Nonlinear Wave Propagation in Fermi Liquids with Resonant Excitations across an Energy Gap: Application to Superfluid ^{3}He

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The propagation of waves in a superfluid Fermi liquid is discussed in the quasiclassical regime on the basis of a nonlinear kinetic equation. We show that the dominant nonlinearity appears in the distribution function (rather than the mean fields) due to resonant excitations across the gap in a small regime of momentum space. The dynamics of these resonantly excited quasiparticles is governed by a Bloch-type equation for a pseudospin vector in particle-hole space. We consider saturation effects and soliton formation in the sound propagation in ^{3}He-*A* in some detail and comment briefly on the *B* phase.

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It is well known that wave propagation in a nonlinear medium, containing, e.g., localized two-level systems which may be resonantly excited by the wave, exhibits striking nonlinear effects such as self-induced transparency and possibly soliton formation.[1] The only essential condition for the existence of these phenomena is the coherence of excitation and recombination processes at the two-level systems. It is therefore natural to ask whether such phenomena may occur even in a single-component system, i.e., when the propagating wave and the local excitations are part of the same many-body system, under conditions where there exist excitations of nonzero energy at zero momentum. Possible candidate systems are semiconductors,[2] where excitations across the energy gap may play the role of the two-level systems, superfluid neutral[3,4] or charged Fermi liquids with the possibility of excitations across the gap, or collective excitations in anisotropic superfluids.[5] In this Letter we consider specifically a neutral anisotropic superfluid Fermi liquid, ^{3}He-*A*, both because (i) there are some experimental data available for this system and because (ii) nonlinear effects are present already at sufficiently low frequencies such that collective order-parameter excitations may be neglected (in the seemingly simpler *B* phase the coupling to collective order-parameter modes is all important). Much of the discussion and the mathematical formulation, however, carries over to any other system of this general type.

In superfluid ^{3}He, quasiparticles are bound into Cooper pairs, thus introducing an (anisotropic) gap $\Delta_{\mathbf{k}}$ into the single-particle spectrum, $E_{\mathbf{k}}=(\xi_k^2+\Delta_{\mathbf{k}}^+\Delta_{\mathbf{k}})^{1/2}$, where $\xi_k=k^2/2m^*-\mu$ is the normal-state quasiparticle energy. An external field of frequency ω and wave vector $\mathbf{q}$, e.g., coupling to the density, will (i) excite particle-hole pairs of energy $\Delta E_{\text{ph}}=E_{\mathbf{k}+\mathbf{q}/2}-E_{\mathbf{k}-\mathbf{q}/2}$ and (ii) break Cooper pairs into a pair of quasiparticles of energy $\Delta E_{\text{pp}}=E_{\mathbf{k}+\mathbf{q}/2}+E_{-\mathbf{k}+\mathbf{q}/2}$. As a function of external field strength one should expect nonlinear effects to appear first in the distribution of resonantly excited quasiparticles, i.e., for momenta such that ΔE_{ph} or ΔE_{pp} is equal to an external field quantum $\hbar\omega$. The nonlinearity will be strongly affected by the coherent recombination of an excited pair. This is where the two-level analogy comes into the picture. Coherent recombination may be spoiled by collision processes, which can be reduced sufficiently by working at lower temperature. Even in the absence of collisions coherence may be lost by the partners of the excited pair if they are drifting away from each other for distances along $\mathbf{q}$ comparable to or greater than the wavelength of the exciting wave.

An additional nonlinear mechanism is due to the possibility for excited pairs to oscillate in the potential wells formed by the wave. The corresponding oscillation frequency is proportional to the square root of the wave amplitude.[6] This requires the resonant pair to move phase-locked with the wave. The effect of quasiparticles' drifting out of the resonant region has also been considered.[7] It may be shown that for normal liquid ^{3}He nonlinear behavior of the former kind[6] sets in only at relatively high power levels.[8] Moreover, in the case of sound waves in liquid ^{3}He, which we will consider exclusively, particle-hole pairs are not excited resonantly as the sound velocity is much greater than the Fermi velocity (this is different for spin waves). The most likely resonant excitation process in superfluid ^{3}He is therefore the coherent excitation and recombination of Cooper pairs. In special circumstances these processes may be strongly coupled to the collective dynamics of the order parameter. We find that a number of interesting nonlinear phenomena occur already at relatively low power levels.

In the following we briefly sketch the derivation of the equations of motion describing the nonlinear time evolution, discuss the method of solution employed, and present some results. A more detailed account will be given elsewhere.[9] We start from the matrix kinetic equation for the distribution function $\mathbf{n}_k$ in particle-hole space (see, e.g., Wölfle[10]),

$$i\,\partial_t \mathbf{n}_k(t,r)+[\boldsymbol{\epsilon}_k(t,r),\mathbf{n}]_- -\tfrac{1}{2}i\{\partial_k\boldsymbol{\epsilon},\partial_r\mathbf{n}\}_+ +\tfrac{1}{2}i\{\partial_r\boldsymbol{\epsilon},\partial_k\mathbf{n}\}_+ -\tfrac{1}{8}[\partial_r^2\boldsymbol{\epsilon},\partial_k^2\mathbf{n}]_- = i\mathbf{I}_k(t,r), \tag{1}$$

which completely describes the nonlinear dynamics of the system in the quasiclassical regime ($\omega \ll \epsilon_F, q \ll k_F$). The additional simplifications we use are the replacement of the collision integral $\mathbf{I}_k$ by a linear relaxation-time expression, and the usual linear expressions for the quasiparticle energy $\boldsymbol{\epsilon}_k$. The brackets in (1) denote commutators and anticommutators, respectively.

In the regime of low excitation intensity, both $\mathbf{n}_k$ and $\boldsymbol{\epsilon}_k$ may be linearized about their equilibrium values. The resulting equations have been solved, e.g., for the case of sound propagation in ^{3}He-A or ^{3}He-B. The calculated sound velocity and attenuation are generally in good agreement with experiment.[10] As the power level of the sound waves is increased, first the quasiparticles which are resonantly excited by the sound wave ($\omega = 2E_k$) will be driven out of the linear regime. It is that "resonant part" of the distribution function which we must describe in a more complete way. The quasiparticles outside the resonant regime in $\mathbf{k}$ space may still be treated in linear approximation. The width of the resonant regime, $\Delta E_{\mathbf{k}}$, is roughly equal to the band width of exciting frequencies. Since in experiment nonlinear wave propagation is usually studied with use of relatively short pulses (length τ) in order to avoid excessive heating, the resonant regime may be estimated by $\Delta E_{\mathbf{k}} \sim 1/\tau$.

In the resonant regime it is advantageous to rewrite (1) in the form of a Bloch equation, which shows the physics of pair excitation and recombination more directly. To this end one performs a Bogolyubov transformation of (1) onto Bogolyubov quasiparticle states. The nonequilibrium Bogolyubov quasiparticle distribution function may be written as $f_{\mathbf{k}} = \frac{1}{2}(1 + w_{\mathbf{k}} t_{\mathbf{k}})$, where $t_{\mathbf{k}} = \tanh(E_{\mathbf{k}}/2T)$. The function $w_{\mathbf{k}}$ is equal to -1 in the ground state and equal to $+1$ for the excited state of a Cooper pair $(\mathbf{k}, -\mathbf{k})$. This suggests that we identify $w_{\mathbf{k}}$ with the third component of a pseudospin vector $\mathbf{S}_{\mathbf{k}}$, $S_{\mathbf{k}3} \equiv w_{\mathbf{k}}$ (equal to the spin polarization). In terms of the components n_{ij} of the original distribution function,

$$S_{\mathbf{k}3} = (1/t_{\mathbf{k}} E_{\mathbf{k}})[n_{12}\Delta_{\mathbf{k}}^{+} + \Delta_{\mathbf{k}} n_{21} + \xi_{\mathbf{k}}(n_{11} - n_{22})]$$

(the quantities n_{ij}, $\Delta_{\mathbf{k}}$ are spin matrices). The off-diagonal components of the Bogolyubov-transformed matrix distribution function describe transitions between the ground and excited states and can be identified with the transverse components of the pseudospin vector, i.e.,

$$S_{\mathbf{k},1} = (|\Delta_{\mathbf{k}}|/t_{\mathbf{k}} E_{\mathbf{k}})[(\xi_{\mathbf{k}}/|\Delta_{\mathbf{k}}|^2)(\mathbf{n}_{12}\boldsymbol{\Delta}_{\mathbf{k}}^{+} + \boldsymbol{\Delta}_{\mathbf{k}}\mathbf{n}_{21}) - (n_{11} - n_{22})],$$

and

$$S_{\mathbf{k},2} = (-i/t_{\mathbf{k}}|\Delta_{\mathbf{k}}|)(\mathbf{n}_{12}\boldsymbol{\Delta}_{\mathbf{k}}^{+} - \boldsymbol{\Delta}_{\mathbf{k}}\mathbf{n}_{21}).$$

Note that $S_{\mathbf{k}}^{+,-} \equiv S_{\mathbf{k},1} \pm i S_{\mathbf{k},2}$ are the amplitudes for excitation and recombination of Cooper pairs. The remaining component of the distribution function does not couple in the limit of wave vector $q \to 0$. In the absence of relaxation processes the length of the vector $\mathbf{S}_{\mathbf{k}}$ is conserved, $\mathbf{S}_{\mathbf{k}}^2 = 1$. The evolution of the state of the system in time is hence described by a trajectory on the surface of the unit sphere. The vector $\mathbf{S}_{\mathbf{k}} = (S_{\mathbf{k}1}, S_{\mathbf{k}2}, S_{\mathbf{k}3})$ obeys the "Bloch equation"

$$\partial \mathbf{S}_{\mathbf{k}}/\partial t = \mathbf{H}_{\mathbf{k}} \times \mathbf{S}_{\mathbf{k}} - \mathbf{C}, \tag{2}$$

where the components of the vector $\mathbf{C}$ are given by $S_{\mathbf{k}_1}/T_2$, $S_{\mathbf{k}_2}/T_2$, $(S_{\mathbf{k}_3}+1)/T_1$, and T_1 and T_2 are the longitudinal and transverse relaxation times, respectively. In writing (2) we have neglected all gradient terms. It is possible, however, to include part of the gradient terms in $\mathbf{H}_{\mathbf{k}}$.[9] Equation (2) describes those states which act like localized two-level systems with respect to the propagation direction $\mathbf{q}$ of the exciting wave. For this to be true, we require that the two partners of an excited Cooper pair do not drift away along $\mathbf{q}$ farther than a wavelength during a characteristic time, τ^*, given by the collision time T_2 or the pulse duration τ, whichever is shorter. Alternatively, and equivalently, we may ask that the momentum q transferred by the phonon in the pair-breaking process be less than the width of the resonant regime, i.e.,

$$q \ll \Delta E_{\mathbf{k}}/\hat{\mathbf{q}} \cdot \nabla_{\mathbf{k}} E_{\mathbf{k}} = [v_F \tau^* (\hat{\mathbf{q}} \cdot \hat{\mathbf{k}})(1 - 4\Delta_{\mathbf{k}}^2/\omega^2)^{1/2}]^{-1}. \tag{3}$$

This condition, which characterizes the "two-level regime," is obviously always satisfied for quasiparticles traveling perpendicular to $\hat{\mathbf{q}}$ and/or at the pair breaking threshold, where $\omega = 2\Delta_{\mathbf{k}}$. Quasiparticles which have been resonantly excited but do not satisfy the above condition are "kicked out" of the resonance regime and cannot recombine coherently. These quasiparticles eventually recombine via collision processes. The above discussion shows that the omission of the gradient terms in (2) is justified within the two-level regime.

We now consider a sound pulse of frequency $\omega \ll 2\Delta_0$ and wave vector $\mathbf{q}$ in ^{3}He-A propagating along $\mathbf{l}$, the gap axis. In this case none of the collective order-parameter modes discussed by Wölfle[11] is excited. The effective magnetic field $\mathbf{H}_{\mathbf{k}}$ consists of a static field component $H_{\mathbf{k}3} = 2E_{\mathbf{k}}$, providing the resonance frequency, and an rf field perpendicular to it. The rf field is proportional to the relative amplitude of the density oscillations $\delta\hat{\rho} = \delta\rho/\rho_0$ associated with the sound wave (ρ_0 is the equilibrium density), i.e., $H_{\mathbf{k}1} = 2h_{\mathbf{k}}\delta\hat{\rho}$, $H_{\mathbf{k}2} = 0$. The energy $h_{\mathbf{k}}$ is given by $h_{\mathbf{k}} = 4\epsilon_F(\Delta_{\mathbf{k}}/\omega)P(\hat{\mathbf{k}} \cdot \hat{\mathbf{l}})$, with the angle-dependent coupling

$$P(\hat{\mathbf{k}} \cdot \hat{\mathbf{l}}) = (\hat{\mathbf{k}} \cdot \hat{\mathbf{l}})^2 - \langle (\hat{\mathbf{k}} \cdot \hat{\mathbf{l}})^2 \mathrm{Re}\lambda_{\mathbf{k}} \rangle_{\hat{\mathbf{k}}} / \langle \mathrm{Re}\lambda_{\mathbf{k}} \rangle_{\hat{\mathbf{k}}}.$$

Here $\lambda_{\mathbf{k}}$ is the Cooper pair susceptibility defined in Refs. 10 and 11, and the angular brackets denote an average over the Fermi sphere.

The nonresonant parts of the distribution function obey linear wave equations or oscillator equations, with the resonant parts acting as driving terms. In the special case considered one needs only one additional variable, or mean field, the relative density change $\delta\hat{\rho}$, which obeys the wave equation

$$[\partial_t^2 - c^2(\omega)\partial_z^2]\delta\hat{\rho} = -(9\omega^2/2F_0^s\epsilon_F)\left\langle \int_{\text{res}} d\xi_k (t_{\mathbf{k}}/2E_{\mathbf{k}})\Delta_{\mathbf{k}} P(\hat{\mathbf{k}}\cdot\hat{\mathbf{l}}) S_{\mathbf{k}1}\right\rangle_{\hat{\mathbf{k}}}. \tag{4}$$

The linear dispersion parts have all been collected into the renormalized sound velocity $c(\omega)$, which contains the correct sound dispersion in the linear regime (see Ref. 10). The coupling to the resonant part of the distribution function, $\mathbf{S}_{\mathbf{k}}$, is inversely proportional to the Landau parameter F_0^s (in the regime $F_0^s \gg 1$).

In order to solve the Bloch equations (2) it is necessary to separate the fast wavelike motion from the slow changes in the envelope function. This is achieved by introduction of the vector $\mathbf{S}' = (u,v,w)$ in the rotating frame as

$$S^{\pm} = S_{\mathbf{k}1} \pm iS_{\mathbf{k}2} = [u_{\mathbf{k}}(t,z) \pm iv_{\mathbf{k}}(t,z)]\exp\{-i[\omega t - Qz + \phi(z,t)]\},$$

and

$$\delta\hat{\rho} = R(t,z)\cos[\omega t - Qz + \phi(z,t)].$$

Here Q is the renormalized wave vector, and $\phi(z,t)$ is a phase function, which in principle affects the pulse form. In the case considered we may put $\phi = \text{const}$. We expect the in-phase amplitude $u_{\mathbf{k}}$ to contribute to the dispersion (thereby renormalizing the wave vector), and the out-of-phase amplitude $v_{\mathbf{k}}$ to determine the attenuation. Also, the normalization condition $u_{\mathbf{k}}^2 + v_{\mathbf{k}}^2 + w_{\mathbf{k}}^2 = 1$ holds. The resulting differential equations for u,v,w now have to be solved near resonance, i.e., for $\Delta\omega = 2E_{\mathbf{k}} - \omega \simeq 0$. In general this can only be done numerically. However, there are two cases of interest where an analytical solution is possible: (i) an "incoherent" regime for $\tau > T_{1,2}$, where collisions provide the limiting factor, and (ii) the fully coherent regime where $\tau \ll T_{1,2}$.

In the incoherent regime (see also Ref. 1) the pseudospin vector in the rotating frame is in a stationary state. The solution, obtained from (2) by dropping the time derivatives of u,v,w, is given by

$$\begin{aligned} v_{\mathbf{k}} &= +h_{\mathbf{k}} R w_{\mathbf{k}} T_2 L(\Delta\omega), \\ w_{\mathbf{k}} &= -1/[1 + T_1\Gamma_{\mathbf{k}}], \end{aligned} \tag{5}$$

where $L(\Delta\omega) = 1/[1 + (\Delta\omega T_2)^2]$ and $\Gamma_{\mathbf{k}}$ is the rate for induced transitions, defined by $\Gamma_{\mathbf{k}} = (h_{\mathbf{k}}R)^2 T_2 L(\Delta\omega)$. Note that $\Gamma_{\mathbf{k}}$ is proportional to the intensity of the wave. The first component $u_{\mathbf{k}}$ is odd in $\Delta\omega$ and leads to small corrections only.

We now substitute the result (5) into (4) and separate into in-phase and out-of-phase components. The power absorption is calculated from the out-of-phase component of (4) by multiplying with $\delta\hat{\rho}$ and integrating over all times. One finds the absorption coefficient as

$$\alpha = \tfrac{9}{2} Q (1/F_0^s) \langle \text{Im}\lambda_{\hat{\mathbf{k}}} P^2(\hat{\mathbf{k}}\cdot\hat{\mathbf{l}})[1 + T_1 T_2 (h_{\hat{\mathbf{k}}}R)^2]^{-1/2}\rangle_{\hat{\mathbf{k}}}, \tag{6}$$

where $Q \simeq \omega/c_1$ and c_1 is the velocity of first sound. In the limit of low intensity, where $\Gamma_{\mathbf{k}} \to 0$, (5) reduces to the known result, expanded to first order in the quantity $\text{Im}\lambda_{\mathbf{k}}$, which describes the pair breaking. This is correct as long as $\omega \ll \Delta$. For sound wave intensity [energy/(area) × (time)] exceeding the threshold value

$$I_{\text{thr}} \equiv \tfrac{1}{2} m\rho_0 c_1^3 R_{\text{thr}}^2 \simeq \hbar^2 (F_0^s c_1 \rho_0 / 6\epsilon_F T_2 T_1),$$

the sound absorption decreases proportional to (intensity)$^{-1/2}$. This result holds for arbitrary orientation of $\hat{\mathbf{q}}$ and $\hat{\mathbf{l}}$, as long as collective modes are not excited resonantly. Such a behavior has been observed in experiment.[4]

In the coherent regime, i.e., for pulses short compared to the collision times T_1, T_2, one can derive a differential equation for the "area" variable A defined by $A(z) = \int_{-\infty}^{\infty} dt\, h_0 R(z,t)$, which describes the evolution of the pulse area $h_0 = h_{\hat{\mathbf{k}}}(\hat{\mathbf{k}} = \hat{\mathbf{l}})$. To this end one integrates the out-of-phase component of (4) over time. Using the solution of the Bloch equations known from the optical resonance problem,[1] one finds $\partial_z A(z) = -\frac{1}{2}\tilde{\alpha}_{\text{lin}} \times \sin A(z)$, where $\tilde{\alpha}_{\text{lin}}$ is the contribution to the sound attenuation coefficient in the linear limit coming from the two-level regime (3). For sound pulse energies per area exceeding the threshold value $E_{\text{thr}}/F \cong \pi^2\hbar^2 \times (F_0^s c_1 \rho_0 / 6\epsilon_F \tau)$ nonlinear behavior occurs. First, mainly quasiparticles outside the two-level regime are resonantly excited. Once these states are depleted, the dynamics is governed by the coherent resonance processes within the narrow band of two-level states. If the remaining pulse area is larger than π, a 2π solitary wave is formed. The envelope function for the density change in the soliton is given[1] by

$$R(z,t) = (2/\tau h_0)\text{sech}[(t - z/v_G)/\tau],$$

with a group velocity $v_G = c_1/(1 + \frac{1}{2}\tau\tilde{\alpha}_{\text{lin}} c_1)$. Note that c_1/v_G is a linear function of τ, as a consequence of the inhomogeneous broadening, i.e., the continuous distribution of resonance frequencies $2E_{\mathbf{k}}$. The soliton solution depends crucially on $h_{\mathbf{k}}$ being independent of the direction of $\mathbf{k}$ within the resonant two-level regime ($E_{\mathbf{k}} = \omega/2$). This is only the case for sound propagation direction $\mathbf{q}$ parallel to $\hat{\mathbf{l}}$. For other orientations the strength of the "rf field" $h_{\mathbf{k}}$ varies with $\mathbf{k}$, i.e., in different portions of the resonant regime $\mathbf{S}_{\mathbf{k}}$ is rotated by varying amounts, which makes it impossible for the different $\mathbf{S}_{\mathbf{k}}$'s to return simultaneously to the initial state, as required for a 2π-soliton solution.

However, the saturation effect in the incoherent regime discussed above still exists even for $\mathbf{q}$ not parallel to $\hat{\mathbf{l}}$. In particular, it is found that in the case that a collective mode (clapping or flapping mode) is excited ($\omega = \omega_0 \simeq \Delta_0$), leading to broad attenuation peaks as a function of temperature or frequency in the linear regime, the absorption in the wings of the peak decreases $\propto$(intensity)$^{-1/2}$ for sufficiently large sound intensities. We predict[9] a sharp collective peak of width $1/T_0$ to remain, provided the sound intensity is well in excess of the threshold given by I_{thr} reduced by a factor

$$r = \tanh^2(\omega/4T)(\hat{\mathbf{q}}\cdot\hat{\mathbf{l}})^2[1-(\hat{\mathbf{q}}\cdot\hat{\mathbf{l}})^2][(\omega-\omega_0)^2+1/T_0^2]/\Delta_0^2,$$

where T_0 is of the order of T_2. This picture holds as long as nonlinear behavior of the collective variables may be neglected.

In the B phase, the situation is more complicated, since the two-level regime for direct excitation of Cooper pairs has too little weight.[9] However, Bogolyubov quasiparticles may be excited indirectly via a collective mode at the pair-breaking edge with frequency $\omega_0^+ = 2\Delta$. This mode in turn may be strongly excited by coupling to the second harmonic of one of the undamped collective modes in the gap (real squashing mode), which at high pressures happens to have a frequency $\omega_2^+ \simeq 1.01\Delta$. In fact, nonlinear behavior of sound propagation reminiscent of self-induced transparency has been seen in experiments probing the frequency regime in the neighborhood of the real squashing mode.[12] We can explain the qualitative features observed, like the initial rise in sound attenuation as a function of intensity, the nonlinear broadening of the attenuation peak, and the dependence of the group velocity of the solitary pulses on the pulse length τ.[9]

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