Discriminating Technicolor Theories through Flavor-Changing Neutral Currents: Slowly Varying or Fixed Coupling Constants?

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On the basis of the analytical study of the ladder Schwinger-Dyson equation of the technifermion self-energy, we find crucial constraints on the running coupling constants in technicolor theories for the flavor-changing neutral currents to be dynamically suppressed. It is unlikely that the slowly varying coupling constants in the asymptotically free technicolor theory can solve the flavor-changing neutral-current problem. Fixed-point theories may be the only viable possibility.

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The flavor-changing neutral-current (FCNC) problem has long been a fatal disease to technicolor (TC) theories.[1] However, a novel solution to this problem has recently been proposed by two of the authors and Matumoto[2] in the scale-invariant TC model in which the dynamical mass of the technifermion, $\Sigma(p)$, is given by the spontaneous-chiral-symmetry-breaking solution of the ladder Schwinger-Dyson (SD) equation: $\Sigma(p)$ in the ladder SD equation asymptotically behaves as

$$\Sigma(p) \sim \Lambda_{\mathrm{TC}}^2/p, \quad p \gg \Lambda_{\mathrm{TC}}, \tag{1}$$

which is communicated through extended TC (ETC) or preonic gauge interactions down to the ordinary fermion (quarks/leptons) mass m_f on the order of

$$m_f \simeq \Lambda_{\mathrm{TC}}^2/\Lambda_S, \tag{2}$$

where Λ_{TC} and Λ_S stand for scale parameters characterizing the TC and the ETC (or subcolor) gauge theories, respectively.

In order that $m_f \simeq 10^2$ MeV for $\Lambda_{\mathrm{TC}} \simeq \frac{1}{3}$ TeV, Λ_S should be $\Lambda_S \simeq 10^3$ TeV, thus yielding FCNC $O(\theta_C^2/\Lambda_S^2) \simeq 5\times 10^{-8}$ TeV^{-2} (θ_C is the Cabibbo angle) in accord with the present experimental limit $< 5\times 10^{-7}$ TeV^{-2}. Equation (2) should be compared with the conventional result,[1] $m_f \simeq \Lambda_{\mathrm{TC}}^3/\Lambda_S^2$, in the usual asymptotically free TC theories in which the asymptotic form of $\Sigma(p)$ is given by $\Sigma(p) \sim \Lambda_{\mathrm{TC}}^3/p^2$ up to logarithms. Moreover, Eq. (1) simultaneously remedies another syndrome, the light pseudo Nambu-Goldstone bosons (PNGB), by raising their masses m_{PNGB} to the order of

$$m_{\mathrm{PNGB}}^2 \sim (1/\Lambda_S^2)(\Lambda_S\Lambda_{\mathrm{TC}}^2)^2/\Lambda_{\mathrm{TC}}^2 = O(\Lambda_{\mathrm{TC}}^2). \tag{3}$$

It should be noted that this "dynamical suppression mechanism" is entirely free of fine tuning. Actually, an asymptotically *nonfree* TC theory, the model of Ref. 2, was shown, through Miransky's renormalization procedure,[3] to have a nontrivial ultraviolet fixed point with a large anomalous dimension $\gamma^* = 1$.[4]

The Appelquist, Karabali, and Wijewardhana[5] made an attempt to fit in the above suppression mechanism, Eq. (1), with the asymptotically *free* TC theories, now in the framework of the "modified" ladder SD equation with the fixed coupling constant simply replaced by a running coupling constant, especially by a slowly varying coupling constant. Suppression of FCNC's in the same framework was first considered numerically by Holdom[6] some time ago.

Now the question is this: Given the (modified) ladder SD equation, can the FCNC's distinguish between the above two conceptually different TC theories, the fixed-point theory (FPT) of Ref. 2 and the asymptotically free theory (AFT) with slowly varying coupling constant of Refs. 5 and 6?

In this Letter, we shall analytically investigate the solution of the modified ladder SD equation and find on rather general grounds severe constraints on the running coupling constants for the FCNC's to be sufficiently suppressed. We first present a formula for the fermion mass m_f which, remarkably enough, is determined by the behavior of $\Sigma(p)$ and the TC coupling constant $\alpha(p)$ *solely at* $p \simeq \Lambda_S$, their behavior for $p < \Lambda_S$ (as well as $p > \Lambda_S$) being totally irrelevant. Then we find that AFT, however slowly the coupling might be varied, does not yield enough suppression factor *unless* at $p \simeq \Lambda_S$ we abandon the asymptotic form, $\Sigma(p) \sim p^{-2}(\ln p)^{A/2-1}$ $(A>0)$, very characteristic of the "asymptotically free" theories. Thus FCNC's require almost fixed coupling constants *over the whole region relevant to* m_f in favor of the FPT of Ref. 2.

Let us start with the formula for m_f in the ETC (or technicolored preon) models[1]:

$$m_f = (1/2\Lambda_S^2)\langle \bar{F}F\rangle, \tag{4}$$

$$\langle \bar{F}F\rangle \equiv \sum_{i=1}^{N} \langle 0|\bar{F}^i F_i|0\rangle = \frac{N}{2\pi^2}\int_0^{\Lambda_S} p^3 dp \frac{\Sigma(p)}{p^2+\Sigma(p)^2}$$
$$= \frac{N}{4\pi^2}\int_0^{\Lambda_S^2} dy \frac{y\Sigma(y)}{y+\Sigma(y)^2}, \tag{5}$$

where N is the number of technifermions F_i communicating to the mass of the ordinary fermion f. $\Sigma(p)$ in (5) is required to be the solution of the modified ladder SD equation[7,8] which, in Euclidean space, takes the form in Landau gauge

$$\Sigma(x) = \frac{\lambda(x)}{x}\int_0^x dy \frac{y\Sigma(y)}{y+\Sigma(y)^2} + \int_x^\infty dy \frac{\lambda(y)\Sigma(y)}{y+\Sigma(y)^2}, \tag{6}$$

where

$$\lambda(p^2) \equiv [3c_2(F)/16\pi^2]g(p)^2 = [3c_2(F)/4\pi]\alpha(p)$$

with $\alpha(p)$ the running coupling constant such that

$$p\,\partial\alpha(p)/\partial p \equiv \beta(p) = -b\alpha(p)^2 - c\alpha(p)^3 + \ldots$$

and $c_2(F)$ the quadratic Casimir for the technifermion representation F, and use has been made of an approximation

$$\lambda((p-q)^2) = \lambda(p^2)\theta(p^2-q^2) + \lambda(q^2)\theta(q^2-p^2).$$

Differentiating (6), we obtain

$$\int_0^x dy \frac{y\Sigma(y)}{y+\Sigma(y)^2} = \frac{\Sigma'(x)}{[\lambda(x)/x]'}, \tag{7}$$

which is substituted into (5) and (4), finally yielding a simple formula for m_f:

$$m_f = \frac{1}{2\Lambda_S^2}\frac{N}{3c_2(F)\pi}\left[\frac{x^2\Sigma'(x)}{\beta(x)-2\alpha(x)}\right]_{x=\Lambda_S^2}, \tag{8}$$

where we noted that

$$\lambda'(x) = [3c_2(F)/4\pi]\alpha'(x) = [3c_2(F)/8\pi]\beta(x)/x.$$

Equation (8) is an amazing formula which expresses m_f in terms of the behavior of $\Sigma(p)$ and $\alpha(p)$ *only at the point* $p = \Lambda_S^2$. Our formula is the direct consequence of the fact that $\Sigma(p)$ is precisely the solution to the modified SD Eq. (6) and enables us to evaluate m_f without recourse to how the running coupling constants are varied for $p < \Lambda_S$. This is sharply contrasted to the numerical analysis of Ref. 5 in which the "slowly varying" property of the coupling constant was considered significant in the relatively low-momentum region,

$$\Sigma(0) < p < \Sigma(0)\exp(1/b\alpha_\mu + c\alpha_\mu^2) \ll \Lambda_S,$$

with $\alpha_\mu \equiv \alpha(\mu) \simeq \alpha_c = \pi/3c_2(F)$ (Ref. 8) the coupling constant at $p=\mu$ where the chiral-symmetry breaking takes place.

By virtue of (8), we are only interested in the asymptotic behavior of $\Sigma(p)$, in which case $[p \gg \Sigma(p)]$ Eq. (6), with $\lambda = A/2t$ $[t = s + 2/b\alpha_\mu \simeq s + 2A$, $s = \ln(x/\mu^2)$, and $A \equiv 3c_2(F)/\pi b]$, is converted into a linearized differential equation (Whittaker's equation)[9]

$$\ddot{G} + [-\tfrac{1}{4} + (A-1)/2t]G = 0, \tag{9}$$

where $G \equiv t^{1/2}\exp(\frac{1}{2}t)\Sigma$ and $\dot{G} = dG/dt$. Solutions to (9) are the Whittaker functions, $W_{A/2-1/2,1/2}(t)$ and $M_{A/2-1/2,1/2}(t)$, which correspond to the spontaneous and the explicit chiral-symmetry breakings, respectively. We may write our spontaneous-breaking solution as

$$\Sigma(p) = \Sigma_\mu e^{-(t-2A)/2}\left[\frac{t}{2A}\right]^{-1/2}\frac{W_{A/2-1/2,1/2}(t)}{W_{A/2-1/2,1/2}(2A)}, \tag{10}$$

where $\Sigma_\mu \equiv \Sigma(\mu)$. Precisely speaking, this normalization is subject to a slight change due mainly to the nonlinearity at $p \simeq \mu$, which, however, does not significantly affect our conclusion. To be definite in numerical estimate, we take a parameter set similar to Ref. 5: $\Sigma(0) \simeq \Sigma(\mu) \simeq \mu \simeq 350$ GeV and $\Lambda_S \simeq 350$ TeV.

Now, in (9) the effects of the running coupling constant are only appreciable, through $t = \ln(x/\mu^2) + 2A$, for values of x such that $\ln(x/\mu^2) > 2A$, i.e., $A < 7$ for $p \simeq \Lambda_S$. The asymptotic behavior of $\Sigma(p)$ in this case[7,8] is obtained from (10) in the limit[10] $t \to \infty$;

$$\Sigma^{\rm AFT}(p) = \Sigma_\mu^{\rm AFT}(\mu^2/x)[1+(1/2A)\ln(x/\mu^2)]^{A/2-1}, \tag{11}$$

which is also normalized as $\Sigma^{\rm AFT}(\mu) = \Sigma_\mu^{\rm AFT} \simeq \Sigma_\mu$.[11]

For $A \gg 7$, on the other hand, Eq. (9) goes over to $\ddot{G} - (s/8A)G = 0$ as $p \simeq \Lambda_S$ $(s \simeq 14)$, which is nothing but the case for the fixed point theory,[3] but in the "weak coupling phase" $(\lambda < \lambda_c = \frac{1}{4})$[12]; $\ddot{G} - (\frac{1}{4} - \lambda)G = 0$ (the limit $A \to \infty$ corresponds to $\lambda \to \lambda_c - 0$). In fact, as $A \to \infty$ with x fixed, the asymptotic form of (10) goes over to that in the FPT of Ref. 2,[13]

$$\Sigma^{\rm FPT}(p) = (\mu/\sqrt{x})\Sigma_\mu^{\rm FPT}, \tag{12}$$

which again is normalized as $\Sigma^{\rm FPT}(\mu) = \Sigma_\mu^{\rm FPT} \simeq \Sigma_\mu$.[14]

We now come to the central problem of this Letter, the evaluation of m_f through our formula (8) in the AFT with slowly varying couplings[5,6] in comparison with FPT.[2] The substitution of (11) and (12) into (8) yields the ratio

$$m_f^{\rm AFT}/m_f^{\rm FPT} = (\mu/\Lambda_S)R, \tag{13}$$

where R essentially comes from the logarithmic factor in (11);

$$R = (A/2)t_S^{A/2}[2t_S - (A-2)/2A][\tfrac{1}{2} + At_S]^{-1},$$

with $t_S \equiv 1+(2A)^{-1}\ln(\Lambda_S^2/\mu^2)$.[15] Typical values of R are $R \simeq 2.7$ ($A=1$), 4.2 ($A=2$), 6.8 ($A=4$), 8.8 ($A=6$), and 9.7 ($A=7$), and hence are at most of order 10^1.[16] Thus $m_f^{\rm AFT}/m_f^{\rm FPT} \lesssim 10^{-2}$.

To be more explicit, we take an example from Ref. 5; the TC gauge group is SU(4) with $N_f=14$ technifermions in the fundamental representation, i.e., $\alpha_c \simeq 0.56$, $b=0.85$, and $c=-0.73$, which corresponds to $A=1/b\alpha_c=2.1$ and $A_{\rm eff} \equiv 1/(b\alpha_c + c\alpha_c^2)=4.0$. Our formula (8) yields $m_f \simeq 3.7\times 10^{-2}$ MeV ($A=2$) and $\simeq 6.1 \times 10^{-2}$ MeV ($A_{\rm eff}=4$), respectively, apart from the factor N ($N<N_f$ in general, since not all F communicate with a single f), thus still smaller than the realistic value by more than two orders of magnitude. Other examples presented in Ref. 5 also lead to similar results.

We thus conclude that AFT with slowly varying coupling constant can only improve FCNC's problem by one order of magnitude compared with the usual strongly asymptotically free ($A<1$) TC models,[1] still two orders' discrepancy being unsolved.

In contrast to the previous analysis[5,6] based on the modified ladder SD Eq. (6), we here made full use of the formula (8) which was neatly derived from precisely the same modified ladder SD Eq. (6) without further approximations. The failure of the slowly varying coupling constant in AFT thus is simply traced to the assumption that the asymptotic form of $\Sigma(p)$, (11), is valid at the point $p=\Lambda_S$. One might be tempted to push away the "asymptotic region" where (11) is valid into the "super-ultraviolet" region $p \gg \Lambda_S$ so that the TC for $\mu \lesssim p \lesssim \Lambda_S$ might behave like the theory with almost fixed coupling constant. But then the trace of "asymptotically free" TC can be seen nowhere; it does not make sense to argue in the region $p>\Lambda_S$ whether TC by itself is asymptotically free or nonfree, since the TC there is already changed into a completely different theory, ETC or TC at the preon level. Actually one might be able to avoid (11) at $p \simeq \Lambda_S$ by letting $A \gg 7$ in (10), which would, however, substantially lead to the FPT itself as we have mentioned.

In conclusion, we may define three kinds of coupling constants in AFT in terms of "$\mu_{\rm asym}$" above which $(p>\mu_{\rm asym})\Sigma(p)$ takes the form (11); (i) running ($\mu_{\rm asym}>\Lambda_S$), (ii) slowlying varying ($\mu_{\rm asym} \lesssim \Lambda_S$) and (iii) almost fixed ($\mu_{\rm asym}>\Lambda_S$) coupling constants. Our analysis in this Letter clearly demonstrated that FCNC's can be sufficiently suppressed only by the almost fixed coupling constant. But this is essentially the case of FPT in Ref. 2 anyway.

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[9]Linearized differential equation reads

$$\ddot{G}+\left[\left(1-\frac{1}{t}\right)-\frac{\ddot{\lambda}-\lambda}{\dot{\lambda}-\lambda}\right]\dot{G}+\left[\frac{\ddot{\lambda}-\dot{\lambda}}{\dot{\lambda}-\lambda}-(\dot{\lambda}-\lambda)-\frac{1}{2}\frac{\ddot{\lambda}-\lambda}{\dot{\lambda}-\lambda}\left(1-\frac{1}{t}\right)+\frac{1}{2}\frac{1}{t^2}+\frac{1}{4}\left(1-\frac{1}{t}\right)^2\right]G=0.$$

In the asymptotic region, this equation is reduced to (9).

[10]The renormalization-group–invariant expression for $\Sigma^{\rm AFT}(p)$ is given by

$$\Sigma(p) \sim \Lambda_{\rm TC}^3 p^{-2}[\tfrac{1}{2}\ln(p^2/\Lambda_{\rm TC}^2)]^{A/2-1},$$

where $\Lambda_{TC}=\mu\exp(-1/b\alpha_\mu)\simeq\mu e^{-A}$. Thus

$$\Lambda_{TC}^3\simeq\Sigma_\mu\mu^2[\tfrac{1}{2}\ln(\mu^2/\Lambda_{TC}^2)]^{-A/2+1}.$$

[11]Actually we have $\Sigma^{AFT}=C(A)\Sigma_\mu$, where $C(A)\equiv e^{-A}(2A)^{A/2-1/2}$. $[W_{A/2-1/2,1/2}(2A)]^{-1}=1$ $(A=1,3)$, 1.3 $(A=5)$, and 1.7 $(A=7)$, etc.

[12]To be precise, there is only one phase (spontaneously broken phase of the chiral symmetry) in AFT, without distinction between "weak" and "strong" coupling phases in contrast to FPT (Refs. 2 and 3). However, there are some similarities between the roles of λ_c $(=\frac{1}{4})$ in AFT (Ref. 8) and the critical point λ_c $(=\frac{1}{4})$ in FPT, particularly in the modified ladder SD equation.

[13]The renormalization-group-invariant expression for $\Sigma^{FPT}(p)$ is given by $\Sigma(p)\sim\Lambda_{TC}^2/p$, where $\Lambda_{TC}=\mu\times\exp[-\pi(\alpha/\alpha_c-1)^{1/2}]$ (Ref. 2). Thus $\Lambda_{TC}^2\simeq\Sigma_\mu^{FPT}\mu$. Precisely speaking, there exists a logarithmic factor in $\Sigma(p)$, i.e., $\Sigma(p)\sim\Lambda_{TC}^2p^{-1}\ln(p/\Lambda_{TC})$, an additional enhancement factor for m_f by $\simeq7$ $(\Lambda_S/\Lambda_{TC}\sim10^3)$.

[14]For $A\rightarrow\infty$, Eq. (10) actually yields, via the saddle-point method,

$$\Sigma(p)\sim[\ln(\sqrt{x}/\mu)+1]^{-1/4}\Sigma_\mu(\mu/\sqrt{x}),$$

the logarithmic factor being different from that in FPT (see Ref. 13). This factor yields 0.6 at $p\simeq\Lambda_S$ $(\Sigma_\mu^{FPT}\simeq0.6\Sigma_\mu)$ but is subject to the enhancement mentioned in Ref. 13. In view of the crude estimate, we may simply set $\Sigma_\mu^{FPT}\simeq\Sigma_\mu$.

[15]In FPT, we simply put $\beta(\Lambda_S)=0$ and $\alpha(\Lambda_S)=\alpha_c=\pi/3c_2(F)$. In AFT, we set $\alpha_\mu=\alpha_c$ (Ref. 8).

[16]We may list the values of R for unrealistic A: $R\simeq15$ $(A=17)$, 22 $(A=100)$, and 24 $(A=\infty)$.