Fractional Quantum Hall Effect at Half-Filled Landau Level in a Multiple-Layer Electron System

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We have studied the excitation spectrum for a one-half filled Landau level in a two-layer system of interacting electrons, by finite-size calculations in a periodic rectangular geometry. The interlayer interaction is found to be important for obtaining a unique ground state and the required gap structure in the spectrum. On the basis of these observations, we predict that the fractional quantum Hall effect might be found at this filling in such a layered system.

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The discovery of the fractional quantum Hall effect (FQHE) in two-dimensional electron systems subjected to a strong perpendicular magnetic field[1] indicated that the effect occurs exclusively at the Landau-level filling factors with *odd* denominators. The theoretical studies of the effect in terms of electrons condensed into an *incompressible* quantum fluid state[2] have been quite successful in explaining several interesting aspects of this phenomenon.[3] These developments have recently taken a new turn by the discovery of FQHE in an *even*-denominator filling factor $\frac{5}{2}$ in the second Landau level.[4] The simplest filling factor with even denominator, $\nu=\frac{1}{2}$, still remains elusive, however. Here $\nu=2\pi l_0^2\rho$ is the Landau-level filling factor with $l_0\equiv(\hbar c/eB)^{1/2}$ the magnetic length and ρ the particle density. Electron pairing for such a state was first suggested by Halperin.[3] For $\nu=\frac{1}{2}$, a Laughlin state describes a system of *bosons* (and interestingly, also describes the ground state of the Heisenberg antiferromagnet in two dimensions[5]). With use of the method of exact diagonalization for finite systems in a periodic rectangular geometry,[6,7] a *cusp* at $\nu=\frac{1}{2}$ was in fact obtained by Yoshioka[8] for a system of particles obeying *Bose* statistics. In a recent study of this filling fraction, Fano, Ortolani, and Tosatti[9] have reported finite-electron-system calculations in a spherical geometry.[10] They obtained the excitation spectrum, which showed very erratic behavior with no clear trend seen. As we shall see below, this is typical for a single-layer system.

In this Letter, we present the results for the collective excitations at $\nu=\frac{1}{2}$ for a *layered* electron system, using the method of exact diagonalization of finite electron systems in a periodic rectangular geometry.[11-13] Multilayer electron systems have been studied earlier experimentally[14] and theoretically,[15,16] as an anisotropic model for an electron gas. In the present work, we have considered the model by Visscher and Falicov[15] where two layers with equal densities of electrons are embedded in an infinite dielectric. We also consider the δ-function-localized electron density in each plane. The electrons move freely in each plane and we consider only Coulombic interaction for electrons in different planes. Tunneling of electrons between two planes is not allowed. Furthermore, the electrons are considered to be in their lowest subband. Recently, experimental systems which can be described reasonably well by the above model have been obtained by different groups in GaAs heterostructures.[14] The Coulomb potential energy between two electrons situated at planes i and j is given as[15,16]

$$(e^2/\epsilon)[(\mathbf{r}-\mathbf{r}')^2+(i-j)^2c^2]^{-1/2},$$

where $\mathbf{r}$ is the two-vector (x,y), ϵ is the background dielectric constant, and c is the interlayer separation. The Fourier transform of the above expression with respect to $\mathbf{r}-\mathbf{r}'$ is

$$v(\mathbf{k};i,j)=(2\pi e^2/\epsilon k)e^{-k|i-j|c},$$

where $\mathbf{k}$ is a two-dimensional in-plane wave vector. In the numerical calculations that follow, we have used the dimensionless parameter $c_s=c/l_0$. For $\nu=\frac{1}{2}$, the magnetic field is usually in the range $B\simeq 10-20$ T and in order to obtain any appreciable effects from the exponential factor in $v(\mathbf{k};i,j)$ in the range of kl_0 accessible in our numerical calculations, we have chosen $c=2l_0$. Results for c which are slightly lower than this value are qualitatively similar and are not presented. The effect of the interlayer interaction is to lift the twofold degeneracy which would otherwise be present. The interlayer coupling is also found to remove the additional degeneracies generally present in a single-layer system at this filling.

The resulting spectrum, as shown below, is strikingly similar to the collective excitations expected for the incompressible fluid state at $\nu=\frac{1}{3}$. Our present study therefore indicates the possibility that FQHE at $\nu=\frac{1}{2}$ might be observable experimentally in a *multilayer* system.

The evaluation of the collective excitations for a finite electron system in a periodic rectangular geometry has been made by a method similar to the one described earlier by Haldane.[11] In the present case, we have generalized the method of Ref. 11 for the two-layer system. Accordingly, we consider a rectangular cell containing two layers of equal numbers of electrons N_e. We ignore, for simplicity, the Landau-level mixing, and impose periodic boundary conditions such that the cell contains an integer number N_s of flux quanta. We also consider the electrons to be in the spin-polarized state, and all the electrons to be in the lowest Landau level. The filling fraction is therefore $\frac{1}{2}$ in both layers. In the earlier analysis of Haldane for a single-layer system, the Hamiltonian conserved the total momentum $\mathbf{k}$, which has only a discrete set of values depending on N_e and the filling fraction. The basis states were chosen to be the eigenstates of the momentum operator.[11] In the case of a two-layer system, the Hamiltonian conserves the *total* momentum as well as the number of electrons in each layer. Therefore, it can be diagonalized for the set of states $|\mathbf{k}_1;L_1\rangle|\mathbf{k}-\mathbf{k}_1;L_2\rangle$, where $|\mathbf{k}_i;L_i\rangle$ is a momentum eigenstate for N_e electrons in a single layer i belonging to the eigenvalue $\mathbf{k}_i$. Here, $L_i=|j_1,\ldots,j_{N_e}\rangle$ labels a Slater determinant of Landau orbitals with momentum $\mathbf{k}_i$. The complexity of the computational method has restricted us presently to a system of four electrons per layer (eight-electron system). The computation of the ten-electron system (five electrons per layer) excitation spectrum is very time consuming and is currently under investigation.

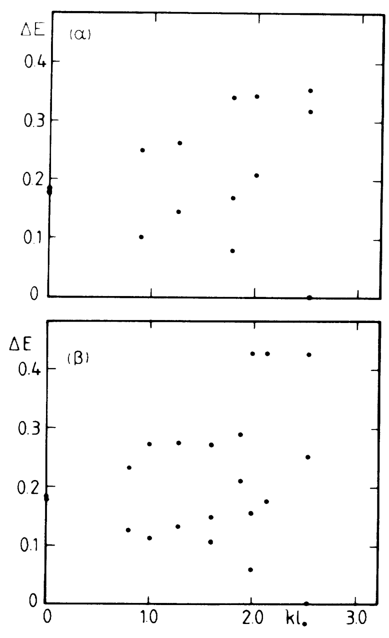

FIG. 1. Low-lying excitation energies as a function of kl_0 for a single layer of electrons at $\nu=\frac{1}{2}$, presented for four-electron system in (α) square ($\lambda=1$) and (β) rectangular ($\lambda=1.25$) geometry.

Let us first discuss the excitation spectrum $\nu=\frac{1}{2}$ for a single-layer system. As shown in Fig. 1, the low-lying excitations for a four-electron system and various values of aspect ratio λ, $\lambda=1$ (square) and $\lambda=1.25$ (rectangular), do not show the well-behaved features generally observed for the filling factors with odd denominators.[13] Although in the single-layer case we could calculate the spectrum for very large systems, the situation does not change qualitatively with the larger systems. As noted earlier by Haldane[11] who studied up to ten-electron systems, the ground state is not found generally at $\mathbf{k}=0$, but at general $\mathbf{k}$ points which move and change discontinuously as the system geometry is varied away from the square geometry. Furthermore, no clear gap structure is apparent in the spectrum. Similar results were also obtained recently by Fano, Ortolani, and Tosatti[9] for six- to twelve-electron systems in a spherical geometry.

The situation changes drastically, however, in the case of the two-layer system. The effect of anisotropy in the system helps, removing all the unwanted features described above from the excitation spectrum. The first important result to note is that the ground state for the layered system is obtained uniquely at $\mathbf{k}=0$ and it remains so even when we change the system geometry by varying the aspect ratio λ. In Figs. 2 and 3, we have presented the excitation spectrum for the eight-electron system (four electrons per layer) for the square ($\lambda=1$) and rectangular ($\lambda=1.25$) geometries, respectively. The

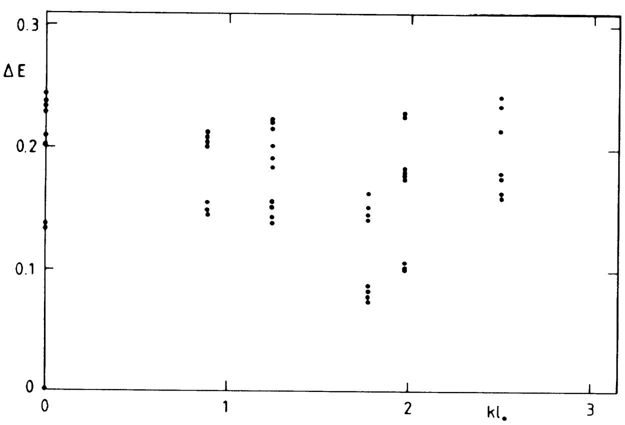

FIG. 2. Excitation spectrum of eight-electron system in a two-layer geometry at $\nu=\frac{1}{2}$ for dimensionless layer separation parameter $c_s=2.0$ and aspect ratio $\lambda=1$.

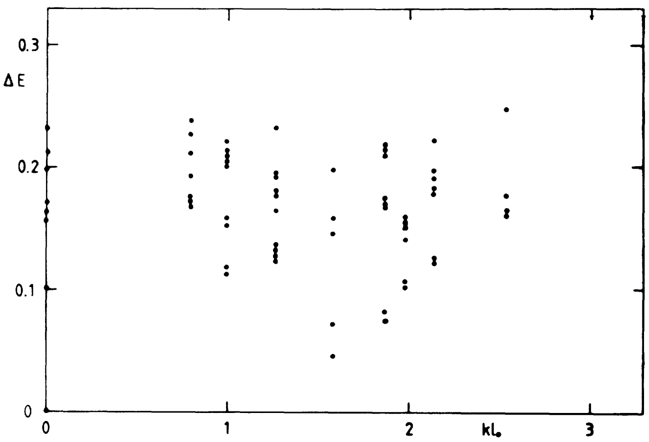

FIG. 3. Same as in Fig. 2, but for the aspect ratio $\lambda = 1.25$ (rectangular cell).

ground-state energy is slightly *lower* for the rectangular geometry; however, the energy difference between the two geometries is very small ($\simeq 0.005$).

The most important result of the present work, however, is a gap structure in the spectrum, as well as the characteristic minimum at a finite kl_0, similar to the magnetoroton minimum of Girvin, MacDonald, and Platzman.[17] For a square geometry, a few energy levels lie very close, but separated from the continuum by a large gap (Fig. 2). These close-lying energy states can be further separated by moving away from the square geometry, as demonstrated in our numerical results in Fig. 3. This is made possible by the fact that the square geometry, as noted by Maksym,[12] has higher degeneracies as compared with the rectangular geometry. The lowest two excitation energies, which are now clearly separated from the higher-energy states for most values of kl_0, could presumably be interpreted as the two eigenmodes in a system of two charge layers. They arise because of the electron correlations in the two layers. One can, in fact assign the parameters $k_\perp(n) = 2\pi n / N_L c$ to the two modes, where $n = 0, \ldots, N_L - 1$, N_L being the number of layers. For a superlattice ($N_L \rightarrow \infty$), $k_\perp(n)$ would be the wave vector perpendicular to the layer. In that case, one would see a *band* of low-lying collective excitation spectrum. Such a spectrum for $\nu = \frac{1}{3}$ has been calculated recently in the single-mode approximation.[18] The lowest-energy excitations in Fig. 3, therefore, exhibit a feature which is quite naturally expected in a two-layer system. As has also been demonstrated in our numerical results, the crucial features in the two-layer system do not change qualitatively with the different geometries considered in the present work, and are very similar to the incompressible fluid state studied by various authors for $\nu = \frac{1}{3}$.

Finally, from the excitation spectrum obtained as above, we can also estimate the energy gap E_g for an infinitely separated quasiparticle-quasihole pair. If we identify the lowest-lying excitations as the *quasiexcitons* (bound state of a quasiparticle and quasihole), E_g is the asymptotic value of the lowest-lying collective dispersion $E(\mathbf{k})$ obtained numerically. As discussed above, the collective mode obtained in the present case has all the necessary characteristics of an incompressible fluid state first proposed by Laughlin[2] to explain FQHE. In that case, we can expect that the quasiparticles and quasiholes will have fractional charge of $e^* = \pm e/m$, with $m = 2$ in the present case. As noted by Kallin and Halperin,[19] for large values of kl_0, the quasiexcitons comprise a quasiparticle and quasihole separated by a large distance, $|\Delta \mathbf{r}| = kl_0^{*2} = kl_0^2 m$, where l_0^* is the effective magnetic length for a particle of charge e^*. For large values of $\mathbf{k}$, we may then have

$$E(k) = E_g - \frac{e^{*2}}{\epsilon |\Delta \mathbf{r}|} = E_g - \frac{e^2}{m^3 \epsilon k l_0^2},$$

from which the gap is estimated. With use of the numerical results for the lowest excitation energy obtained for the maximum value of kl_0 available in our present numerical work, the gap is evaluated to be[20] $\simeq 0.209$. The energy gap obtained earlier in this manner[13] for finite electron systems in a periodic rectangular geometry provided quite reliable results at $\nu = \frac{1}{3}$. It is interesting to note that the energy gap estimated for this filling is much higher than the corresponding gap for $\frac{1}{3}$ filling, $E_g \simeq 0.1$, as estimated by various authors.[3-6] The gap will be reduced somewhat if we consider the finite-thickness corrections in a more realistic situation.[7,21] However, considering the magnitude of the gap, it is expected to be still large enough to be measured experimentally.

In conclusion, we have presented the excitation spectrum for $\frac{1}{2}$ filling of the lowest Landau level in a system of two layers of interacting electrons subjected to a strong perpendicular magnetic field. While in the single-layer case, the ground state is obtained at a finite value of $\mathbf{k}$, which is strongly geometry dependent, introduction of another layer of interacting electrons helps to reorganize the excitation energies, particularly for the $\mathbf{k} = 0$ state. The spectrum has several features characteristic of an incompressible fluid state. The observation of a unique ground state and the appearance of a gap structure in the spectrum raises the interesting possibility that FQHE might be observed experimentally in such a layered system. The energy gap could also be estimated from the activation energies as measured earlier for odd-denominator filling fractions.[22] From the theoretical point of view, the present small-system calculations provide only a qualitative understanding of the ground state and excited states for $\nu = \frac{1}{2}$, but larger system results are required to confirm our conclusions on a quantitative level. Many-body, Laughlin-type approaches are also to be developed. Finally, experimental observation of the properties of this filling factor described in the present work would not only provide a stimulus for obtaining an interesting array of even-denominator filling fractions, as seen for the odd-denominator filling fractions, it would also have a direct impact on the standard methods currently in use to explain FQHE.

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