Nonlinear Realization of Heavy Fermions and Effective Lagrangean

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We show that when a member of the quark doublet obtains a mass much larger than any other scale, this standard-model doublet becomes a nonlinear realization of SU(2). While the S-matrix elements are SU(2) invariant, the Green's functions are not. We explicitly display the one-loop effective Lagrangean with external Higgs scalars.

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This Letter reports some of the results we have obtained so far in our study of effects of heavy fermions within the framework of the standard electroweak theory. The motivation for us to look into the present problem is as follows: The pattern of masses of the various fermion families seems to indicate that in the fourth family, if in fact there is one more family, the mass of the up-member quark will be very large. In fact, perhaps the top quark in the third family is already heavier than W and Z. If so, our analysis should apply. Let us then generically call this heavy fermion top, and its lighter partner bottom.

Now, in the standard model, with only one Higgs doublet, the way to give a large mass to a fermion is to raise its Yukawa coupling to the Higgs boson. There are many questions we can ask in this situation.

We know that if we just raise the Yukawa couplings of all members of a multiplet so as to give large masses to all of them, we do not get decoupling. Instead, we have the residual Wess-Zumino terms[1] in the effective low-energy theory. We are going one step further: What will be the situation when we remove only a member of a multiplet? In a gauged theory, we have a U(1) anomaly and the theory is no longer renormalizable. We want to find out how serious is this nonrenormalizability.

On a more practical side, we want to obtain explicitly an effective Lagrangean, which is the most compact way to summarize all the heavy-mass effects. It is the perturbative aspect of this problem which will be emphasized and partly solved in this note. Our assumption here is that the Yukawa coupling H is much larger than the SU(2)⊗U(1) couplings $g_{1,2}$ and the other Yukawa couplings, but not so large that we need to discard perturbation theory.[2] We want to investigate low-energy physics. In other words, when M_t is the largest scale in a problem, compared with all other masses and external momenta, we want to find out how $\mathcal{L} \to \mathcal{L}_{\text{eff}}$.

As a start, we want to deal with a simplified model, in which we have no gauge fields. We have a Higgs doublet and one quark family (the gauged theory will be discussed later):

$$\mathcal{L}_{\text{L}} = -(\bar t,\bar b)_L \gamma^\mu \frac{1}{i}\partial_\mu \begin{pmatrix} t \\ b \end{pmatrix}_L - \bar t_R \gamma^\mu \frac{1}{i}\partial_\mu t_R - \bar b_R \gamma_\mu \frac{1}{i}\partial^\mu b_R - \partial_\mu \bar\phi\, \partial^\mu \phi$$
$$- \left[H(\bar t,\bar b)_L \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} t_R + \text{H.c.} \right] - \left[h(\bar t,\bar b)_L \begin{pmatrix} -\phi^+ \\ \phi^{0\dagger} \end{pmatrix} b_R + \text{H.c.} \right] - \frac{\lambda}{2}(\bar\phi\phi)^2 - \mu^2(\bar\phi\phi).$$

Let us look at the formal solutions at the tree level. We have for the mass of the t quark $M_t = Hv$, $v = \langle\phi^0\rangle = \langle\phi^{0\dagger}\rangle$,

$$\begin{pmatrix} t_R \\ t_L \end{pmatrix} = \frac{1}{M_t^2 + p^2} \begin{pmatrix} -\gamma\cdot p & M_t \\ M_t & -\gamma\cdot p \end{pmatrix} \begin{pmatrix} -H[\phi^+ b_L + (\phi^{0\dagger} - v)t_L] \\ -H(\phi^0 - v)t_R + h\phi^+ b_R \end{pmatrix} + \begin{pmatrix} t_{R_0} \\ t_{L_0} \end{pmatrix},$$

where t_{R_0} and t_{L_0} are solutions of the homogeneous equation

$$\begin{pmatrix} \gamma\cdot p & M_t \\ M_t & \gamma\cdot p \end{pmatrix} \begin{pmatrix} t_{R_0} \\ t_{L_0} \end{pmatrix} = 0.$$

There are similar equations for b and ϕ. By iterating these equations to obtain $t = t(t_0, b_0, \phi_0)$ etc., we obtain all the tree diagrams.

Here, we are interested in the situation when t_L and t_R cannot be produced. We should set $t_{L_0} = 0$, $t_{R_0} = 0$ for $M \gg p, m$ or $H \gg h, \lambda$, in which case, we have

$$\begin{pmatrix} t_R \\ t_L \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -[\phi^+ b_L + (\phi^{0\dagger} - v)t_L]/v \\ -(\phi^0 - v)t_R/v \end{pmatrix} + O(H^{-1}),$$

or

$$t_R=0,\quad t_L=-(\phi^+/\phi^{0\dagger})b_L\equiv(t_L)_{\text{nonlinear}}.$$

The last is a constraint $(\phi^{0\dagger},\phi^+)(^t_b)_L=0$, which is a SU(2)-invariant statement. To put it differently, under a SU(2) rotation $\exp(i\boldsymbol{\tau}\cdot\delta\boldsymbol{\alpha})$ we can check explicitly that

$$(\tilde{t}_L)'_{\text{nonlinear}}=(\tilde{t}_L)_{\text{nonlinear}}+i\delta\alpha_3(\tilde{t}_L)_{\text{nonlinear}}+(-i\delta\alpha_1+\delta\alpha_2)\tilde{b}_L$$

which shows that we have a *nonlinear realization of SU(2)*.

Note that we can expand ϕ^0 around its vacuum expectation value to make $(\phi^0)^{-1}$ meaningful. We can show that the nonlinear (NL) Lagrangean [$\tilde{t}_L=-(\tilde{\phi}^+/\tilde{\phi}^{0\dagger})\tilde{b}_L$; all quantities denoted by symbols with tildes over them correspond to classical solutions]

$$\mathcal{L}_{\text{NL}}=-\bar{\tilde{b}}_L\gamma^\mu i^{-1}\partial_\mu\tilde{b}_L-\bar{\tilde{b}}_R\gamma^\mu i^{-1}\partial_\mu\tilde{b}_R-\bar{\tilde{t}}_L\gamma^\mu i^{-1}\partial_\mu\tilde{t}_L-[h(\bar{\tilde{b}}_L\tilde{\phi}^{0\dagger}-\bar{\tilde{t}}_L\tilde{\phi}^+)\tilde{b}_R+\text{H.c.}]-\tfrac{1}{2}\lambda(\bar{\tilde{\phi}}\tilde{\phi})^2-\mu^2\bar{\tilde{\phi}}\tilde{\phi}$$
$$-\partial_\mu\bar{\tilde{\phi}}\partial^\mu\tilde{\phi}+(\bar{\eta}_R\tilde{b}_R+\bar{\eta}_L\tilde{b}_L+\bar{\tilde{\phi}}J_\phi+\text{H.c.})$$

reproduces all the tree results with only ϕ and b as external lines to the accuracy of $O(H^{-1})$.

Let us turn to the problems of quantum corrections.[3] We have the generating functional

$$\lim_{M_t\to\infty}e^{iW(J,\eta)}=\lim_{M_t\to\infty}\int d\phi\,d\bar{\phi}\,db\,d\bar{b}\,dt\,d\bar{t}\exp\left[i\int d^4x[\mathcal{L}_{\text{L}}+(\bar{\eta}_Rb_R+\bar{\eta}_Lb_L+\bar{\phi}J_\phi+\text{H.c.})]\right]$$
$$=\int d\phi\,d\bar{\phi}\,db\,d\bar{b}\exp\left[i\int d^4x[\mathcal{L}_{\text{eff}}+(\bar{\eta}_Rb_R+\bar{\eta}_Lb_L+\bar{\phi}J_\phi+\text{H.c.})]\right]$$

naively.

Note that we have made a subtle interchange of integration and the large-mass limit. This is a rather dangerous interchange, because the external momenta of a loop with a top quark inside may be some internal momenta of some bigger loops. The machinery which justifies this interchange after some rearrangement is the Zimmermann's algebraic identity.[4] In words, there are oversubtractions supplied by an algebraic identity which give meaning to the interchange.

We must digress here. By definition, any diagram which contributes to the effective Lagrangean has to have at least one heavy internal line. Calculationally, however, we want to exploit SU(2) invariance. It is better first to include some parts of diagrams which have only light internal lines and then subtract them out later. (For example, in the scalar sector, these contribute to terms which have up to four external derivatives; see later.)

We treat the "effective Lagrangean" which includes these extra pieces in a loop expansion. We have seen that the solution with $\mathcal{L}_{\text{NL}}$ gives us all the tree graphs, correct to order $O(H^{-1})$. This is a good point around which to do quantum fluctuation. We write $b_L=B_L+\tilde{b}_L$, $b_R=B_R+\tilde{b}_R$, $\phi=\Phi+\tilde{\phi}$, $t_L=T_L+\tilde{t}_L$, $t_R=T_R$. Quantities with lower-case letters and tildes are solutions of the nonlinear Lagrangean. Quantities which are capitalized, such as B, Φ, and T's are quantum fluctuations. Note that because $\tilde{b}$, $\tilde{\phi}$, and $\tilde{t}$ are *not exact* solutions of the Lagrangean $\mathcal{L}_{\text{L}}$, we have terms linear in fluctuations. Specifically, we have

$$\mathcal{L}_{\text{L}}=\mathcal{L}_{\text{NL}}+\left\{\bar{B}_R\left(\frac{\delta\mathcal{L}_{\text{L}}}{\delta\bar{b}_R}+\eta_R\right)\bigg|_{\text{L}\to\text{NL}}+\bar{B}_L\left(\frac{\delta\mathcal{L}_{\text{L}}}{\delta\bar{b}_L}+\eta_L\right)\bigg|_{\text{L}\to\text{NL}}+\bar{T}_L\frac{\delta\mathcal{L}_{\text{L}}}{\delta\bar{t}_L}\bigg|_{\text{L}\to\text{NL}}+\bar{T}_R\frac{\delta\mathcal{L}_{\text{L}}}{\delta\bar{t}_R}\bigg|_{\text{L}\to\text{NL}}\right.$$
$$\left.+\bar{\Phi}\left(\frac{\delta\mathcal{L}_{\text{L}}}{\delta\bar{\phi}}-\partial_\mu\frac{\delta\mathcal{L}_{\text{L}}}{\delta\,\partial_\mu\bar{\phi}}+J_\phi\right)\bigg|_{\text{L}\to\text{NL}}+\text{H.c.}\right\}+\text{higher orders in fluctuations.}$$

Upon using the equations of the nonlinear model, we have

$$e^{iW}=\exp\left[i\int d^4x\,\mathcal{L}_{\text{NL}}\right]\int d\Phi\,d\bar{\Phi}\,dB\,d\bar{B}\,dT\,d\bar{T}\exp\left[i\int d^4x\,\mathcal{L}'\right],$$

where

$$\mathcal{L}'=\{(\bar{B}_L\tilde{\phi}^-+\bar{T}_L\tilde{\phi}^0)(\tilde{\phi}^0)^{-1}(-\gamma^\mu i^{-1}\partial_\mu\tilde{t}_L+h\tilde{\phi}^+\tilde{b}_R)+(\Phi^0\bar{\tilde{t}}_L+\Phi^-\bar{\tilde{b}}_L)(\tilde{\phi}^0)^{-1}(-\gamma^\mu i^{-1}\partial_\mu\tilde{t}_L+h\tilde{\phi}^+\tilde{b}_R)+\text{H.c.}\}$$
$$+\text{higher orders in fluctuation.}$$

As we noted before, $(\tilde{\phi}^0,\tilde{\phi}^-)$ and $(\tilde{t},\tilde{b})_L$ transform like isodoublets. If we make a simultaneous isospin rotation of (Φ^0,Φ^-) and $(T,B)_L$, we can easily see that terms which are higher order in fluctuations are all isospin invariant. The combinations $\bar{B}_L\tilde{\phi}^- + \bar{T}_L\tilde{\phi}^0$ and $\Phi^0\bar{\tilde{t}}_L + \Phi^-\bar{\tilde{b}}_L$ are isosinglets. Then, the only term which is apparently noninvariant is

$$X = (-\gamma^\mu i^{-1}\partial_\mu \tilde{t}_L + h\tilde{\phi}^+\tilde{b}_R)/\tilde{\phi}^0.$$

However, if we use the equations of motion, we have, under SU(2), $\delta X = -(i\delta\alpha_1 + \delta\alpha_2)\eta_L/\tilde{\phi}^0$ which has no one-particle pole. Hence, W due to this part is in fact *SU(2) invariant on shell.*

The conclusion of this discussion is that when including the relevant light-particle diagrams we should end up with a SU(2)-invariant S matrix. For the Green's-function generating functional W, we have monomials built up from SU(2)-invariant quantities, which are formed by $(\tilde{t},\tilde{b})_L$, b_R, and $(\tilde{\phi}^0,\tilde{\phi}^-)$ and powers of X. We shall call these effective vertices.

The coefficients which multiply these monomials depend on powers of M_t and $\ln M_t$. We need a power-counting procedure to determine what this dependence can be and when we can drop terms. By definition a diagram has to have some heavy fermion lines to generate effective vertices. Also, the structure of the interaction is such that we do not have power infrared singularity due to light quarks and bosons. This means that M_t is the scale of the integral in inverse powers. A standard power counting establishes that for a given loop order (L), the maximum number of derivatives (N) which can appear in effective vertices with n_F external bottom lines is

$$N = 2(L+1) - n_F/2.$$

There is something quite noticeable in this relation. The number of external boson lines does not appear. This means that we can have any arbitrary number of them.

One way to calculate the effective Lagrangean is to use the external-field technique, with the aid of SCHOONSCHIP.[5] The calculational details will be reported elsewhere and here we only write down the one-loop results with external scalars: $\bar{\phi} = (\phi^{0\dagger},\phi^+)$,

$$\text{“}\mathcal{L}_{\text{eff}}^{\text{scalar}}\text{”} = -\text{“}V\text{”} + \text{“}\mathcal{L}_{\text{eff}}^{(2)}\text{”} + \text{“}\mathcal{L}_{\text{eff}}^{(4)}\text{”},$$

where

$$\text{“}V\text{”} = (16\pi^2)^{-1}\{(h^4+H^4)[\epsilon^{-1} + \Gamma'(1) - \ln\pi + 1 - \ln(\bar{\phi}\phi/\mu^2)] - H^4\ln H^2 - h^4\ln h^2\}(\bar{\phi}\phi)^2,$$

$$\begin{aligned}\text{“}\mathcal{L}_{\text{eff}}^{(2)}\text{”} = (16\pi^2)^{-1}(&-\{H^2[\epsilon^{-1}+\Gamma'(1)-\ln\pi-\ln H^2] + h^2[\epsilon^{-1}+\Gamma'(1)-\ln\pi-\ln h^2]\}\partial_\mu\bar{\phi}\partial^\mu\phi \\ &+ (h^2+H^2)(\ln\bar{\phi}\phi/\mu^2)\partial^\mu\bar{\phi}\partial_\mu\phi + [h^2\ln(H^2/h^2)]\partial_\mu\bar{\phi}\partial^\mu\phi \\ &+ \tfrac{1}{6}(h^2+H^2)[(\partial^\mu\bar{\phi}\phi)(\partial_\mu\bar{\phi}\phi) + (\bar{\phi}\partial^\mu\phi)(\bar{\phi}\partial_\mu\phi)]/\bar{\phi}\phi + [\tfrac{5}{6}h^2 - h^2\ln(H^2/h^2) + \tfrac{5}{6}H^2](\bar{\phi}\partial_\mu\phi)(\partial^\mu\bar{\phi}\phi)/\bar{\phi}\phi),\end{aligned}$$

$$\begin{aligned}\text{“}\mathcal{L}_{\text{eff}}^{(4)}\text{”} = \tfrac{1}{16}\pi^{-2}\{&\tfrac{1}{6}I^{\mu\nu,\mu\nu} - \tfrac{1}{30}I^{\mu\nu,0}_{0,\mu\nu} - \tfrac{1}{30}I^{\mu\nu,0}_{\mu\nu,0} - \tfrac{5}{18}I^{\mu\nu,\nu}_{\mu,0} - \tfrac{4}{9}I^{\mu\nu,\nu}_{0,\mu} - \tfrac{7}{90}I^{\mu\nu,0}_{\nu,\mu} + [-\tfrac{17}{60} + \tfrac{1}{6}\ln(H^2/h^2)]I^{\nu,\mu}_{\nu,\mu} + \tfrac{1}{2}C_{12}I^{\nu,\mu}_{\mu,\nu} \\ &+ \tfrac{1}{2}C_{13}I^{\mu,\mu}_{\nu,\nu} + \tfrac{1}{9}\,{}_{\mu,0}I^{\nu\mu,0}_{\nu,0} + \tfrac{1}{9}\,{}_{0,\mu}I^{\nu\mu,0}_{\nu,0} + \tfrac{2}{9}\,{}_{0,\mu}I^{\mu\nu,0}_{0,\nu} + \tfrac{2}{9}\,{}_{\nu,0}I^{\mu,\mu}_{\nu,0} + \tfrac{1}{2}C_{21}\,{}_{0,\mu}I^{\nu,\nu}_{\mu,0} + \tfrac{1}{3}\,{}_{\nu,0}I^{\nu,\mu}_{\mu,0} + \tfrac{1}{2}C_{24}\,{}_{0,\nu}I^{\nu,\mu}_{\mu,0} \\ &+ [\tfrac{5}{9} - \tfrac{1}{3}\ln(H^2/h^2)]\,{}_{\nu,0}I^{\nu,\mu}_{0,\mu} - \tfrac{1}{12}\,{}^{\nu,0}_{\nu,0}I^{\mu,0}_{\mu,0} - \tfrac{1}{3}\,{}^{0,\nu}_{\nu,0}I^{\mu,0}_{\mu,0} + [-\tfrac{7}{36} + \tfrac{1}{6}\ln(H^2/h^2)]\,{}^{0,\nu}_{0,\nu}I^{\mu,0}_{\mu,0} \\ &+ [-\tfrac{1}{18} - \tfrac{1}{6}\ln(H^2/h^2)]\,{}^{0,\nu}_{\nu,0}I^{\mu,0}_{0,\mu} + \text{H.c.}\},\end{aligned}$$

in which we have used dimensional continuation to regulate, with $\epsilon = 2 - n/2$, $\Gamma'(1) = -0.577215\ldots$, and μ is the subtraction scale. The divergences can be minimally subtracted by wave-function renormalization of ϕ and coupling renormalization of λ. We have also introduced a compact notation $I^{\mu\nu,\mu\nu}$, etc. Here, each pair of indices acts on a bilinear $\bar{\phi}\phi$. Thus,

$$I^{\mu\nu,\mu\nu} = \partial_\mu\partial_\nu\bar{\phi}\,\partial^\mu\partial^\nu\phi/\bar{\phi}\phi, \text{ etc.;}$$

clearly, the power of $\bar{\phi}\phi$ in the denominator of each term is determined by dimensional consideration. We have the following relations:

$$2C_{12} + C_{24} = \tfrac{8}{15}, \quad 2C_{13} + C_{21} = \tfrac{1}{9},$$

and

$$C_{12} + C_{13} = \tfrac{17}{45} - \tfrac{1}{3}\ln(H^2/h^2).$$

It is an amusing fact that in this model no physical process can be used to disentangle C_{12}, C_{13}, C_{21}, and C_{24} further.

As remarked earlier, to obtain the true effective Lagrangean, we should subtract off all the contributions from diagrams with only light internal bottom lines. Here, only neutral scalars can be emitted. We obtain

$$\mathcal{L}_{\text{light}}^{\text{scalar}} = -V_{\text{light}} + \mathcal{L}_{\text{light}}^{(2)} + \mathcal{L}_{\text{light}}^{(4)}.$$

V_{light} can be deduced from “V” by replacing $\bar{\phi} \to \phi^{0\dagger}$, $\phi \to \phi^0$ and setting $H \to 0$; $\mathcal{L}_{\text{light}}^{(2)}$ from “$\mathcal{L}^{(2)}$” in the same fashion; and $\mathcal{L}_{\text{light}}^{(4)}$ from “$\mathcal{L}^{(4)}$” by taking one-half the values for the various coefficients. The $\ln(H^2/h^2)$ terms cancel as they should. Then the true effective Lagrangean is

$$\mathcal{L}_{\text{eff}}^{\text{scalar}} = \text{“}\mathcal{L}_{\text{eff}}^{\text{scalar}}\text{”} - \mathcal{L}_{\text{light}}^{\text{scalar}}.$$

We have deliberately displayed $\ln(H^2/h^2)$ terms. These correspond to ϵ^{-1} divergences of the nonlinear Lagrangean $\mathcal{L}_{\text{NL}}$. The other $\ln H^2$ should be absorbed into wave-function and coupling renormalizations of the original linear Lagrangean $\mathcal{L}_{\text{L}}$.

One can also look into the situation when the Higgs scalar is heavy as well, $\lambda \sim H$. This will result in another constraint $\bar{\phi}\phi = v^2$ which can be used to reduce the number of invariants.[6]

Let us mention briefly the (gauged) standard model. Our calculational procedure is based on the external-field technique and therefore is particularly suited for the background gauge-field formulation. We shall report in other publications our results with external fermions.[7]

Here we shall take into account the mixing of the "top" quark with other light fermions to complete the full $SU(2)_L \times U(1)_Y$ gauge structure. Our approach as exemplified earlier will enable us to identify easily those processes which the standard model predicts to be enhanced by heavy-quark effects. This is of special interest in B^0-$\bar{B}^0$ mixing, where recent evidence points to a rather heavy top quark. In fact, the existing formulas were derived under the assumption $m_t \ll m_W$, while our approach is for $m_t \gg m_W$. Clearly, new operator structures should exist and will give new contributions to the low-energy theory. Another place where we will be able to make definite statements about the theoretical situation is radiative decays of B mesons and their enhancement due to heavy-quark effects.

The counter-term structure of the effective Lagrangean and apparently "nonrenormalizable" aspects, such as anomaly and Wess-Zumino terms, will be treated later. We shall then succeed in displaying all the heavy-fermion effects to this order in the standard model.

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