Precision Measurement beyond the Shot-Noise Limit

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An improvement in precision beyond the limit set by the vacuum-state or zero-point fluctuations of the electromagnetic field is reported for the measurement of phase modulation in an optical interferometer. The experiment makes use of squeezed light to reduce the level of fluctuations below the shot-noise limit. An increase in the signal-to-noise ratio of 3.0 dB relative to the shot-noise limit is demonstrated, with the improvement currently limited by losses in propagation and detection and not by the degree of available squeezing.

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The quantum nature of the electromagnetic field leads to limitations on the sensitivity of precision measurement of amplitude or phase changes of the field. The quantum fluctuations responsible for enforcing a lower limit on the "noise" in an optical experiment are succinctly expressed in terms of uncertainty products that follow from the commutation relations between conjugate field operators. The fundamental limit encountered in optical physics has been the so-called "shot-noise" limit (SNL), which represents a level of fluctuations for which the minimum uncertainty allowed by quantum mechanics is achieved for the uncertainty product and for which the variances for each of two conjugate operators are equal. This symmetric distribution of fluctuations is characteristic of the vacuum state of the field (zero-point fluctuations) or of a coherent state (approximated by a single-mode laser).

Although the vacuum fluctuations of the field have been the practical limit on precision optical measurement, these fluctuations are of course not a limit in principle since quantum states with variance less than that of the vacuum state can be employed. In particular, the use of squeezed states to circumvent the SNL has been discussed for many years in the theoretical literature.[1-8] Squeezed states are characterized by a phase-dependent distribution of quantum fluctuations such that the dispersion in one of two quadrature components of the field drops below the level set by the vacuum state. In a measurement with squeezed light, the signal that one wishes to detect is encoded on the field variable with reduced fluctuations. The detection scheme is arranged to be largely insensitive to the increased fluctuations in the conjugate variable that are required by the uncertainty relation.

In this Letter we report an experiment in which an improvement in the signal-to-noise ratio of 3.0 dB relative to the SNL has been achieved in an optical measurement with squeezed states. The experiment follows the work of Caves on precision interferometry[6] and employs squeezed light in a Mach-Zehnder interferometer for the detection of phase modulation in propagation along the two arms of the interferometer.[9] The observed increase in sensitivity in the experiment is currently limited by simple linear losses in propagation and detection, and not by the available degree of squeezing from our source. Thus one might anticipate that these rather modest initial results can be substantially improved as the losses in the experiment are reduced.

The experimental arrangement for the use of squeezed states in interferometry is shown in broad outline in Fig. 1. A Mach-Zehnder interferometer is formed by the two beam splitters (m_1, m_4) and the highly reflecting mirrors (m_2, m_3). A coherent field $\hat{E}_1$ at 1.06 μm is injected into the input port m_1, and the fields from the two paths through P_1 and P_2 are recombined at the output port m_4 to produce (complementary) interference fringes as a function of phase difference along the two arms. For the simplest case of 50-50 beam splitters, the fluxes incident upon the detectors D_1 and D_2 are

$$I_{1,2} = \tfrac{1}{2}\xi\langle \hat{E}_1^\dagger \hat{E}_1\rangle[1 \pm \cos\phi], \qquad (1)$$

where ϕ is the phase difference for propagation along the

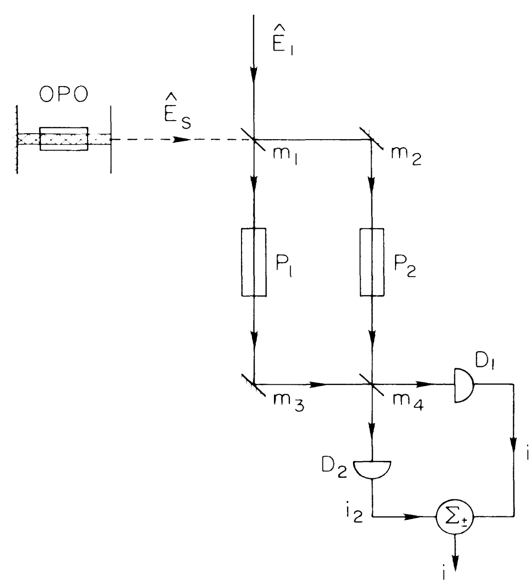

FIG. 1. Diagram of the principal elements of the apparatus for interferometry with squeezed states.

two arms of the interferometer, ξ is an efficiency factor $(0 \le \xi \le 1)$ that incorporates possible losses in propagation through the interferometer, and the units of $\hat{E}_1$ are chosen such that $I_{1,2}$ expresses the flux in photons/sec emerging from m_4. The low-frequency (dc–50 kHz) and high-frequency (>500 kHz) components of the photocurrents from the two photodiodes are amplified by separate electronics. For our measurements the operating point of the Mach-Zehnder interferometer is actively stabilized (servo gain unity at 1.2 kHz) near a point $\phi_0=(2p+1)\pi/2$ $(p=0,1,2,\ldots)$, i.e., at the half-power point of the fringe, with use of an error signal derived from the low-frequency channels.

The elements (P_1,P_2) in the two arms of the Mach-Zehnder are deuterated potassium dihydrogen phosphate (KD*P) phase modulators, each with half-wave voltage $V'=1800$ V at 1.06 μm. A voltage $V(t)=V\cos\Omega t$ is applied to the modulator P_1 to produce a phase dither at $\Omega/2\pi=1.6$ MHz, with the phase $\phi(t)=\phi_0+2\delta\cos\Omega t$ and $\delta=\pi V/2V'$. Expanding (1) for small δ, we find a signal at frequency Ω in the difference photocurrent i of rms amplitude $i_s=\sqrt{2}e(\xi\alpha\langle\hat{E}_1^\dagger\hat{E}_1\rangle)\delta$. The noise current i_n against which this signal must be detected is the "shot noise" associated with the total power reaching the detectors (D_1,D_2). That is, $i_n^2=2e^2(\xi\alpha\langle\hat{E}_1^\dagger\hat{E}_1\rangle)B$, with B the detection bandwidth and α the detector quantum efficiency. The signal-to-noise ratio Ψ_v in the case of a coherent-state input $\hat{E}_1$ and a vacuum-state input $\hat{E}_s$ is thus

$$\Psi_v \equiv i_s^2/i_n^2=\xi\alpha P\delta^2/B, \tag{2}$$

where $P\equiv\langle\hat{E}_1^\dagger\hat{E}_1\rangle$ is the power in photons/sec incident upon m_1. A signal-to-noise ratio of unity, $\Psi_v=1$, implies $\delta_v=[B/\xi\alpha P]^{1/2}=1/\sqrt{N}$, with N as the mean number of photoelectrons detected in the measurement interval B^{-1}. This limit on the minimum detectable phase change is the SNL and is the best sensitivity possible for inputs of a coherent state $\hat{E}_1$ and a vacuum field $\hat{E}_s$.

To achieve sensitivity beyond the SNL, a squeezed field $\hat{E}_s$ is injected in place of the vacuum field into the normally open input of m_1.[6] As indicated in Fig. 1, the squeezed light in our experiments is produced by an optical parametric oscillator (OPO), which is described in detail by Wu and co-workers.[10] For the present purposes, note that the field generated by the subthreshold OPO is characterized by a spectrum of squeezing $S(\nu,\theta)$, where ν is the frequency offset from the optical carrier, and θ selects the quadrature phase. The angle θ is defined relative to the phase of the coherent field $\hat{E}_1$ inside the interferometer, with $\theta=0$ corresponding to fluctuations in the amplitude quadrature $[S(\theta=0)\equiv S_+]$, and $\theta=\pi/2$ corresponding to fluctuations in the phase quadrature $[S(\theta=\pi/2)\equiv S_-]$. For the subthreshold OPO, the field is a squeezed vacuum state with

$$S_\pm(\bar{\nu})=\pm 4r/[\bar{\nu}+(1\mp r)^2], \tag{3}$$

where $\bar{\nu}\equiv\nu/\Gamma_0$ and $\Gamma_0/2\pi\sim 7$ MHz in our experiments.[11] The ratio $r=(p_2/p_0)^{1/2}$ with p_2 the pump power driving the OPO and p_0 the threshold pump power.

Although there are a number of issues of both fundamental and technical natures to be addressed for the use of squeezed states in precision interferometry,[6,12–14] we present only the simplest analysis that is approximately valid for the regime of operation of our interferometer. For the case of a modestly squeezed input $\hat{E}_s$, the calculation of the signal current proceeds exactly as before. However, the noise in the difference photocurrent is now given by $i^2(\nu,\theta)=i_n^2[1+\zeta S(\nu,\theta)]$, with corresponding signal-to-noise ratio

$$\psi(\nu,\theta)=\psi_v/[1+\zeta S(\nu,\theta)], \tag{4}$$

where the factor ζ expresses the efficiency with which the squeezed light is propagated and detected, with $\zeta=\rho T_0\alpha\eta^2\xi$. Here ρ and T_0 are the efficiencies for escape from the cavity of the OPO and propagation to the beam splitter m_1, while η is the heterodyne efficiency. From Eq. (4) we see that noise reductions $(S<0)$ below the level of fluctuations set by a vacuum-state input $(S=0)$ lead to an improvement in signal to noise beyond the SNL by the factor $[1+\zeta S_-]^{-1}\gg 1$ for $\zeta\simeq 1$ and $S_-\rightarrow -1$. On the other hand, enhanced fluctuations in the conjugate quadrature amplitude produce a large degradation in signal to noise by the factor $[1+\zeta S_+]^{-1}\ll 1$, emphasizing the need for precise control of θ.

Figures 2 and 3 document our observations of the detection of phase modulation $\phi(t)$ with and without squeezed light. In both figures the level of fluctuations Φ of the difference photocurrent i is displayed as a function of time for fixed analysis frequency $\nu/2\pi=1.6$ MHz, analysis bandwidth $B=100$ kHz, and two postdetection video filters of time constants $\tau_1=1.5\times 10^{-4}$ sec and $\tau_2=5.0\times 10^{-4}$ sec. Phase modulation at 1.6 MHz is applied with P_1 and is gated on and off with a square wave of repetition rate 50 Hz (Fig. 2) or 70 Hz (Fig. 3). The off state represents the noise level in the absence of the signal field with $\Phi=10\log_{10}R$, where $R\equiv 1+\zeta S$. In Fig. 2(a) this is a noise level set by the laser input $\hat{E}_1$ together with a vacuum-state input $\hat{E}_s$ to the left port of m_1; in Fig. 2(b) it is a noise level set by a squeezed input to m_1 [$S<0$ in Eq. (4)]. The decrease in noise level shown in Fig. 2(b) of more than 3 dB relative to the vacuum or shot-noise level is apparent. The on states in Figs. 2(a) and 2(b) represent the total level of signal plus noise when the phase modulation at 1.6 MHz is gated on, with $\Phi=10\log_{10}R'$, where R' includes the signal contribution from the phase modulation. The only difference between Figs. 2(a) and 2(b) is that in 2(b) squeezed light has been injected into the normally open input of m_1. In particular both the input power P and the phase modulation $V(t)$ are identical for the two traces. The figure thus represents an improvement in signal to noise of 3.0 dB relative to the SNL for the detection of differential phase changes in the Mach-Zehnder interferometer.

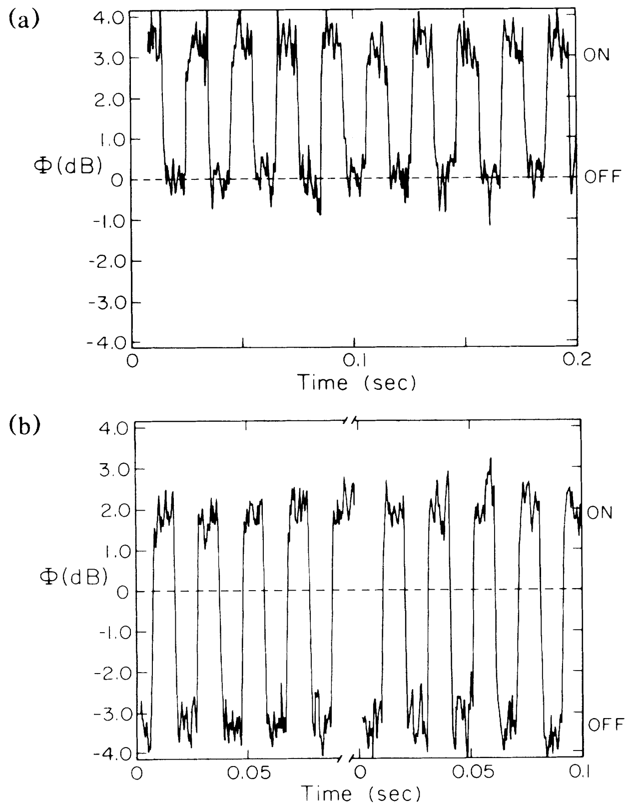

FIG. 2. Level of fluctuations Φ of the difference photocurrent i vs time for fixed analysis frequency $\nu/2\pi=1.6$ MHz, analysis bandwidth $B=100$ kHz, and two video filters of time constant $\tau=1.5\times10^{-4}$ sec and $\tau_2=5\times10^{-4}$ sec. The dashed line gives the vacuum level obtained with no phase modulation and with the squeezed input to m_1 blocked. The ON/OFF square-wave sequence is obtained by gating of the phase modulation to P_1 on and off at a 50-Hz repetition rate. That is, the ON level is obtained with both the modulation signal and noise present; the OFF level is with noise and no signal. The trace in (a) is taken with a vacuum-state input for the field $\hat{E}_s$; the trace in (b) is recorded with a squeezed-state input for $\hat{E}_s$ with the phase θ adjusted for a minimum noise level. An improvement of 3.0 dB in signal-to-noise ratio is achieved in (b) relative to (a). Note that (b) is composed of two traces which are part of the same record obtained during a search for the optimum in both the degree of squeezing S and the local oscillator phase θ.

This observed improvement is in reasonable agreement with that predicted by Eq. (4). Assuming operation at $r^2\sim0.4$, we find $10\log_{10}(\Psi/\Psi_v)=-3.7$ dB for the measured values $\xi=0.94$, $\rho=0.85$, $T_0=0.97$, $\alpha=0.89$, and $\eta=0.93$ appropriate to Fig. 2. While our measurements are conducted without phase-sensitive detection of the modulation at Ω, we note that coherent detection would provide an additional 3 dB of improvement in signal-to-noise ratio both for the case of a vacuum-state input and for a squeezed-state input.

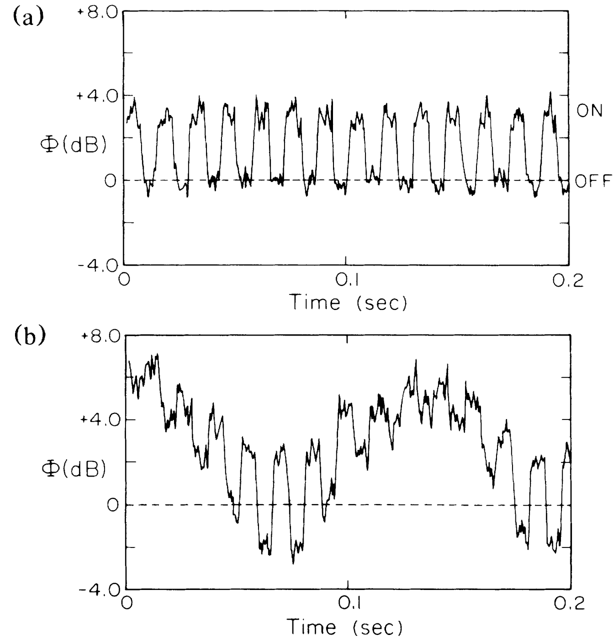

FIG. 3. Level of fluctuations Φ of the photocurrent i vs time. Parameters are similar to those in Fig. 2 with the exception that the phase angle θ is slowly swept with a linear ramp. (a) Vacuum-state input for the field $\hat{E}_s$; (b) Squeezed-state input $\hat{E}_s$. The variation of θ produces alternately a degradation and an improvement in signal to noise as first the increased $(S>0)$ and then the decreased $(S<0)$ fluctuations of the squeezed state are combined with the coherent field $\hat{E}_1$.

The dependence of the detected signal on the phase angle θ is displayed in Fig. 3, which is of the same format as Fig. 2 except that the phase between the fields $\hat{E}_1$ and $\hat{E}_s$ is slowly scanned instead of being held at a fixed value for minimum noise. Figure 3(a) is taken for a vacuum-state input; Fig. 3(b) is for a squeezed state. As expected from Eq. (4) for phase angles such that $S(\theta)<0$, the noise level is reduced, while for $S(\theta)>0$ there is a large degradation in signal-to-noise ratio. This degradation becomes even more pronounced for larger degrees of squeezing until the signal modulation is lost altogether near the maximum noise levels, which for operation closer to threshold are observed to be greater than 8 dB above the SNL. The measured efficiencies for Fig. 3 are $\xi=0.90$, $\rho=0.95$, $T_0=0.97$, $\alpha=0.89$, and $\eta=0.85$. For all our work the fringe visibility is approximately 0.96.

The dashed lines shown in Figs. 2 and 3 are obtained from multiple-trace averages and give the level of fluctuations for a vacuum-state input $\hat{E}_s$ with no phase modulation. That this is indeed the vacuum level is confirmed following the procedures discussed in Ref. 10. Operation at the half-power points of the output fringe has the advantage of preserving the virtues of the balanced homodyne scheme in suppressing excess local oscillator noise. In the current experiments the inherently quiet laser at 1.6 MHz together with the suppression provided by the subtraction arrangement (Σ_-) reduces the contribution of excess local oscillator noise to below 0.1%. The signal shown in Fig. 2(a) was generated by a modulation amplitude $\delta \simeq 7.6\times10^{-6}$, which agrees with the value $\delta_v = 7.8\times10^{-6}$ inferred from the parameters of the experiment ($P = 800\ \mu\mathrm{W}$, $B = 100$ kHz).[15]

In conclusion, we have demonstrated an improvement in sensitivity for optical measurements beyond the limit set by the vacuum-state or zero-point fluctuations of the electromagnetic field. Phase modulation is detected in a Mach-Zehnder interferometer with an improvement in signal-to-noise ratio of 3.0 dB relative to the SNL. We note that this increase in sensitivity is currently limited by losses in propagation and detection and not by the degree of available squeezing. Indeed, the field emitted by the OPO exhibits a degree of squeezing $S^{\theta}_{-} = \rho S_{-} \simeq -0.8$ for the conditions of Fig. 2, indicating the possibility of improvements in sensitivity of 7 dB if the efficiency factors (T_0, α, η, and ξ) can be increased toward unity. Although we have employed a Mach-Zehnder configuration, the results are generally applicable to other types of interferometers, in particular to the arrangements under development for gravity-wave detection.[16]

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[1]V. Braginsky, Y. I. Vorontsov, and K. S. Thorne, Science **209**, 547 (1980).

[2]J. H. Hollenhorst, Phys. Rev. D **19**, 1669 (1979).

[3]H. Takahasi, in *Advances in Communication Systems: Theory and Applications,* edited by A. V. Balakrishnan (Academic, New York, 1965), Vol. 1, p. 227.

[4]H. P. Yuen, Phys. Rev. A **13**, 2226 (1974).

[5]H. P. Yuen and J. H. Shapiro, IEEE Trans. Inf. Theory **24**, 657 (1978), and **26**, 78 (1980).

[6]C. M. Caves, Phys. Rev. D **23**, 1693 (1981).

[7]D. F. Walls, Nature (London) **306**, 141 (1983).

[8]"Squeezed States of the Electromagnetic Field," J. Opt. Soc. Am. B **4** (to be published) (special issue).

[9]H. J. Kimble, Bull. Am. Phys. Soc. **32**, 1280 (1987).

[10]Ling-An Wu, H. J. Kimble, J. L. Hall, and H. Wu, Phys. Rev. Lett. **57**, 2520 (1986); Ling-An Wu, Min Xiao, and H. J. Kimble, in Ref. 8.

[11]M. J. Collett and C. W. Gardiner, Phys. Rev. A **30**, 1386 (1984); C. W. Gardiner and C. M. Savage, Opt. Commun. **50**, 173 (1984).

[12]R. S. Bondurant and J. H. Shapiro, Phys. Rev. D **30**, 2548 (1984).

[13]J. Gea-Banacloche and G. Leuchs, to be published, and in Ref. 8.

[14]B. Yurke, P. Grangier, and R. E. Slusher, in Ref. 8.

[15]Hewlett-Packard Application Note No. 150-4, Spectrum Analyzer Series (Hewlett-Packard Corp., Palo Alto, CA, 1974), Chap. 2.

[16]K. S. Thorne, in *300 Years of Gravitation,* edited by S. W. Hawking and W. Israel (Cambridge Univ. Press, Cambridge, 1987).