Exact Construction of (0,2) Calabi-Yau Manifolds

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(Received 6 November 1987)

It is shown that (0,2) Calabi-Yau manifolds as exact solutions at the string tree level can be obtained from a class of (2,2) Calabi-Yau manifolds, which are constructed as blown-up Z_n orbifolds.

PACS numbers: 11.17.+y

Since its proposal,[1] the status of (0,2) Calabi-Yau manifolds has gone through different phases. Such manifolds can be obtained from (2,2) Calabi-Yau manifolds which have no $(\mathbf{27}\,\mathbf{27}^*)^K$ terms (with $K \geq 2$) in the superpotential. Here **27** and $\mathbf{27}^*$ are the massless-field representations of the E_6 gauge group. In this case there are flat directions in the effective potential with $\langle S_i\rangle = \langle S_j^*\rangle \neq 0$. Here S_i and S_j^* are the fields of $\mathbf{27}_i$ and $\mathbf{27}_j^*$ multiplets, respectively, which are singlets of the SO(10) or SO(5) or some other subgroup of E_6. Thus, for such models one is able to have a gauge group $G_0 \subset E_6$, i.e., spin and gauge connection are not identified any more; however, the supersymmetry is still preserved.

In Ref. 1 constraints were given for which $(\mathbf{27}\,\mathbf{27}^*)^K$ terms (with $K \geq 2$) are absent in the superpotential to any order in the σ-model perturbation, i.e., to any order in the α'/R^2 expansion. Here α' is the string tension and R is the radius of the compactified space. Actually, it seems to be a general feature[2] of smooth (2,2) compactifications that to any order in α'/R^2 *all* the nonrenormalizable terms of the superpotential are absent. This statement has been proven[3] for any symmetric orbifold[4] with at least (0,2) world-sheet supersymmetry, and is thus valid for smooth compactifications on (2,2) blown-up orbifolds[5,6] as well.

However, the world-sheet instantons[7] (when they exist) generically induce[8] $(\mathbf{27}\,\mathbf{27}^*)^K$ terms proportional to $\exp(-R^2/\alpha')$. Thus there are generically *no* flat directions with the gauge group $G_0 \subset E_6$, i.e., (0,2) smooth compactifications are generically not a vacuum solution of the string theory.

The smooth (0,2) compactifications were resurrected[9,10] by the study of constraints on (2,2) Calabi-Yau compactifications for which the $(\mathbf{27}\,\mathbf{27}^*)^K$ terms are not induced to the lowest order (and possibly to all orders) in the instanton contribution.

In this note I would like to present explicit constructions of Calabi-Yau manifolds for which *all* the terms $(\mathbf{27}\,\mathbf{27}^*)^K$ are *absent at the string tree level;* i.e., by studying blowing-up[5,6] (2,2) Z_N orbifolds.

The blown-up orbifolds are obtained[5,6,11,12] from the corresponding orbifolds by our giving nonzero vacuum expectation values to the massless scalar fields—"moduli"—associated with the orbifold singularities, i.e., the so-called blowing-up modes, whose potential is flat.[11,12] Scattering amplitudes in the repaired Calabi-Yau background—and hence also parameters of the effective Lagrangean—can be calculated by the insertion of successively larger numbers of background blowing-up modes into orbifold amplitudes. Although perturbative in the blowing-up vacuum expectation values, the method enables one to obtain explicit values[5,6,13] for parameters of the blown-up orbifolds, thus giving *exact* results at the string tree level.

All such orbifolds possess the local conformal invariance[14-16] in the right-moving sector, with vertices in different "pictures" having different ghost numbers for the bosonized right-moving superconformal ghost ϕ. Tree-level amplitudes involve collections of vertices such that the total ghost number equals -2.[15] The simplest form of the vertex operator for such a space-time fermion is the $-\frac{1}{2}$ picture, while that for a space-time boson is the -1 picture. The picture-changing formalism enables one to obtain vertices in other pictures. For example, the vertex for a space-time boson in the 0 picture is obtained in the following way[15]:

$$V(z)_0 = \lim_{w\to z} \exp(\phi) T_{\mathrm{F}}(w) V(z)_{-1}. \tag{1}$$

Here $V(z)_{-1}$ is the corresponding vertex operator in the -1 picture and

$$T_{\mathrm{F}} = \partial_z X^i \psi^{*i^*} + \partial_z X^{*i^*} \psi^i + \partial_z X^\mu \phi^\mu \tag{2}$$

is the world-sheet supersymmetry generator[15]—the stress-energy tensor for orbifolds. X and ψ are the string bosonic and fermionic coordinates, respectively; the indices $(i,i^*)=(1,2,3)$ and $\mu=(1,2,3,4)$ denote the three complex internal and the four space-time dimensions, respectively. The right-moving $N=2$ superalgebra of (2,2) models incorporates a $\mathrm{U}(1)_{r,l}$ current algebra, generated by $J_{r,l} = -\sqrt{3}\partial H_{r,l}$. $H_{r,l}(z)$ is a free right-moving and left-moving scalar field, respectively. For orbifolds, world-sheet symmetry is enlarged to $[\mathrm{U}(1)\otimes\mathrm{U}(1)\otimes\mathrm{U}(1)]_{r,l}$. Thus, instead of two conserved charges $H_{r,l} \equiv \sum_{i=1}^3 (H_i)_{r,l}$ there are six separately conserved charges, $(H_i)_{r,l}$, $i=1,2,3$. The H_r charges are fixed for vertices for massless chiral multiplets. For example, one finds that $H_r = +1$ (-1) for vertices in the -1 picture and $H_r = -\frac{1}{2}$ $(+\frac{1}{2})$ for vertices in the $-\frac{1}{2}$ picture with the space-time chiral multiplet having positive (negative) chirality.

The separate $(H_i)_r$ charges are the phases of the discrete rotation θ acting on the three compactified coordinates. For example, for Z_3 orbifolds $\theta=(\omega,\omega,\omega)$ in its diagonal form with $\omega=\exp(2\pi i/3)$. This in following determines $(H_i)_r$ charges of the singly twisted sector (g^1) to be $(\frac{1}{3},\frac{1}{3},\frac{1}{3})$ and $(\frac{1}{3}-\frac{1}{2},\frac{1}{3}-\frac{1}{2},\frac{1}{3}-\frac{1}{2})$ in the -1 and $-\frac{1}{2}$ pictures, respectively. In Table I[4] I give $(H_i)_r$ charges of the singly twisted sector for all the Z_N orbifolds with space-time supersymmetry. $(H_i)_r$ charges completely specify the right-moving part of the vertex operators in the -1 and $-\frac{1}{2}$ pictures. For example, for Z_3 the right-moving part of V_{-1} for the singly twisted sector has the form

$$\exp(-\phi)\textstyle\prod_j \exp[i(H_j)_r/3]\sigma\exp(ik_\mu X^\mu).$$

Here σ denotes the twist field creating the singly twisted vacuum.

On the other hand, the parts of the vertices which carry the information of the left-moving sector should be constructed explicitly, because of the lack of the local superconformal invariance in the left-moving sector; i.e., the picture-changing formalism does not apply. However, the (2,2) world-sheet supersymmetry enables one to find a general prescription for the vertex operators of the "moduli," i.e., the "blowing-up" modes corresponding to the blowing up of the fixed points (moduli of the twisted sectors) *and* the massless modes corresponding to the deformation of the six torus, T^6 (moduli of the untwisted sector). The moduli transforming as (1,1) (b) and (1,2) (b^*) forms appear along with **27** and **27***, respectively.[17]

It turns out[6] that the left-moving sector of these vertex operators is the vertex operator in the 0 picture. For example, the blowing-up mode b of the singly twisted sector of Z_3 orbifold which sits at 27 fixed points transforms as (1,1) form and has a vertex operator of the type

$$\lim_{w^*\to z^*}\sum_{i^*}\partial_{z^*}X^{*i^*}e^{-i(H_i)_l}(w^*)\prod_j e^{i(H_j)_l/3}(z^*).$$

TABLE I. $(H_i)_r$ charges of the singly twisted sector in the -1 picture for Z_N orbifolds with space-time supersymmetry.

N	$(H_1)_r$	$(H_2)_r$	$(H_3)_r$
3	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
4	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
6	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
6′	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$
7	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{4}{7}$
8	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{5}{8}$
8′	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$
12	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{1}{2}$
12′	$\frac{1}{12}$	$\frac{1}{3}$	$\frac{7}{12}$

One can also construct the left-moving sector for the vertex operator of **27** and **27*** of E_6. For the parts of **27** (**27***) transforming as **16** (**16***) of SO(10), the vertex operator corresponding to the left-moving sector can be represented as vertex operators in the $-\frac{1}{2}$ picture with $H_l=-\frac{1}{2}$ $(H_l=+\frac{1}{2})$. It can be obtained by use of (1); however, one suppresses the ghost charge ϕ, the space-time indices μ, $w,z\to w^*,z^*$, and $(H_i)_r\to(H_i)_l$. For example, **16** of the twisted sector of the Z_3 has $(H_i)_l=-\frac{1}{6}$, $i=1,2,3$.

The calculation of parameters of the effective Lagrangean of a particular theory reduces to the study of the corresponding amplitude of the massless states emitted from the string propagating in this particular background. When one is calculating amplitudes which probe the terms of the superpotential, it is most convenient to calculate[5,6] the corresponding Yukawa-type n-point function.

$$A=\langle V_{-1/2}V_{-1/2}V_{-1}V_0\cdots V_0\rangle, \tag{3}$$

with $V_{(-1/2,-1,0)}$ corresponding to vertices in different pictures which represent emission of massless fields from the positive chirality supermultiplets. One sees from (1) and (2) that only terms of V_0 with $H_r=0$ contribute to (3).[11,12] Namely, for $V_{(-1/2),(-1)}$, $H_r=-\frac{1}{2},1$, respectively, and thus only the terms of V_0 proportional to $\partial X^i\psi^{*i^*}$ survive in such amplitudes in order to conserve the total H_r charge.

Such an amplitude should also be invariant under the automorphisms, i.e., the point group of discrete rotations θ^m, of the lattice. Thus, if one can show that the amplitude (4) which corresponds to $(27\,27^*)^K$ with arbitrary insertion of moduli is not invariant under a discrete rotation θ, such an amplitude is *identically* zero at the string tree level.

In order to show this, I shall analyze first the $(H_i)_{r,l}$ charges of **27*** as well as b^*, i.e., moduli which transform as (one- and two-) forms. In Ref. 6 it has been shown that **27** (b^*) appears in the untwisted sector of the theory if $\theta_{i_0}=-1$, i.e., the discrete rotation θ rotates the i_0^{th} complex coordinate by 180°. This implies

$$(H_{i_0})_l=\begin{cases}-\frac{1}{2}, & \mathbf{16}^*,\\ 0, & b^*,\end{cases} \qquad (H_{i_0})_r=1,\quad -1\text{ picture.} \tag{4}$$

Here **16*** refers to the part of **27*** that transforms as **16*** of SO(10).

On the other hand,[6] **27*** (b^*) appear only in such m-twisted sectors for which $\theta_{i_0'}^m=+1$, i.e., the $i_0'^{\text{th}}$ direction is left fixed by θ^m. Then the charges are the following:

$$(H_{i_0'})_l=\begin{cases}+\frac{1}{2}, & \mathbf{16}^*,\\ 0, & b^*,\end{cases} \qquad (H_{i_0'})_r=0,\quad -1\text{ picture.} \tag{5}$$

By inspecting Table I, one sees that the i_0th direction and i_0'th direction *are the same* for all but the $Z_{6'}$ orbifold. Thus, for such (blown-up) orbifolds, one can now examine properties of the corresponding amplitude with respect to the $(H_{i_0})_{l,r}$ charge and the X_{i_0} world-sheet coordinate only.[18] It turns out that with use of Eqs. (4) and (5) and a separate conservation of $(H_i)_{l,r}$ charges, the amplitude (3) is

$$A = \langle (\mathbf{27}\,\mathbf{27}^*)^K b^L b^{*M} \rangle \propto \langle \partial_z X_{i_0}^a \partial_{z^*} X_{i_0}^b \partial_{z^*} X_{i_0}^{*c} \rangle, \quad (6)$$

with $a = c = b + 2d - 1$ where d and b denote the number of $\mathbf{27}^*$ and b^* from the untwisted sector, respectively. c depends[6] on the sum of $(H_{i_0})_{l,r}$ charges for the $\mathbf{27}$'s and b's and is thus specific to each particular Z_N model.[19]

$A \equiv 0$, because it is not invariant under θ_{i_0}. Namely, for the models with $\theta_{i_0} = -1$, (6) changes sign under θ_{i_0}, while for the models with $\theta_{i_0} \neq -1$, $d = b \equiv 0$, and (6) again picks up a phase under the action of θ_{i_0}.

Thus at the string tree level, the above (blown-up) (2,2) orbifolds have flat directions with $G_0 \subset E_6$.[20] This is only the feature of the class-Z_N blown-up orbifolds, for which the appearance of $\mathbf{27}^*$ (b^*) from the untwisted and twisted sectors is tied to a particular value of the discrete rotation acting on only *one* particular direction. This symmetry, however, breaks already for $Z_{6'}$ (blown-up) orbifolds and presumably for other (blown-up) orbifold constructions, as well.

I would like to thank Michael Dine for a useful discussion. I would also like to thank Lance Dixon for many discussions related to the topic presented in this paper. This work was supported by the U.S. Department of Energy through Grant No. DE5-22418-281.

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[17]In the "orbifold terminology" the definition for **27** (**27***) is just the opposite from the one in "Calabi-Yau terminology."

[18]The above orbifolds, i.e., those with **27***'s and $i_0 = i_0'$, one can always be compactified on a six-torus, T^6, which is obtained by continuously deforming $T_0^6 = T^4 \otimes T_{i_0}^2$, i.e., T_0^6 is a direct product of mutually orthogonal four- and two-tori.

[19]For the sake of completeness $c = N_l + N_r - d$, where N_l, N_r are the sums of $(H_{i_0})_{l,r}$ changes given in -1 picture for sectors with **27**'s and b's.

[20]Note that for symmetric torus configurations the couplings of matter singlets to some **27** and some **27*** can be absent as well. We checked this for the Z_4 blown-up orbifolds.