Universal Seesaw Mechanism?

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The idea that fermions acquire their masses via a universal seesaw mechanism can provide a plausible explanation for the mass hierarchy $m_{e,u,d} \simeq 10^{-4} M_W$. A minimal $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ grand-unifiable realization is presented. Whereas the fermionic representation is enlarged to include $SU(2)_L \otimes SU(2)_R$ singlets, the Higgs system contains none of the conventional scalars of left-right-symmetric models. An alternative way to account for the superlightness of neutrinos emerges.

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A remarkable, yet still somewhat mysterious, feature of the Glashow-Weinberg-Salam electroweak scheme[1] is that spontaneous symmetry breaking and fermion masses are both triggered by one and the same Higgs scalar. On the one hand, it is useful to have some guidance as to how to choose the otherwise arbitrary Higgs system. On the other hand, it seems as if naturalness is lost in the following sense. If the electron and the W bosons acquire mass via a common vacuum expectation value, why is it that

$$m_e \simeq 10^{-5} M_W? \tag{1}$$

Further, this mass hierarchy is numerically not so different from

$$m_{\nu_e} \lesssim 10^{-5} m_e, \tag{2}$$

but somehow it has not received due attention. The latter hierarchy is not confronted in the minimal electroweak model.

Whereas the difficulty associated with $m_e \ll M_W$ persists when $SU(2)_L \otimes U(1)_Y$ is enlarged[2] to $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$, the $m_{\nu_e} \ll m_e$ problem can be addressed by invoking the so-called seesaw mechanism.[3] The higher the mass scale of the now mandatory ν_R (which is not the case in the standard electroweak model), the lighter is ν_L. The price, however, is a richer Higgs system which necessarily includes[4]

$$\phi(2,2)_0 + \phi(3,1)_{-2} + \phi(1,3)_{-2}. \tag{3}$$

What physics gives rise to $m_e/M_W \ll 1$? Is there an alternative way to account for $m_{\nu_e}/m_e \ll 1$? Are these two hierarchies correlated?

In an attempt to deal with the above questions, we have adopted an unorthodox approach. Rather than sticking to a fixed fermionic set while complicating the Higgs system, we hereby enlarge the fermionic representation but simplify the Higgs system to its limits. In particular, none of the conventional scalars specified by Eq. (3) are present! Yet, the electron does acquire mass, via a universal seesaw mechanism, and at the same time the superlightness of the neutrinos follows without any further input. The scheme lends itself to $SU(5)_L \otimes SU(5)_R$ grand unification.[5]

Consider an $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ electronuclear model, and allow for the following fermionic content. In addition to the standard *complex* representation

$$q_L(3,2,1)_{1/3}, \quad q_R(3,1,2)_{1/3}, \qquad l_L(1,2,1)_{-1}, \quad l_R(1,1,2)_{-1}, \tag{4}$$

let there be an exotic *real* representation

$$u^H_{L,R}(3,1,1)_{4/3}, \quad d^H_{L,R}(3,1,1)_{-2/3}, \qquad \nu^H_{L,R}(1,1,1)_0, \quad e^H_{L,R}(1,1,1)_{-2}. \tag{5}$$

The various $(B-L)$ charges have been carefully adjusted such that each ordinary fermion f has its own nonmirror f^H companion with matching $SU(3)_C \otimes U(1)_Q$ assignments $[Q = T_{3L} + T_{3R} + \frac{1}{2}(B-L)]$. By such assignments, we have unfortunately ruined the ingenious interpretation of $B-L$ as the fourth color.[2]

It is true that postulating the extra fermionic structure seems, at first glance, superfluous and quite unnatural. Do we have a compelling theoretical reason for doing it? The positive answer is provided by a recently proposed left-right-symmetric, yet flavor-chiral Georgi-Glashow-type,[6] $SU(5)_L \otimes SU(5)_R$ grand-unification scheme. While $SU(3)_C \equiv SU(3)_{L+R}$ and $U(1)_{B-L} \equiv U(1)_{L+R}$, $SU(2)_{L,R}$ are contained in $SU(5)_{L,R}$, respectively. It is straightforward to verify that the above fermionic representation is in fact

$$\psi_L(\mathbf{10} \oplus \mathbf{5}^* \oplus \mathbf{1};\mathbf{1}) + \psi_R(\mathbf{1};\mathbf{10} \oplus \mathbf{5}^* \oplus \mathbf{1}). \tag{6}$$

$\nu_{L,R}(1;1)$ are optional here, but mandatory in the $SO(10)_L \otimes SO(10)_R$ generalization. The underlying unification is, however, beyond the scope of the present paper.

The associated Higgs system is minimal. It consists of two complex doublets

$$\phi_L(1,2,1)_{-1} + \phi_R(1,1,2)_{-1}. \tag{7}$$

We cannot think of any simpler left-right–symmetric generalization of the standard Weinberg-Salam doublet. In particular, note that the only Higgs scalar appearing in all conventional $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ models, namely $\phi(1,2,2)_0$, the standard source of quark and/or lepton masses, is absent. Also missing are $\phi(1,3,1)_{-2} + \phi(1,1,3)_{-2}$, the conventional sources for inducing neutrino Majorana masses. The simplicity of the Higgs system has an immediate low-energy consequence. Since the charged scalar degrees of freedom are incorporated to make $W^{\pm}_{L,R}$ massive, the two (real) remnant physical scalars are necessarily electrically neutral.

The most general renormalizable Higgs potential involving $\phi_{L,R}$ is given by

$$V = \sum_{i,j=L,R}\sum \lambda_{ij}(\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) - \sum_{i=L,R} n_i^2 \phi_i^\dagger \phi_i. \tag{8}$$

The above potential, however, is too constrained, so that the imposition of the discrete left-right symmetry implies either $v_L = v_R$ or alternatively $v_L = 0$, where $v_{L,R} \equiv \langle \phi_{L,R} \rangle$. To gain a tree-level hierarchy $v_L \ll v_R$, we heave to assume that left-right symmetry is *explicitly* broken, thus allowing for

$$g_L \neq g_R. \tag{9}$$

This should be regarded, however, as an effective consequence of physics beyond $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U_{B-L}$, where left-right symmetry is only *spontaneously* broken. Invoking $SU(5)_L \otimes SU(5)_R$ unification ideas, we do anticipate in fact[5] that $SU(2)_L$ and $SU(2)_R$ would not share a common evolution (that is if only 3,4 generations exist).

Let us examine now the quark sector. On top of the off-diagonal Yukawa couplings

$$\mathcal{L}_{\text{Yukawa}} = (Y_{uL}\bar{q}_L \phi_L u_R^{\text{H}} + Y_{dL}\bar{q}_L \phi_L^\dagger d_R^{\text{H}} + Y_{uR}\bar{u}_L^{\text{H}} \phi_R^\dagger q_R + Y_{dR}\bar{d}_L^{\text{H}} \phi_R q_R) + \text{H.c.}, \tag{10}$$

there are the $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$-invariant mass terms

$$\mathcal{L}_{\text{mass}} = (x_u \bar{u}_L^{\text{H}} u_R^{\text{H}} + x_d \bar{d}_L^{\text{H}} d_R^{\text{H}}) + \text{H.c.} \tag{11}$$

Once the gauge symmetry is spontaneously broken down to $SU(3)_C \otimes U(1)_Q$, the tree-level mass matrices

$$M_u = \begin{pmatrix} 0 & Y_{uL} v_L \\ Y_{uR} v_R^* & x_u \end{pmatrix}, \tag{12a}$$

$$M_d = \begin{pmatrix} 0 & Y_{dL} v_L^* \\ Y_{dR} v_R & x_d \end{pmatrix} \tag{12b}$$

make their appearance. It is convenient to extract the eigenmasses and the $f_{L,R} - f_{L,R}^{\text{H}}$ mixing angles by diagonalizing $MM^\dagger$ and $M^\dagger M$, respectively. By naturalness, we expect

$$Y_{L,R} v_{L,R} \sim m(W_{L,R}), \tag{13}$$

whereas the so-called "survival hypothesis" suggests (but does not imply)

$$Y_{L,R} v_{L,R} < x. \tag{14}$$

With this in mind, the quark eigenmasses become

$$m_{u,d} \sim v_L v_R / x < m(W_L), \tag{15}$$

and the $f_{L,R} - f_{L,R}^{\text{H}}$ mixing angles are of order $m(W_{L,R})/x$, respectively. This is the essence of our model. The mass hierarchy $m_{u,d} \simeq 10^{-4} m(W_L)$ may signify physics beyond the $SU(2)_R$ breaking scale. The fact that our model is $SU(5)_L \otimes SU(5)_R$ embeddable suggests that x presumably marks the mass scale associated with $SU(3)_L \otimes SU(3)_R \to SU(3)_C$ or $U(1)_L \otimes U(1)_R \to U(1)_{B-L}$.

As far as the leptons are concerned, the discussion goes along the same lines, up to the neutrino puzzle,[7] of course. Naturalness means $Y_{\nu L,R}\, v_{L,R} \sim m(W_{L,R})$, and there is no group-theoretical reason why x_ν, and the additional bare Majorana mass term, should be quite different from $x_{e,u,d}$. Still, with $v_L \ll v_R \ll x$ already established, we now proceed to show how the neutrino superlightness follows naturally, with no further assumptions.

Taking into account the fact that both ν_L^{H} and $\bar{\nu}_L^{\text{H}}$ transform trivially under $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$, the emerging neutrino mass matrix, in the $(\nu_L, \bar{\nu}_L, \nu_L^{\text{H}}, \bar{\nu}_L^{\text{H}})$ space, reads

$$M_\nu = \begin{pmatrix} 0 & 0 & Y_{\nu L} v_L & \tilde{Y}_{\nu L} v_L \\ 0 & 0 & Y_{\nu R} v_R^* & \tilde{Y}_{\nu R} v_R^* \\ Y_{\nu L} v_L & Y_{\nu R} v_R^* & x_1 & x \\ \tilde{Y}_{\nu L} v_L & \tilde{Y}_{\nu R} v_R^* & x & x_2 \end{pmatrix}. \tag{16}$$

The lowest eigenmasses are

$$m(\nu_L^{\text{phy}}) \sim v_L^2/x, \qquad m(\nu_R^{\text{phy}}) \sim v_R^2/x, \tag{17}$$

establishing the original seesaw result

$$m(\nu_L^{\text{phy}})\, m(\nu_R^{\text{phy}}) \simeq m_e^2. \tag{18}$$

Note that both $m(\nu_{L,R}^{\text{phy}})$ are lighter by a factor of $m(W_R)/x$ than the corresponding masses predicted by left-right–symmetric models. If $v_R/x \approx v_L/v_R$, for instance, ν_R^{phy} may be as light as the W_L bosons. It should also be noted that had we introduced just a single ν_L^{H}, we would have faced $m(\nu_L^{\text{phy}}) = 0$. Such a strict masslessness is kinematical ($\det M_\nu = 0$), as in the standard model.

The low-energy effects can be described by an effective Lagrangean. Such a Lagrangean includes dimension-five terms of the generic form $\sim(1/x)\phi^2\bar{f}f$. With the usual symmetry breaking, the various fermionic masses are recovered. This establishes an interesting link with the conventional left-right-symmetric scheme: The conventional scalars are bilinears of the new scalars, namely $\phi(2,2)_0 \sim \phi_L^\dagger \phi_R$, $\phi(3,1)_{-2} \sim \phi_L^2$, and $\phi(1,3)_{-2} \sim \phi_R^2$.

In the gauge-boson sector, two characteristic features are encountered.

(i) It is straightforward to derive

$$m(W_{L,R}) = \tfrac{1}{2} g_{L,R} v_{L,R}, \tag{19}$$

primarily because $W_L^\pm$ and $W_R^\pm$ do *not* mix. In the absence of the conventional $\phi(1,2,2)_0$, no scalar transforms nontrivially under both $SU(2)_L$ and $SU(2)_R$.

(ii) The neutral (mass)^2 matrix

$$M^2 = \frac{1}{4}\begin{pmatrix} g_L^2 v_L^2 & 0 & -g_L \tilde{g} v_L^2 \\ 0 & g_R^2 v_R^2 & -g_R \tilde{g} v_R^2 \\ -g_L \tilde{g} v_L^2 & -g_R \tilde{g} v_R^2 & \tilde{g}^2 (v_L^2 + v_R^2) \end{pmatrix} \tag{20}$$

gives rise to a massless photon

$$A = \sin\theta\, W_L^3 + \cos\theta(\sin\xi\, W_R^3 + \cos\xi\, W^0) \sim \tilde{g}(g_R W_L^3 + g_L W_R^3) + g_L g_R W^0, \tag{21}$$

and two heavy Z's ($\sin\xi \to \tan\theta$ in the left-right-symmetric limit). The standard $\Delta I = \frac{1}{2}$ formula

$$m(Z)\cos\theta \simeq m(W_L), \tag{22}$$

is accompanied by the less familiar

$$m(Z')\cos\xi \simeq m(W_R). \tag{23}$$

In conventional left-right-symmetric models for light neutrinos, $SU(2)_R$ is broken by $\langle \phi(1,1,3)_{-2} \rangle$, so that $m(Z')$ is heavier by a factor of $\sqrt{2}$.

To summarize, we have presented a simple alternative to the conventional $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ scenario, with the following distinctive features: (i) No conventional scalar is introduced. The complication of the Higgs system has been traded for an extended fermionic representation, such that each quark and lepton has an $SU(2)_L \otimes SU(2)_R$-singlet companion. (ii) Whereas $v_L \ll v_R$ assures the emergence of $SU(2)_L \otimes U(1)_Y$ at low energies, $v_R \ll x$ accounts for $m_{e,u,d} \sim v_L v_R / x < m(W_L)$ via a universal seesaw mechanism. (iii) With $v_L \ll v_R \ll x$ established, it follows with no additional assumptions that $m(v_L^{\text{phy}}) \sim v_L^2/x$ and $m(v_R^{\text{phy}}) \sim v_R^2/x$. (iv) The scheme is $SU(5)_L \otimes SU(5)_R$ unifiable. We are aware of the fact that this model is still not fully realistic. Higher-generational fermions are much heavier than e,d,u, with the extreme of $m_t \sim M_W$. The easy way out is to translate the $m_e \ll m_\mu \ll m_\tau$ hierarchy into a reversed $x_e \gg x_\mu \gg x_\tau$ hierarchy. But what is really needed is a multigenerational generalization, with quark and/or lepton masses ranging from $v_L v_R / x$ to v_L. Such multigenerational attempts are now in progress.

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