Chaotic Transients and Multiple Attractors in Spin-Wave Experiments

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A long-lived transient form of chaos was observed by rf perpendicular pumping of spheres of yttrium iron garnet in the region of the first-order Suhl instability. The average temporal lengths of these transients as functions of pumping power were fitted by a theory involving capture of the chaotic trajectory by multiple periodic attractors.

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The unusual behavior of the ferromagnetic resonance line and spin waves in yttrium iron garnet (YIG) when subjected to high-power radio-frequency (rf) magnetic fields has been a long-standing problem.[1,2] Suhl,[3] in an analysis of spin-wave instabilities, explained why the resonance line saturates at relatively low rf powers, but detailed explanations of the behavior of the spin waves in this system have had to wait for recent developments in the field of nonlinear dynamics.[4-8]. Recent experiments[9-11] have shown that many of the behavioral patterns found in nonlinear flows and maps can also be found in ferromagnetic and ferrimagnetic materials in parameter regimes beyond the onset of instabilities. For a review of applications of nonlinear dynamics to solid-state experiments see Jeffries[12] and Zettl.[13]

We have applied certain methods of nonlinear dynamics to explain long-lived chaotic transients that we have observed in spheres of YIG. We note that Gorman, Widmann, and Robbins[14] have made some qualitative observations of chaotic transients in a convection-loop experiment using nonlinear-dynamics concepts. We present here a detailed quantitative study of such transients in YIG, with comparison to recent theoretical results.[15-17] The average time lengths of these transients, varying by more than 5 orders of magnitude, form a very stable feature of a system that otherwise displays great sensitivity to initial conditions. Our findings suggest multiple quasiperiodic attractors in prechaotic regimes of microwave power which are distinguishable in size, dimension, and period type. Furthermore, the unstable chaotic attractor appears to overlap the basins of attraction of more than one of these quasiperiodic attractors.

At small rf driving fields which are perpendicular to the dc magnetic field, the spins precess uniformly about the dc field. At rf fields above the Suhl instability, energy is transferred from the uniform mode of precession into spin-wave modes. For the first-order Suhl instability, the rate of transfer of energy into the spin waves goes as first order in the amplitude of the uniform mode and the first spin waves to be excited, corresponding to the smallest critical field, have a frequency half that of the driving frequency. We have chosen the rf frequency so that half the frequency is within the spin-wave band for $k=0$, making it possible to excite a large number of spin-wave modes and decreasing the rf field necessary for the onset of the Suhl instability. When two or more spin waves are excited, their interaction produces a difference frequency that modulates the amplitude of the detected spin-wave signal. These "auto-oscillations," with frequencies ranging from 5 to 400 kHz, are what we detect here.

In the experimental setup a 20-mil YIG sphere was held with Apezion M grease inside a quartz tube so that the YIG sphere was between an excitation coil and a perpendicular pickup coil. An electromagnet provided a dc field perpendicular to both coils, and parallel to the easy axis of the YIG sphere. Microwave power was provided by an HP 8341A synthesized sweeper and modulated with an HP 11665B modulator biased by a square-wave generator so that the ratio of power on to power off was greater than 40 dB. The signal transmitted through the YIG sphere was detected by a crystal detector and amplified. The signal was digitized at 3 MHz with a LeCroy TR8828C digitizer with eight-bit resolution to produce a time series and downloaded to a VAX 11/780 computer or stored in a Norland 3001 waveform analyzer. All measurements were made with the system tuned to the center of the ferromagnetic resonance line at 760 G and 2.5 GHz. The YIG sphere was weakly coupled to the excitation coil. To insure reproducibility, two different 20-mil spherical samples were used.

At $t=0$ the microwave power was turned on and left on. Initially the spin-wave system behaved chaotically, but after some time changed abruptly into a quasiperiodic waveform. The base line of the quasiperiodic waveform decayed exponentially leaving only the quasiperiodic motion as the asymptotic behavior of the system. These three types of behavior, chaos, exponential decay to a quasiperiodic waveform, and final stable quasiperiodic waveform, may be seen in Fig. 1.

The sampling of transients under a wide range of uniformly distributed initial conditions is essential in obtaining good average transient times.[15] While all experimentally controlled parameters were kept the same for a given microwave power, the individual transient lengths fit the distribution $\mathcal{P}(t) \sim \exp(-t/\langle t \rangle)$, where $\langle t \rangle$ was

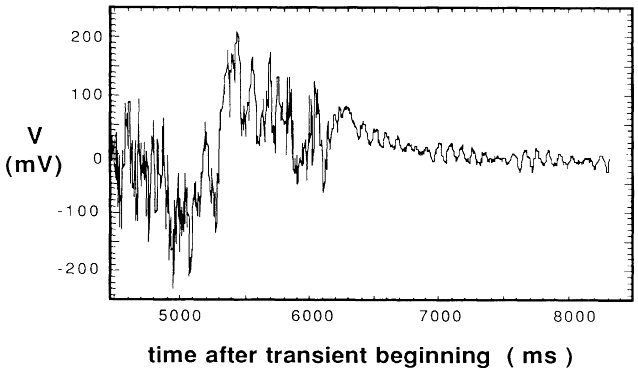

FIG. 1. Detected power as a function of time after driving microwave power was turned on, showing chaos, sudden appearance of a quasiperiodic waveform with an exponentially decaying base line, and stable quasiperiodic waveform.

the average transient length, indicating that immeasurably small fluctuations in these parameters caused the system to be started under a wide variety of initial conditions.[15] In fact, when the system was started in a different region of phase space by our simply stepping up the microwave power level by about 6 dB (instead of turning it on from zero, as with the data displayed here) the same average chaotic transient lengths were recorded.

The length of time of the chaotic transient averaged over forty episodes at each power is plotted in Fig. 2 as a function of microwave power above the critical field for the onset of the Suhl instabilities.[3] The relation of incident microwave power and rf magnetic field at the sample depends on the exact geometry of the sample and driving coil, and so we consider instead the ratio of the incident microwave power to the microwave power at the onset of the Suhl instability by representing microwave power in decibels above the instability. In this paper, capital P refers to relative power units while lower case p refers to decibels. The amplitude of the rf magnetic field at the onset of the Suhl instability agreed well with previous work.[2] There appear to be two distinct regions in the plot of average transient length: one below and one above 18.4 dB. While the general form of the curve of average time versus power in these two regions appears similar, there is a break in the curve at the boundary between these two regions. The average transient lengths presented here vary over 5 orders of magnitude.

Phase-space analysis was done by our embedding the time series of data $(V_1, V_2, V_3, \ldots)$ from each transient-decay–quasiperiodic episode in a d-dimensional phase space by the usual method of constructing vectors from the series by picking a delay of n data points and forming the d-uple $(V_i, V_{i+n}, V_{i+2n}, \ldots, V_{i+(d-1)n})$ for all points V_i in the data.[18] This enabled us to reconstruct the attractor corresponding to the given time series. The information (or pointwise) dimensions[19] of the chaotic

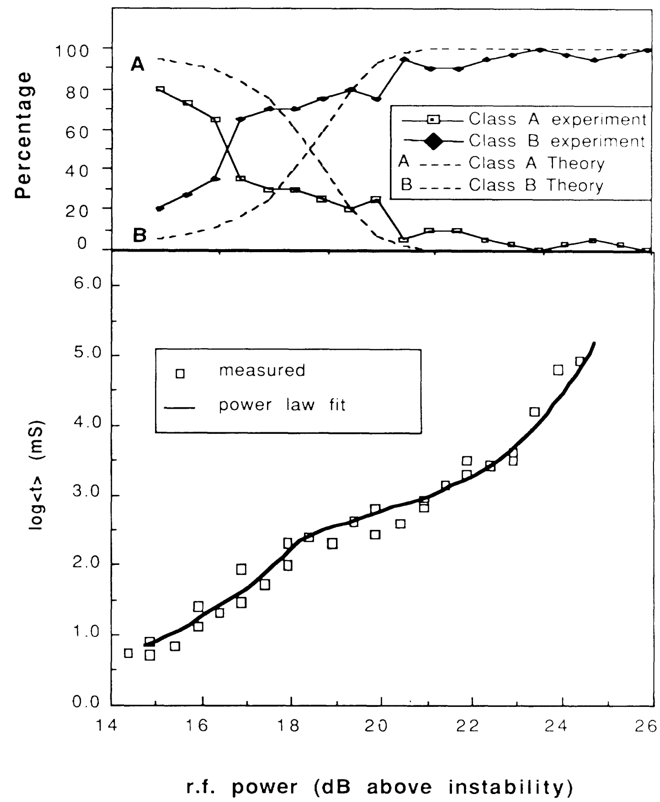

FIG. 2. Lower graph: average temporal lengths of chaotic transients at different microwave powers above the Suhl instability. The line though these points is a fit by theory. Upper graph: distribution of the two distinct types of final nonchaotic attractor as determined by an examination of phase-space trajectories from the data and as determined from rate equations for average transient times. The x axis is the same as in the lower graph.

attractors were calculated by use of the method of Termonia and Alexandrowicz.[20] This was done for embedding spaces of dimensions 2 to 10 and the slope of the log(neighbor-order) versus log(distance) was seen to saturate. The dimensions of the chaotic attractors were found to be 2.7 ± 0.2 for all microwave powers.

At the end of the chaotic transient a quasiperiodic waveform suddenly appeared. The average voltage of this quasiperiodic waveform (taken over an integral number of approximate periods long after the end of the chaotic transient) was used as the center in phase space of the final quasiperiodic attractor. The distance in phase space (averaged over one approximate period) between all points after the end of the chaotic transient and this center decayed according to the relation $V \sim \exp(-\alpha t) + V_0$, where V_0 corrected for the nonzero base line. The time constant α was $(2.5 \pm 0.5) \times 10^3$ sec^{-1}, or a decay time of $(4 \pm 1) \times 10^{-4}$ sec for all time series. This may be compared with a spin-lattice relaxation time of 7×10^{-8} sec in YIG[21] or an inverse linewidth ($\Delta H/\gamma$) for this experiment of 5.5×10^{-7} sec.

Although the chaotic transient decays into many different quasiperiodic waveforms, depending very sensitively on the initial conditions, the average behavior of this system follows a regular pattern. In general these final waveforms are not reproducible in detail, but reconstruction of the corresponding attractor in phase space reveals that they fall into two distinct classes: class-A attractors, which have a pointwise dimension of 4, approximate periods between 1×10^{-5} sec and 5×10^{-5} sec; and class-B attractors, which have a pointwise dimension of 5, are also quasiperiodic, and are smaller in amplitude than the class-A attractors.[22] The radius in phase space of a class-A attractor is less than one-quarter that of the initial chaotic attractor, while the radius of the class-B attractor is half or less than that of the class-A attractor. The Fourier spectra of both class-A and -B attractors consisted of discrete lines, while the Fourier spectrum of the chaos was a continuous band. Figure 2 shows the percentage of chaotic transients decaying into class-A and class-B attractors. At the lowest microwave power, most of the final nonchaotic attractors are of type A. As power is increased, the system quickly begins to fall more often into a class-B attractor. Figure 3 is a three-dimensional phase-space map of a typical time series for this system. The chaotic portion of the time series extends over most of the phase space. Near the center is the toroidal quasiperiodic attractor to which the system evolves.

The above behavior follows the general scheme for chaotic transients according to Grebogi, Ott, and Yorke,[15-17] in which a stable chaotic attractor changes into an unstable chaotic attractor when it collides with the boundary of its basin of attraction at some critical value P_c of the system driving parameter. In our case, the system driving parameter is the microwave power and the stable chaotic attractor exists above some critical power. Below the critical power, according to this theory, some portion of the chaotic attractor overlaps the basins of attraction of stable nonchaotic attractors, the aperiodic or quasiperiodic final states in our case. Depending very sensitively on initial conditions, the system will initially spend some length of time following the chaotic attractor. As soon as the system trajectory reaches the region where the chaotic attractor overlaps the basin of attraction of the nonchaotic attractor, the system will leave the chaotic attractor and move toward the nonchaotic attractor. Grebogi, Ott, and Yorke[15-17] have determined from numerical experiments and heuristic arguments that the average length of time that the system spends on the chaotic attractor should depend on the system driving parameter P (here the microwave power in relative power units) as $\langle t\rangle = K/(P_c-P)^{\gamma}$.

This result should be true for a system where the chaotic attractor overlaps the basin of attraction of one nonchaotic attractor. In this spin-wave system, however,

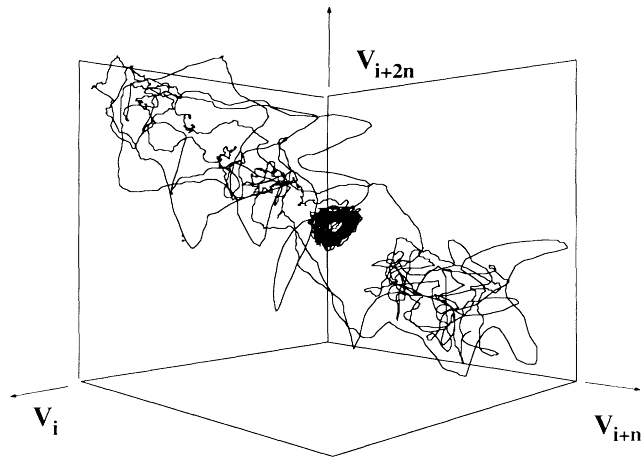

FIG. 3. Three-dimensional phase-space plot with time delay $n=4$ (see text) showing the chaotic attractor filling most of the phase space and changing into a quasiperiodic attractor, which appears as a dark toroidal region near the center of the plot.

there are two distinct types of nonchaotic attractors present after the end of the chaos: one more prevalent below the break in the plot in Fig. 2 and one more prevalent above. The system trajectory while in the transient is long, convoluted, and entangled, suggesting that the overlap of the metastable strange attractor with the basin of attraction of either final, nonchaotic attractor is small. This suggests that there are two independent parallel and competing paths for an exit from the chaotic regime. Therefore we fitted the transient length versus power curve as the decay of the chaotic transient into two different attractors using the function

$$\langle t\rangle = K_1K_2/[K_2(P_{c1}-P)^{\gamma_1}+K_1(P_{c2}-P)^{\gamma_2}].$$

The fit is very good, with $p_{c1}=21.8$ dB and $\gamma_1=13.0$ and $p_{c2}=25.9$ dB and $\gamma_2=6.1$. These values were used to plot the "theoretical" distribution of final attractor types in Fig. 2 from rate equations based on the above expressions for $\langle t\rangle$.

The general behavior of the system fits well with the following scenario. At the lowest powers studied, the chaotic trajectory in this spin-wave system overlaps the basins of attraction of two nonchaotic attractors. At these low powers the system falls into the class-A attractor (corresponding to γ_1 and p_{c1}) more often than the class-B attractor. As power is increased, the overlap between the chaotic attractor and the class-A attractor's basin of attraction decreases more rapidly than the overlap between the chaotic attractor and the class-B attractor's basin, resulting in the larger γ_1 value. Near 17 to 18 dB there is a crossover in rates at which the system falls into the class-A attractor or the class-B attractor, and the distinct change in the transient length versus power curve appears. Long-lived chaotic transients which are excited fit an extended Grebogi-Ott-Yorke scheme.[15-17] The average lifetimes of these transients are a very stable feature of a system that is otherwise very sensitive to initial conditions and parameter changes.

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