Spatiotemporal Observation of the Soliton-Antisoliton Collision in a Josephson Transmission Line

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The first experimental observation of a soliton-antisoliton annihilation process in solid state is reported. By precise control of the collision point of a fluxon and an antifluxon in a Josephson transmission line, the annihilation process is observed not only in time but also in space by the Josephson sampler which is coupled to the center of the transmission line. The observed line shapes agree well with the results of numerical calculations which show that a fluxon and an antifluxon fade off in the breather decay mode after collision, and that a fluxon behaves as a relativistic particle.

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The fundamental role of solitons in the physics of nonlinear waves has been well recognized.[1] Much of the recent activity is motivated by the successes of molecular dynamics in polyacetylene[2] and α-helical protein,[3] and of fluxon dynamics in Josephson junctions.[4–6] Although the solitons in these systems are modeled by several equations (for example, Korteweg–de Vries equation, ϕ^4 equation, sine-Gordon equation, etc.[7]), little information has been experimentally obtained concerning the nature of individual solitons and their interactions to verify the validity of these models. Only in their superconducting Josephson transmission line has the line shape of a soliton recently been observed by use of a conventional sampler[4] and Josephson sampler.[5,6] Several authors have studied the interaction between solitons by means of theoretical analyses,[8] numerical computations,[9] and analog simulations on electric nonlinear circuits.[10] Recently, Matsuda reported the observation of a fluxon-antifluxon collision deduced by measurements of delay times for propagating fluxons.[11] However, the actual dynamics of the interaction between solitons in time and space coordinates has never been directly observed in solid state, mainly because it has been difficult to obtain the real soliton wave forms in time and space coordinates with high resolution and with high sensitivity.

In this Letter we report the first direct observation of a soliton-antisoliton collision in solid state in time and space coordinates by use of a Josephson sampler. We also report a new experimental observation of Lorentz contraction of solitons. We compare experimental results with numerical computations, and discuss their implications for the application of soliton dynamics in a quantum computer.

Figure 1(a) shows a simplified block diagram of the measuring circuit system. The superconducting circuit comprises four components: a Josephson sampler, a discrete Josephson transmission line (JTL), and two identical fluxon generators (FGs). Figure 1(b) shows the equivalent circuit of the Josephson sampler. The sampler consists of a sampling pulse generator and a sampling gate resistively coupled to each other. When the SQUID switches to the voltage state, a sharp sampling current pulse is injected into the sampling gate. The timing of the sampling pulses is controlled by a mechanical delay, MD-1, which produces a delay of 800 ps maximum. The sampling gate is also connected to the center of the discrete JTL through a resistance of 3.4 Ω. According to a numerical calculation, a current fed from the JTL to the sampling gate has 350 μA of current peak when a fluxon and an antifluxon collide with each other at the center of the JTL. The sampling pulses are designed to have about 400 μA amplitude, which is sufficient to observe a fluxon (antifluxon) by this method.

Figure 1(c) shows the equivalent circuit of the discrete JTL under investigation. The JTL is composed of 31 Josephson junctions of 4×4 μm^2. The length Δx between two consecutive junctions is designed to be 60 μm, and the total length of the JTL is 1804 μm. The width of the overlap region is 14 μm, as shown in the inset of Fig. 1. In this structure the bias current I_b flows uniformly and normal to the length of the JTL. The details of the structure description will be reported elsewhere.[12] The superconducting circuit is fabricated by a conventional Pb-alloy technology on a Nb ground plane. $AuIn_2$ films are used as resistors, which have a sheet resistance of 0.6 Ω/□. In this experiment, the critical current density and the capacitance of junctions are 2.625 kA/cm^2 and 6.875 $\mu F/cm^2$, respectively, as measured by the monitor SQUID (the two-junction interferometer) on the same chip. The two fluxon generators (FG) shown in Fig. 1 are used to introduce fluxons into the JTL. When a current pulse is injected into the edge of the JTL with I_b supplied, fluxons are created and propagate along the JTL. The number of fluxons is controlled by I_b.

First we tried to observe a single fluxon propagating from FG-1. Figure 2(a) shows the experimental result of the current wave form of a fluxon for $I_b = 8.37$ mA. The pulse shape shown in Fig. 2(a) suggests that the sampler has a time resolution less than 4 ps. It is found that a single fluxon passes through the JTL when I_b is

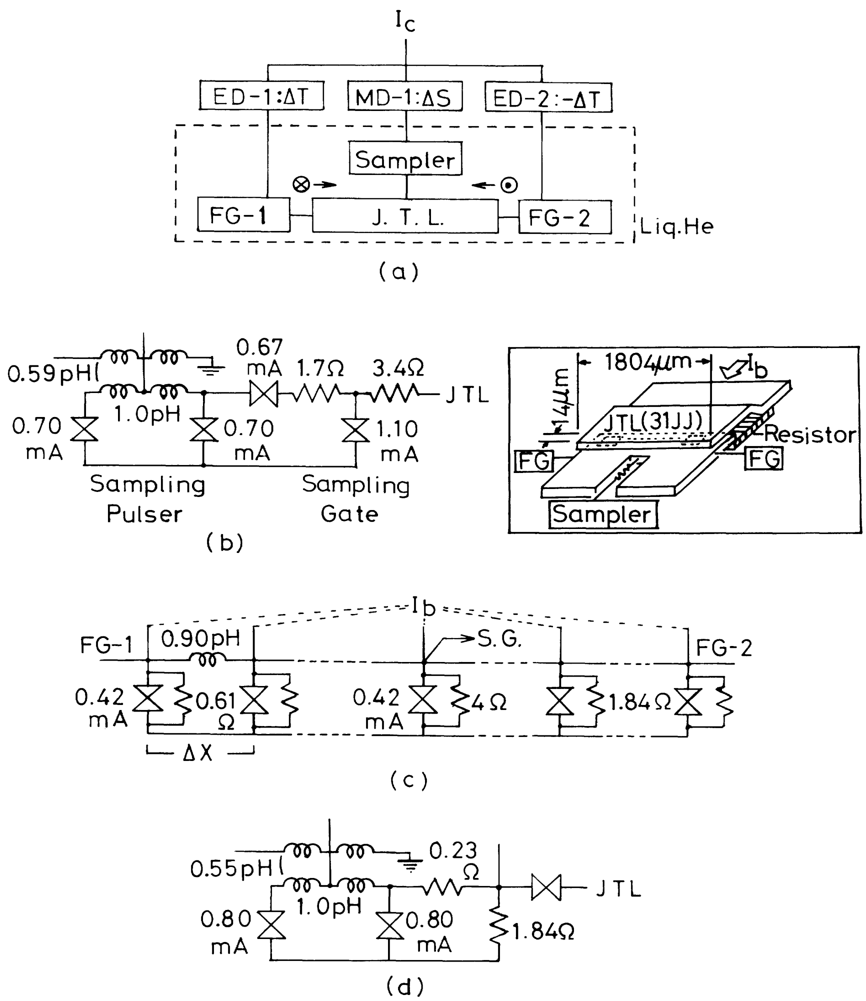

FIG. 1. (a) Simplified block diagrams of the measuring system including a superconducting circuit. Electric delays (ED-1, ED-2) play an important role in control of the position where a fluxon and an antifluxon collide with each other. (b)–(d) Equivalent circuits of a Josephson sampler, a discrete Josephson transmission line (JTL) under investigation, and a fluxon generator (FG) which generates an ultrashort pulse. Inset: Schematic configuration of JTL.

between 7.46 and 8.37 mA. No fluxon propagates for $I_b < 6.65$ mA, and the whole JTL switches to the voltage state after a single fluxon propagation for 9.39 mA $< I_b < 10.5$ mA. Figure 2(b) shows the bias-current dependence of the width and the height of a traveling pulse. The pulse width decreases, while the pulse height increases, with increasing I_b, and the product of the pulse height and the pulse width is nearly constant. This is consistent with the theoretical results for the sine-Gordon solitons of relativistic nature.

Now let us consider the interaction between a fluxon and an antifluxon. The line bias current I_b is fixed to be 8.37 mA while we measure the collision between a fluxon and an antifluxon. In order to observe the wave forms of fluxons in space, two electric delays (ED-1, ED-2) are introduced to the measuring system. A delay is achieved by the addition of an offset current to I_c,[13] since a fluxon is introduced into the JTL just when the control current I_c exceeds the threshold of the SQUID of FG-1 (FG-2). ED-1 produces a delay time ΔT for a fluxon to be introduced from FG-1, and ED-2 advances the timing by ΔT for an antifluxon to be introduced from FG-2. A fluxon and an antifluxon travel along the JTL in opposite directions under a proper value of I_b. The delay time ΔS of MD-1, which controls the sampler, is fixed while we observe a voltage wave form in space coordinate. When $\Delta T = 0$, these two fluxons collide with each other at the center of the JTL. If $\Delta T > 0$, the collision point moves towards FG-1 by a distance $v\Delta T$, where v is a power-balance velocity of a fluxon. The sampler reads out the voltage value at the position which is $v\Delta T$ distant from a colliding position. By the scanning of ΔT, a voltage distribution along the JTL is measured at a fixed time delay ΔS. Moreover, one can obtain the time evolution of the

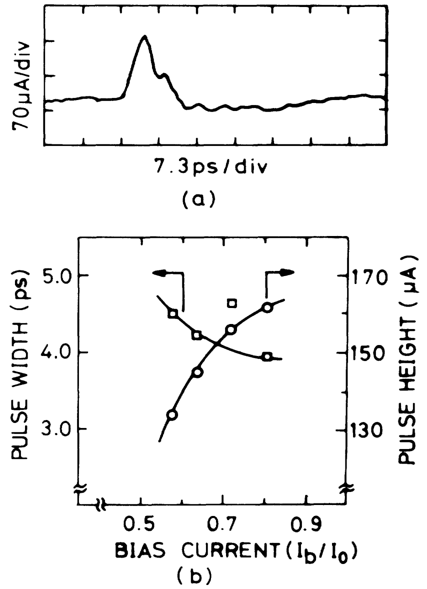

FIG. 2. (a) Current wave form of a traveling fluxon observed under the line bias current $I_b = 8.37$ mA. (b) Bias-current dependence of the width and the height of the traveling pulse. Bias current is normalized by the critical current of the JTL, 13 mA. A fluxon behaves as a relativistic particle because the product of the width and the height is nearly constant independent of the bias current.

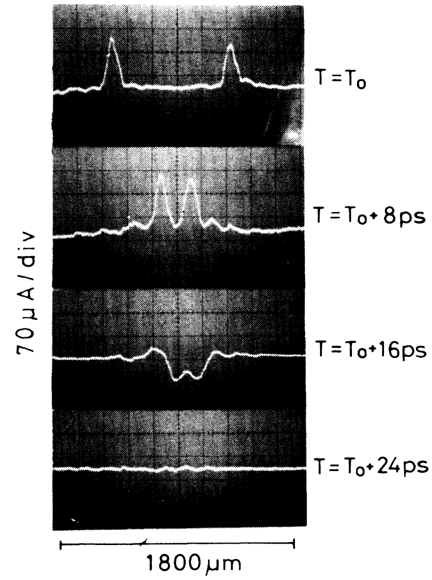

FIG. 3. A fluxon-antifluxon annihilation process observed by a new measuring system in space and time. After the collision, the two fluxons fade off by the breather decay mode.

voltage distribution by scanning ΔS.

Figure 3 shows the experimentally observed collision and annihilation of a fluxon and an antifluxon. The horizontal axis represents the space coordinate along JTL. The length scale was calibrated with use of the fluxon velocity of propagation $v = 4.2\times10^7$ m/s which was numerically estimated by the measured circuit parameters. The electric delay time ΔT is translated into the length along the horizontal axis shown in Fig. 3. In the top picture a free fluxon from FG-1 and a free antifluxon from FG-2 are observed. After 8 ps from the first picture, these fluxons start interacting, producing characteristic ripple waves behind them (second picture). At T_0+16 ps, the amplitude of the colliding fluxons is negative (third picture). The negative amplitude indicates that a fluxon and an antifluxon are in a breather decay mode in which a fluxon-antifluxon pair is bound together in a damped oscillatory state. The oscillatory wave in the breather mode decays dissipating the energy at the shunt resistances. And at T_0+24 ps, the collision is almost completed and two fluxons are annihilated (the last picture). The estimated time from collision to annihilation is roughly 16 ps.

The product LI_0 is estimated to be $0.2\Phi_0$ from the measured circuit parameters, where L (=0.9 pH) is the unit inductance of the JTL. According to the analysis of the static characteristics of the JTL, a fluxon at rest extends over 5 units (=300 μm) at $LI_0 = 0.2\Phi_0$. On the other hand, the speed of light c_d in the JTL is given by the expression $c_d = \Delta x/(LC_j)^{1/2}$, and is estimated to be 6.0×10^7 m/s. Here C_j is the capacitance of a Josephson junction of area 4×4 μm^2. Considering the actual velocity of a traveling fluxon $v = 4.2\times10^7$ m/s $= 0.7c_d$ and

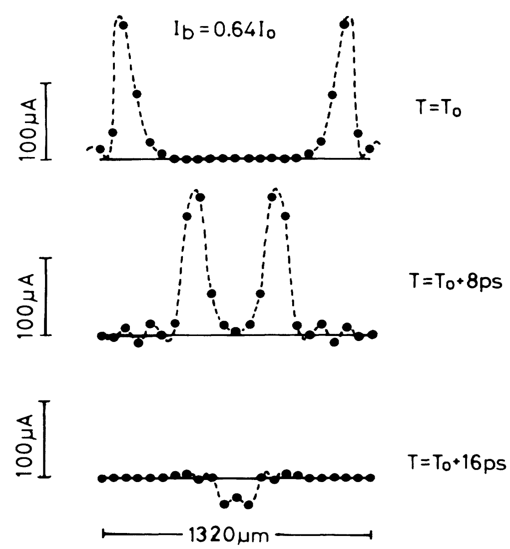

FIG. 4. Numerical results of the fluxon-antifluxon annihilation. These results agree well with the experimental results shown in Fig. 3. The circles denote the positions of each junction.

Lorentz contraction $l = l_0[1-(v/c_d)^2]^{1/2}$, the length occupied by a traveling fluxon is estimated to be 210 μm. The experimental result in Fig. 3 shows that a fluxon extends over 200 μm, and agrees well with the numerical results. It is evident from these results and Fig. 2(b) that a fluxon behaves as a relativistic particle as is expected.

Figure 4 shows the voltage distribution during the fluxon-antifluxon collision by numerical calculation. In the calculation, we use the measured values shown in Fig. 1 as circuit parameters. Numerical results agree well with the experimental results, except for the fact that the front of the soliton is more abrupt for the experimentally observed shape than the numerical simulation.

A fluxon-antifluxon annihilation can be regarded as one of logic operations in the phase-mode logic circuit,[14] in which fluxons are employed as information bits. The results obtained in this experiment may contribute to practical applications of the phase-mode circuits. However, a fluxon extending over 200 μm is too large to be employed as an information bit. Moreover, the length of 400 μm over which a fluxon and an antifluxon interact with each other is also too large to be employed as a logic gate. In order to reduce the size of the phase-mode circuit, the kinetic inductances[15] given by the Josephson junction must be effectively used instead of the spatial inductances of the superconducting strip lines.

In conclusion, we have observed dynamics of a fluxon-antifluxon interaction in time and space on the discrete JTL by a modified Josephson-sampling technique. A fluxon and an antifluxon moving in opposite directions annihilate each other by the breather decay mode after collision. Experimental results show that a fluxon behaves as a relativistic particle.

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[1] For example, S. E. Trullinger, V. E. Zakharov, and V. L. Pokrovsky, *Solitons* (North-Holland, Amsterdam, 1986).

[2] W. P. Su, J. R. Schrieffer, and A. J. Heeger, Phys. Rev. Lett. **42**, 1698 (1979).

[3] A. S. Davydov, Phys. Scr. **20**, 387 (1979).

[4] A. Matsuda and T. Kawakami, Phys. Rev. Lett. **51**, 694 (1983).

[5] H. Akoh, S. Sakai, A. Yagi, and H. Hayakawa, IEEE Trans. Magn. **21**, 737 (1985).

[6] A. Fujimaki, K. Nakajima, and Y. Sawada, J. Appl. Phys. **61**, 5471 (1987).

[7] A. C. Scott, F. Y. F. Chu, and D. W. McLaughlin, Proc. IEEE **61**, 1443 (1973).

[8] N. F. Pedersen, M. R. Samuelsen, and D. Welner, Phys. Rev. B **30**, 4057 (1984).

[9] K. Nakajima, Y. Onodera, T. Nakamura, and R. Sato, J. Appl. Phys. **45**, 4095 (1974).

[10] For example, K. Lonngren and A. C. Scott, *Solitons in Action* (Academic, New York, 1978).

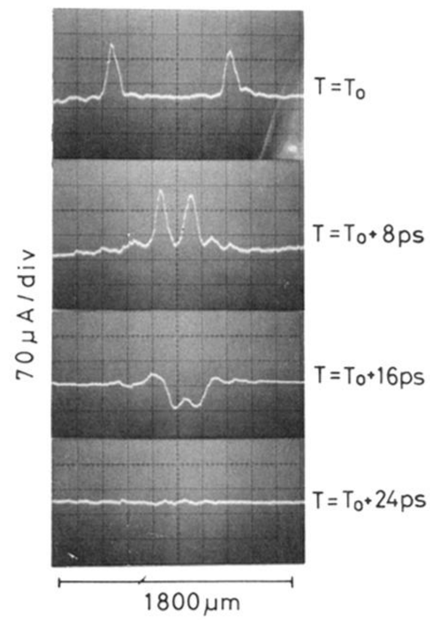
[11] A. Matsuda, Phys. Rev. B **34**, 3127 (1986).

[12] K. Nakajima, A. Fujimaki, and Y. Sawada, to be published.

[13] P. Wolf, B. J. Van Zeghbroeck, and U. Deutsch, IEEE Trans. Magn. **21**, 226 (1985).

[14] K. Nakajima, G. Oya, and Y. Sawada, IEEE Trans. Magn. **19**, 1201 (1983).

[15] T. Yamashita, Y. Ogawa, and Y. Onodera, J. Appl. Phys. **50**, 3547 (1979).



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