Causes for Nuclear Collective Flow Revealed by Its Mass Dependence

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(Received 13 August 1987)

The energy, multiplicity, and mass dependences of the exclusive observable "flow" for heavy-ion reactions are studied in a model which includes mean-field but lacks compression effects. The energy and multiplicity dependences of recently published Plastic-Ball data are qualitatively reproduced but the mass-number dependence is not. This as well as the comparison with other models at present favors viscous hydrodynamics with an ideal-gas equation of state and a closely related microscopic reaction mechanism as candidates for yielding a "flow" compatible with the Plastic-Ball analysis.

PACS numbers: 25.70.Np

Exclusive measurements of particles emitted in high-energy collisions of two massive nuclei exhibit sideward flow.[1] There is generally agreement that at least part of this arises from the presence of the nuclear mean field. However, whether the flow mainly originates from the attractive or the repulsive contributions to the force is subject to discussion.[2,3] This question is closely related to the fundamental problem of how to gain insight into the equation of state of nuclear matter far away from its ground-state properties.

The recently proposed average transverse momentum per nucleon as a function of rapidity[4] is a sensitive indicator of sideward flow. Still, this observable is influenced by unwanted spectator-particle contributions. To avoid these a closely related quantity has been introduced[5]: the "flow" which is essentially the slope of the average transverse momentum taken at midrapidity. The dependences of the flow on multiplicity M, beam energy E_0, and mass number A have to some extent been experimentally studied.[5] In the present work it is argued that measurements of this type provide a crucial test for the origin of the sideward flow.

We base our investigation on Boltzmann's theory of dilute gases with two separate mean force fields. These fields arise from two distinct classes of nucleons which in the nucleus-nucleus center-of-mass (c.m.) frame move in forward and backward directions, respectively, implying transparency of a high-energy heavy-ion reaction. Through both of these moving potentials, each participant nucleon receives a momentum transfer. The net result $\boldsymbol{\delta}_n(\mathbf{b})$ with the number of binary collisions n and the impact parameter vector $\mathbf{b}$ is, in general, different from zero. Within reasonable approximations we obtain the time-integrated solution of the Boltzmann equation, i.e., the nuclear momentum distribution for given impact parameter, $P(\mathbf{b};\mathbf{p})$, in closed form.[6]

The average transverse momentum per nucleon with the (x,z) plane as reaction plane is

$$\left\langle \frac{p_x}{N} \right\rangle(\mathbf{b},p_z) = \frac{\int dp_x\, dp_y\, p_x P(\mathbf{b};\mathbf{p})}{\int dp_x\, dp_y\, P(\mathbf{b};\mathbf{p})}. \tag{1}$$

The flow is defined by

$$f(\mathbf{b}) = \left[\frac{d\langle p_x/N \rangle}{d(p_z/p_0)} \right]_{p_z=0}, \tag{2}$$

where p_0 is the initial c.m. momentum per nucleon. For both (1) and (2) we obtain analytic expressions[2] whose key quantity is the net transverse momentum transfer $\boldsymbol{\delta}_n$ discussed above.

The results of our model calculations for the specific reaction Nb+Nb are displayed in Fig. 1. In part (a) the average transverse momentum per nucleon for central collisions is shown as a function of the c.m. rapidity $y_{\text{c.m.}}$. The differences between the curves with and without momentum-dependent force terms are typically of the order of (20–30)%.[8] Figure 1(b) displays the flow as a function of the beam energy per nucleon without multiplicity restriction, again with and without momentum dependence. The flow is seen to vary smoothly with energy, exhibiting a broad maximum at roughly 500 MeV per nucleon. In Fig. 1(c) the dependence of the flow on the impact parameter to nuclear radius b/R is shown under the assumption of a linear relationship with the ratio of multiplicity to maximum multiplicity M/M_{max}.[9] Observe that the flow possesses a maximum at $b \approx R$ corresponding to an intermediate multiplicity of $M \approx 0.6 M_{\text{max}}$ [cf. Fig. 1(d)]. The shape of the flow has a strong similarity with that of the flow angle calculated within the same approach.[8] This is plausible because it turns out that for our model

$$f \approx y_0 (r-1) \sin\theta_f \cos\theta_f. \tag{3}$$

Here, y_0 is the initial c.m. rapidity, r is the aspect ratio (i.e., the ratio of the largest to the smallest principal axis of the kinetic energy flow tensor), and θ_f is the flow angle (i.e., the angle between the largest principal axis and the beam axis). Relation (3) holds for beam energies $\lesssim 400$ MeV per nucleon. In our model the aspect ratio is a slowly varying function of the impact parameter whereas the flow angle changes much faster, with its maximum value at $b = R$.[8]

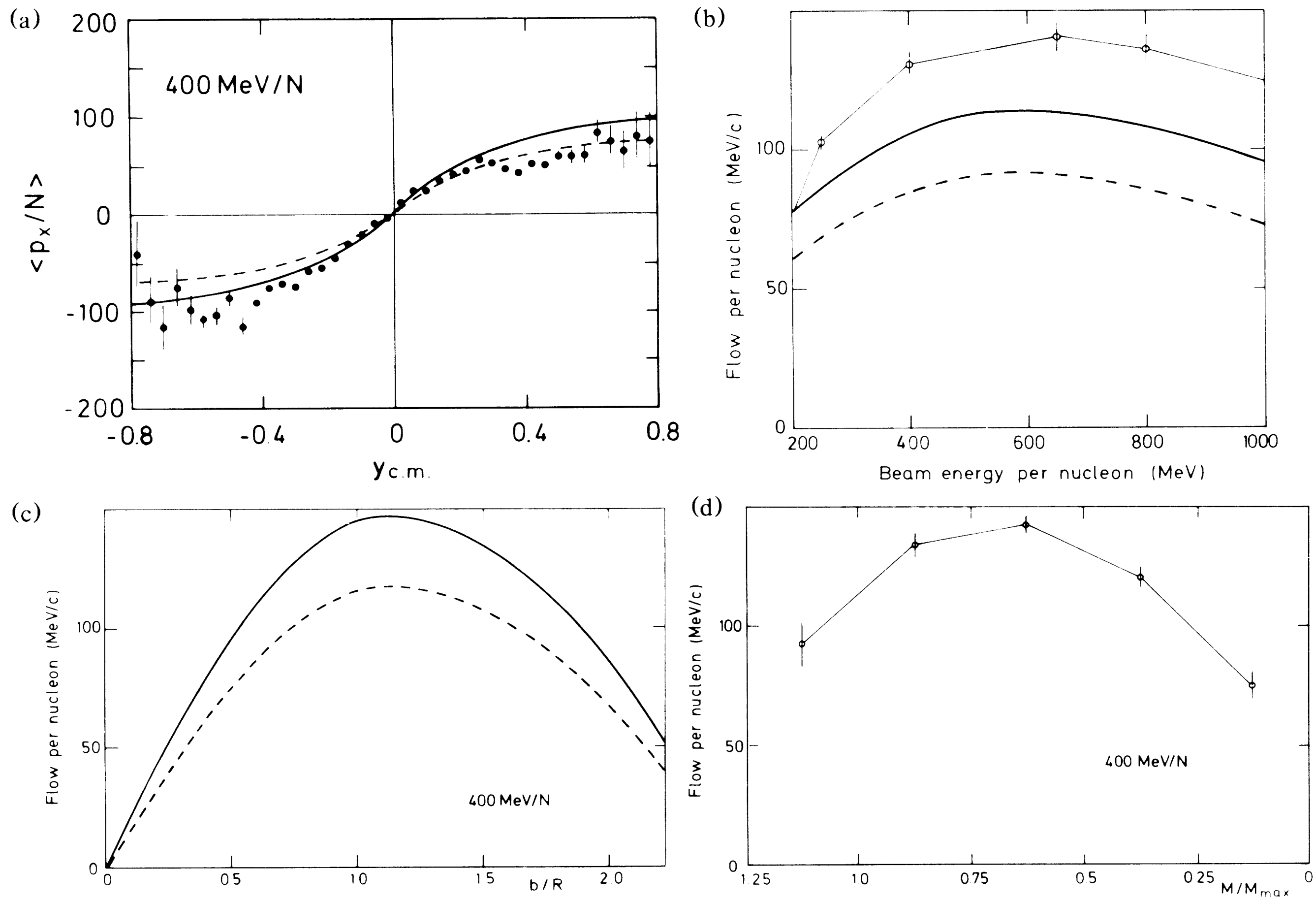

FIG. 1. Mean transverse momentum per nucleon $\langle p_x/N\rangle$ and flow per nucleon for the reaction Nb on Nb. The solid and dashed lines represent the model calculations with and without momentum dependence, respectively. The circles denote the experimental results (Refs. 5 and 7) and in (b) and (d) are connected by lines to guide the eye.

According to (3) the flow f is a combination of both the deviation of the aspect ratio from unity and of the flow angle such that f vanishes (i) for spherical events, (ii) for zero flow angle, and (iii) for a flow angle of 90°. The physics content of (i)–(iii) is as follows. Case (i) occurs in the global thermal ("fireball") model, case (ii) in cascade models with a dominance of quasifree scattering ("first-collision" models), and case (iii) in the hydrodynamical model of Buchwald *et al.*[10] at zero impact parameter. Hence it appears that the essence of relation (3) is rather general even though the precise form of (3) may differ for different model prescriptions.[11]

Our numerical results for fixed mass number exhibit a behavior qualitatively similar to that of the corresponding experimental data denoted by the circles in Fig. 1, keeping in mind, however, that the detector response possibly influences the energy dependence of the measured flow.[5] In addition, the mass-number dependence of the flow has been measured[5] by our considering apart from Nb+Nb the systems Ca+Ca and Au+Au. In Table I we display for all three systems both the experimental and the calculated mean values for the flow as functions of the beam energy per nucleon. When we compare Ca+Ca with Nb+Nb the measured flow roughly has a mass-number dependence of $A^{2/3}$, whereas when we compare Nb+Nb with Au+Au, at 400 MeV per nucleon it exhibits an approximate $A^{1/3}$ dependence and an

TABLE I. Observed (Ref. 5) and calculated flow in (MeV/c)/nucleon for minimum-bias events as functions of beam energy for various systems. The numbers in parentheses are computed without a momentum-dependent force term.

Reaction \ E_0 (MeV/nucleon)	400	650	800
Ca+Ca	75	⋯	⋯
Nb+Nb	130	140	135
Au+Au	159	161	150
Theory	105 (85)	112 (90)	107 (85)

increasingly weaker dependence with increasing beam energy. This is at variance with our mass-number-independent–model results (last row of Table I). Hence we conclude that our theory can account for only part of the flow.

Next we discuss the performance of various other models in comparison with the data of Ref. 5 as far as results are available. Under certain conditions, in particular for an ideal-gas equation of state, nonviscous hydrodynamics leads to an A-independent and to an $E_0^{1/2}$-dependent flow as a scaling analysis[12] reveals. This A independence is in clear contradiction to the data of Ref. 5. On the other hand the method of Ref. 12 shows that under the same conditions *viscous* hydrodynamics yields A and E_0 dependence of the flow compatible with those of the measured one. This is based on the fact that once it is possible to separate all scaling parameters in the initial conditions, in the equation of state, and in the hydrodynamical equations, the solutions of the latter and the quantities constructed from them such as the dimensionless flow $\tilde{f}=f/p_0$ only depend on one single quantity, namely the Reynolds number N_{Re} whose mass-number and energy dependences can be calculated.

Regarding the numerical transport models which include mean-field effects[3,13–15] we are not aware of systematic studies of the mass-number dependence of the flow. Again, if we consult the scaling analysis[12] the energy dependence of the flow in these calculations is $E_0^{1/2}$ just like in nonviscous hydrodynamics, which is in agreement with the data in fixed high-multiplicity bins. It would be interesting to extend the alternative low-energy numerical transport approach of Vinet *et al.*[16] to higher energy for the study of the mass and energy dependences of the flow.

The conservative cascade model (i.e., no mean field included) yields a flow angle whose A and E_0 dependences qualitatively agree with those of experiment[7] but whose size is too small. According to the authors of Ref. 9 the flow angle arises in their cascade from the action of the pressure built up inside the compressed central region on the outer portions of the overlap zone. The flow angle is reduced by viscosity forces or more generally by off-equilibrium effects. No calculations for the flow are presented but on the basis of relation (3) we expect similar features as for the flow angle, in particular a too small size of the flow.

We conclude from the above considerations that at the present stage there are two candidates qualitatively compatible with the data of Ref. 5. One is viscous hydrodynamics supplemented by an ideal-gas equation of state. A further test of its applicability would be to check experimentally for a large variety of systems and incident energies whether the dimensionless flow $\tilde{f}(A,E_0)$ remains constant for such combinations of A and E_0 for which the Reynolds number $N_{\mathrm{Re}}(A,E_0)$ is constant. So far we did not mention the size of the viscous hydrodynamic flow in comparison with the measured one simply because corresponding numerical hydrodynamic calculations are not available at present. To obtain a reasonable size is, however, probably not difficult since there is some freedom in the choice of the viscosity coefficients (bulk and kinematical) which are not well known. One could also turn things around and use the size of the flow as a means to determine these coefficients.

The second candidate emerges from the following picture. In the early stage of the collision the dilute-gas limit as described by the Boltzmann equation with two separate mean force fields applies and a mass-number-independent background flow develops. At a later, high-temperature, stage the two fields are completely destroyed and the cascade picture with only binary nucleon-nucleon collisions dominates. In this stage the mass-number-dependent portion of the flow is being built up. It adds to the background flow, the sum yielding the experimentally observed flow. A test of the presence of the background flow would be to study experimentally systems lighter than Ca+Ca. From cascade-model calculations with the nucleons moving in their respective potential wells there are hints that such a background flow in fact exists.[17,18] This could further be checked with the numerical transport models[13–16] employing an attractive mean field only, parametrized such that it is destroyed at high temperature. Related calculations leading to a nonzero transverse average momentum and hence to a nonzero flow have recently been reported.[19]

The latter microscopic picture is closely related to viscous hydrodynamics with an ideal-gas equation of state but it is more general in the sense that not all of its off-equilibrium features can be phrased in terms of viscosity alone. In any case, if either one of the two candidates stands the tests against future experiments of the types proposed above, the fundamental task of extracting the compressional part of the nuclear-matter equation of state will remain very difficult.

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