effect is found in numerical simulations.[8,9] A restoration of this symmetry would allow the plasma-mode vestige of the Goldstone boson, should it survive, to have a nonzero gap frequency, and would require the formation of chiral multiplets of states related by the larger symmetry. The chiral multiplet structure is determined explicitly by the valence-quark assignments for the various states. Thus, for example, the plasma modes π and σ must fall in a singlet multiplet, the a_1 and ρ in a multiplet, and the nucleon must either be parity doubled or have zero gap frequency.

To find the plasma chiral multiplets, should they exist, requires a study of the low-lying modes of excitation. They are defined by correlations of local operators $A(\mathbf{x},t)$ and $B(\mathbf{x},t)$:

$$S_{AB}(\mathbf{x},t)=\langle A(\mathbf{x},t)B(0,0)\rangle-\langle A(\mathbf{0},0)\rangle\langle B(\mathbf{0},0)\rangle, \quad (1)$$

where the average is taken over the Gibbs ensemble at temperature T. Unfortunately, lattice gauge theory has not been formulated in such a way that numerical simulations can measure finite-temperature, real-time response. Nonetheless, it is possible through numerical simulation to obtain indirect information about the excitations by a study of the static screening lengths, i.e., correlations between static operators at large distances and high temperatures:

$$S_{AB}(z) = \langle \bar{A}(z)\bar{B}(0)\rangle - \langle \bar{A}(0)\rangle\langle \bar{B}(0)\rangle, \tag{2}$$

where the time and transverse-plane averages are given by

$$\bar{A}(z) = \lim_{L\to\infty} \int_0^\beta d\tau \int_{-L}^{L} dx \int_{-L}^{L} dy \frac{A(x,y,z,-i\tau)}{4L^2\beta}. \tag{3}$$

The large-distance behavior of this correlation,

$$S_{AB}(z) \underset{|z|\to\infty}{\sim} b\exp[-\mu(T)\,|z|\,], \tag{4}$$

gives a screening "mass" or inverse screening length $\mu(T)$.

How is the screening phenomenon related to a real-time excitation? If we consider the dispersion relation of one of the normal modes of the plasma, given by $f(k,\omega,T)=0$, where $k=|\mathbf{k}|$ and ω are the real momentum and real frequency of the real-time response (1), then the screening mass obtained in (4) is found by analytic continuation to be a solution to $f(\pm i\mu(T),0,T)=0$ for the longest-range mode in the channel determined by the quantum numbers of the operators A and B.[3] Just as the plasmon in an ordinary electrodynamic plasma is associated with the phenomenon of Debye screening, so we expect low-lying excitations of the plasma to be related to the screening effects that we measure.

It is easily shown that when a symmetry is not spontaneously broken, i.e., the Gibbs ensemble is invariant under the symmetry transformation, hadron channels related by symmetry must have identical spectral properties, and, in particular, the screening masses must reveal the multiplet structure.

In this Letter we report the highlights of our results without giving details of the numerical analysis. A complete report is currently in preparation.[10] What follows describes our analysis briefly, and gives some of our results, and then we summarize our conclusions.

Simulations were carried out at three values of the bare-quark mass, namely $m=0.05$, 0.075, and 0.10 in lattice units, at each of three values of the gauge coupling adjacent to the phase transition, namely $\beta=5.10$, 5.175, and 5.25. In the chiral limit the first value of β is expected to lie in the low-temperature ($T<T_c$) phase, and the last two, in the high-temperature ($T>T_c$) phase. The bulk of the measurements were carried out on the smaller lattice, namely 6×10^3.

Screening masses were measured for a variety of color-singlet channels with the quantum numbers of the π, σ, ρ, b_1, and a_1 mesons and both parity states of the lowest-lying baryon. These masses were measured by our calculating quark propagators on lattices obtained by doubling of the gauge-field configuration in a spatial direction.

A careful analysis is required to determine the systematic and statistical errors in the screening masses. Systematic errors arise from finite-size effects, a neglect of higher spectral components, and a variety of correlations.[10]

Shown in Fig. 1 is a sample plot of the static correlation (2) for the π-meson channel in the high-temperature phase. The fit is by the form

$$S_\pi(z) = b\exp(-\mu z) + b\exp[-\mu(L-z)], \tag{5}$$

where L is the length of the lattice in the z direction. In this case the fit begins at $z=4$ and runs to the largest value of z, so as to minimize the effect of higher spectral components. The fit to the data running to $z=20$ gives a screening mass of 0.765 with a χ^2 of 29 for fifteen degrees of freedom. The fit to the data from the 6×10^3 lattice alone with a far larger data sample gives a screening mass of 0.754 ± 0.003 with $\chi^2=20$ for five degrees of freedom in the lattice units. (In this and other channels, only values from the smaller lattice are used in the extrapolation to the chiral limit.)

Thus the static correlation shows a remarkably clean

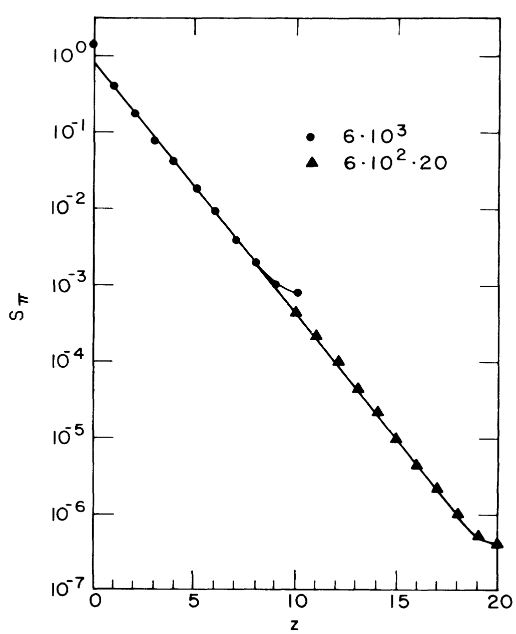

FIG. 1. The static correlation (2) in the pion channel, averaged over x and y, as a function of the separation z in the high-temperature phase. The points are from the numerical simulation at $\beta=5.25$ and quark mass $m=0.05$ in lattice units. The errors are smaller than the symbol sizes.

fit to a single spectral component over six decades. If there were a $q\bar{q}$ component in this channel it would have a continuum threshold screening mass of at least twice the lowest fermion Matsubara frequency, namely $2\pi T$. In the present lattice units $T=\frac{1}{6}$, and so this threshold corresponds to a screening mass higher than 1. Since the pionic spectral component seen in Fig. 1 is obviously of a considerably lower screening mass, it cannot be due to such a quark continuum. However, we cannot rule out the possibility that such a continuum occurs in addition to the pionic mode as a higher spectral component.

Other channels were analyzed in a similar way. The fitting functions for the other channels include oscillating terms according to the requirements of the staggered-fermion scheme.[11] Figure 2 plots the results for the π and σ mesons at $\beta=5.25$ as a function of bare-quark mass. The data are consistent with the expected degeneracy in the chiral limit. The data are also consistent with the conclusion that the pionic mode is not a Goldstone boson for $T>T_c$.

Results for other hadronic channels at the three values of β were similarly extrapolated to zero quark mass. Where statistically allowed, the intercepts of the expected multiplets were equated. Results for extrapolations to the chiral limit of all of the channels are collected in Fig. 3. The shaded region in this figure indicates the possible location of the phase transition in the chiral limit.[8,12] It is evident that the ρ and a_1 fall into the expected multiplets above the phase transition and that the lowest-lying baryonic mode chooses parity doubling, instead of a zero mass. We also find some indication in the $\beta=5.25$ data that the two helicity states of the vector/axial-vector meson are split.

Our results are consistent with a restoration of an $SU(N)\otimes SU(N)$ chiral symmetry. In particular, we find good numerical evidence in the chiral limit of our model (1) that there is a cleanly identifiable pion plasma mode in the high-temperature phase; (2) that the expected π-σ and ρ-a_1 multiplets occur in the plasma; and finally (3) that there are parity-doubled baryon plasma modes with finite screening length for $T>T_c$.

A partial restoration of the U(1) axial symmetry, associated with the diminution of the gauge anomaly, would require a further parity doubling of the π-σ multiplet.[13] However, we have not examined the relevant channels, and so cannot draw any conclusions regarding the fate of the U(1) symmetry.

Our results strongly suggest the existence of hadronic modes in the plasma screening spectrum. Whether these modes are also important as real-time excitations of the plasma is an urgent question that will probably not be answered soon by lattice gauge theory. Nonetheless, their appearance in the screening spectrum deals a serious blow to the naive deconfinement picture and requires a reconsideration of several of its predictions of the experimental plasma signature.

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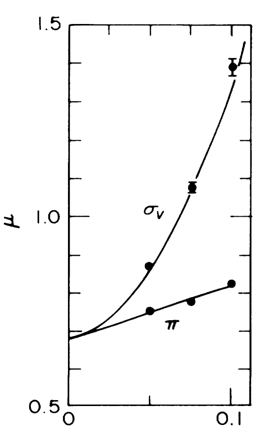

FIG. 2. Screening masses for the π and σ modes in the high-temperature phase ($\beta=5.25$) as functions of the bare quark mass. All masses are in lattice units. The curves show a linear extrapolation of the π and quadratic extrapolation of the σ screening masses to a common intercept in the chiral limit.

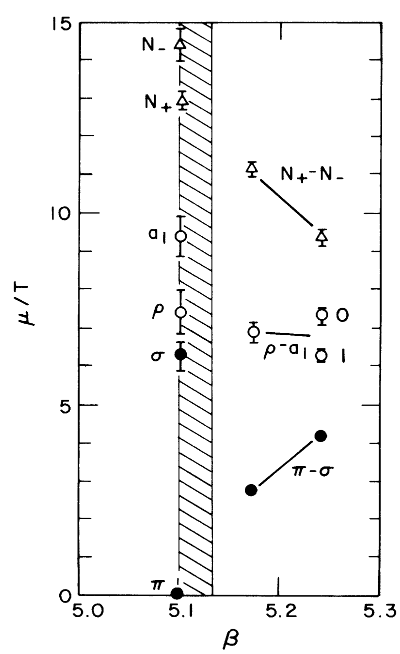

FIG. 3. Screening masses, as extrapolated to the chiral limit, for the π, σ, ρ, and a_1 meson plasma modes, and the lowest even-parity (N_+) and odd-parity (N_-) baryon plasma modes, plotted as functions of the gauge coupling β, expressed in units of the temperature. An increase in β corresponds to an increase in temperature. The shaded region indicates the possible location of the phase transition. The notation 0 and 1 beside the highest-temperature vector/axial-vector meson mass indicates the helicity assignment.

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