Nonlinear Interaction between a Warm Electron Beam and a Single Wave

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We have experimentally observed the growth and saturation of a single wave on a warm-beam, slow-wave–structure system. When the wave growth is sufficiently weak, the time-averaged beam distribution function develops a localized plateau. When the growth is stronger, the entire distribution function can become distorted. The electric field at saturation scales as the square of the linear growth rate of the wave.

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In the linear theory of electron plasma waves propagating in warm collisionless plasmas, it is well known[1,2] that if there is a negative slope in the electron velocity distribution function at the phase velocity of the wave, the wave exhibits Landau damping. If the slope is positive, the wave exhibits Landau growth. However, when the plasma-wave oscillations are nonlinear, only the damping of a single launched wave is well understood theoretically[3] and experimentally.[4] The growth of a single launched wave to nonlinear saturation is not well understood in the warm-plasma case.

The canonical experimental situation in which wave growth is studied is the case of a low-density warm electron beam streaming through a background plasma. Normally, an entire spectrum of modes becomes unstable and grows. The standard theoretical description in this case is quasilinear theory.[5] The case of the growth of a single wave on a warm-beam–plasma system is rather difficult to realize experimentally. Nevertheless, an experiment to address this basic problem is of interest. There have been a number of relevant numerical and theoretical works. In 1971 the problem was studied numerically by Fried *et al.*[6] The related problem of single-wave saturation in a low-density, cold-beam–plasma system was studied by Drummond *et al.*, by O'Neil, Winfrey, and Malmberg, and by Onischenko *et al.*[7] These authors predicted on the basis of a trapping model that in the temporally growing case, the saturation electric field, $E_{\rm sat}$, would scale with the linear growth rate, ω_i, as $E_{\rm sat} \propto \omega_i^2$. In a spatially growing system this corresponds to $E_{\rm sat} \propto k_i^2$. We will call this scaling "trapping scaling." In addition, the closely related case of the saturation of a marginally unstable wave in the low-density, warm-beam case was studied theoretically by Simon and Rosenbluth[8]; this case has been studied numerically in a recent work by Denavit.[9]

A useful feature of the weak-warm-beam–plasma interaction is that if the beam is of sufficiently low density, the background plasma behaves as a linear dielectric and acts only to support the wave. We exploit this feature in our experiment by replacing the plasma with a helical slow-wave structure.[10] In the weak-beam limit, the equations that describe the interaction of the warm beam and the slow-wave structure are formally identical[11] to those that describe the warm-beam–plasma interaction. The low inherent noise level of the slow-wave structure enables us to launch a single wave at a level which is not only well below saturation, but also well above the background noise level. With this device we have experimentally observed the growth and saturation of a single wave in a warm-beam, slow-wave–structure system. If the growth rate is sufficiently weak, saturation occurs when a localized plateau is formed in the beam distribution function. If the growth rate is sufficiently strong, the entire beam distribution function can be distorted at saturation. We have also found that the wave electric field at saturation, $E_{\rm sat}$, scales with the spatial linear growth rate, k_i, as $E_{\rm sat} \propto k_i^\alpha$, where $\alpha = 1.94 \pm 0.20$, in excellent agreement with the $\alpha = 2$ prediction of trapping scaling.

The interaction between the beam electrons and the growing wave is characterized by the ratio of the particle autocorrelation length to the spatial growth length, $\eta_p = k_i \omega / k^2 \Delta v_p$. We are interested in the case in which this ratio is less than unity. In the opposite limit, $\eta_p \gg 1$, the beam is considered to be cold and the Landau formula does not apply. If $\eta_p \gg 1$ and if beam density is sufficiently low, then the interaction is well described by the small-cold-beam–plasma theory.[7] Here ω and k are the angular frequency and wave number of the mode. Δv_p represents the width of the distribution function; in our experiment we define $\Delta v_p = v_{75} - v_{25}$, where v_{75} and v_{25} are the velocities at which the unperturbed-beam parallel-energy distribution function has decreased down its positive slope to 75% and 25% of its maximum value, respectively.

The experimental apparatus[12] consists of an electron beam which is confined by a strong magnetic field ($B_z = 440$ G) and directed along the axis of a helical slow-wave structure. The electron beam enters the slow-wave structure with an axial velocity spread. This spread in axial velocities is produced[13] by our passing a cold electron beam through three parallel, closely spaced, wire mesh grids. All three grids can be biased; the middle grid is normally biased at a high positive potential,

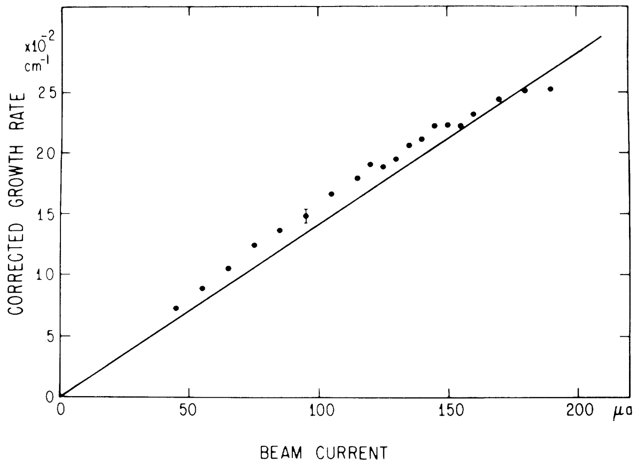

FIG. 1. Corrected linear growth rate vs beam current. Cathode voltage, $V_0=65$ V; frequency, $f=65$ MHz; $V_S=2$ kV. Bias on the entrance grid of the spreader, $V_{S1}=0$; bias on the exit grid of the spreader, $V_{S3}=0$.

V_S. Because of the strong electric fields near the grid wires, the electrons nonadiabatically scatter some of their axial kinetic energy into perpendicular kinetic energy. Thus, by adjusting V_S we can control the axial velocity spread of the beam. An important property of this method of producing an axial velocity spread is that the shape of the electron velocity distribution is independent of the beam current over the range of beam currents used in the experiment (5–800 μA). The helix is rigidly held together by a support structure and is enclosed by a glass vacuum tube. Outside of the glass tube are axially movable electrostatic probes which are used to transmit and receive radio-frequency waves. The helix and probe assembly are enclosed by an axially slotted 3.8-cm-radius cylinder which defines the rf ground and acts as a waveguide beyond cutoff for the frequencies used in the experiment. This ensures that for these frequencies, the only traveling waves that can exist in the tube propagate on the helix. Most of the beam is collected immediately after passing through the helix. A small fraction of the beam passes through a hole in the collecting electrode, then through a discriminator tube, and is then separately collected. By biasing the discriminator tube and measuring the current that gets through it, we determine[14] the time-averaged beam velocity distribution function.

The linear growth of a single wave on a warm beam is described theoretically by the Landau formula. In Fig. 1, we plot the corrected linear growth rate, k_1-k_{i0} versus the beam density. Here k_i is the measured linear growth rate and k_{i0} ($k_{i0}<0$) is the measured damping rate of the wave in the absence of the beam. This wave damping is due to dissipation in the helix support structure. The beam density was varied by the variation of the beam current with the beam voltage kept fixed. The

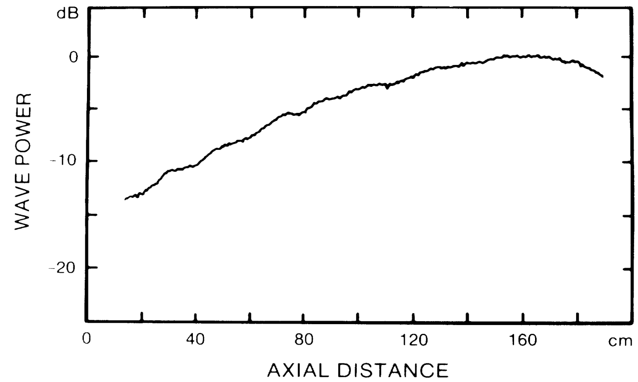

FIG. 2. Wave power vs axial distance. Beam voltage, $V_0=120$ V; beam current, $I_0=440$ μA; $V_S=3$ kV, $V_{S1}=115$ V, $V_{S3}=0$; $f=50.0$ MHz; $\eta_p=0.098$.

points are the measured values and the line is given[11] by

$$k_i-k_{i0}=\frac{\pi ne^2\omega R(\partial f/\partial v)}{m},$$

where n is the density of electrons per unit length, $-e$ and m are the electron charge and mass, R is essentially[10] the impedance of the helix and $\partial f/\partial v$ is the measured slope of the unperturbed distribution function evaluated at the wave phase velocity. The agreement is seen to be quite good.

In Fig. 2 we exhibit the growth and nonlinear saturation of the single wave. Here we plot the logarithm of the wave power as a function of axial distance, z. The wave power is seen to grow exponentially nearly 15 dB and then saturate at around $z=160$ cm. The weak slow oscillations visible in the plot are due to a beat between the forward wave on the helix and a passive mode on the beam.

If the growth rate is weak in the sense that $\eta_p \ll 1$, a localized plateau forms in the beam distribution function. In Fig. 3(a) we show the time-averaged parallel-energy distribution function of the beam with no wave launched (dotted line) and with a launched single wave which has grown to saturation (solid line). As the wave grows and saturates, the distribution function is seen to develop a localized plateau. The arrow marks the parallel energy corresponding to the phase velocity of the wave. If we increase η_p, the region in parallel energy over which the distribution function becomes distorted increases. If η_p is sufficiently large to cause the distortion to involve the high-energy edge of the distribution, the entire beam distribution becomes distorted at saturation. The value of η_p at which this occurs can be made smaller if the wave is launched at a phase velocity closer to the edge of the beam distribution. In Fig. 3(b) we show an example of such a distortion of the beam.

We next present our results on the scaling of the saturation electric field with the growth rate. The basic idea of the trapping scaling can be understood from the fol-

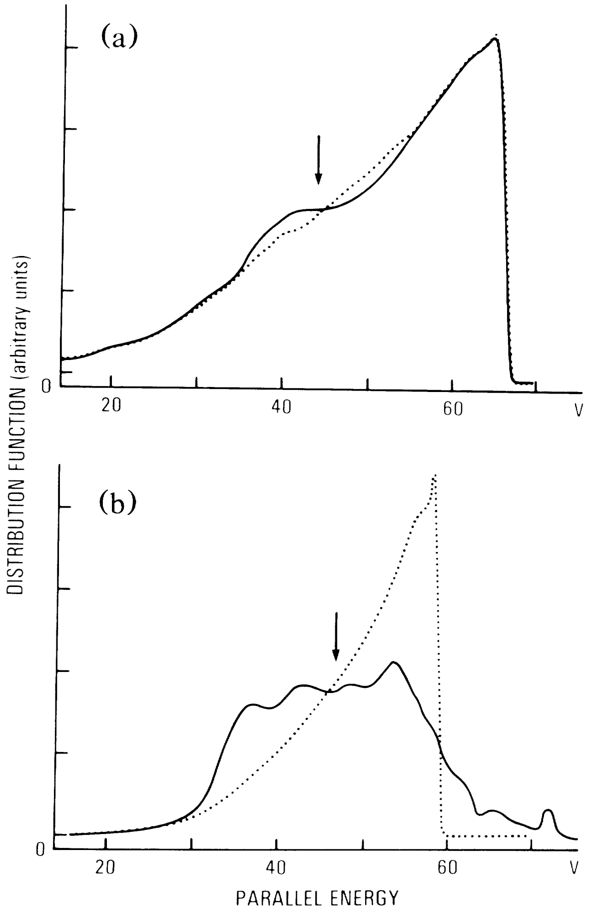

FIG. 3. (a) Time-averaged beam parallel-energy distribution function at saturation (solid line) and with no launched wave (dotted lines). $V_0=65$ V; $I_0=200$ μA; $V_S=3$ kV; $V_{S1}=V_{S3}=0$; $f=65$ MHz; $\eta_p=0.037$. (b) Same as (a) but with $\eta_p=0.14$; $V_0=-60$ V; $I_0=110$ μA; $V_S=2$ kV; $f=55$ MHz.

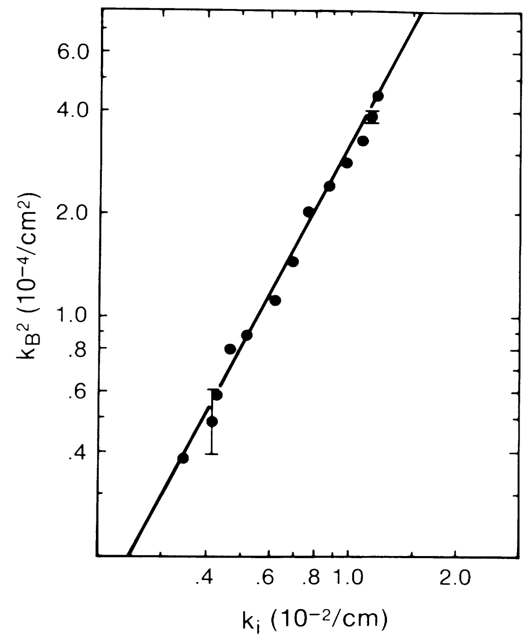

FIG. 4. Electric field at saturation vs growth rate. $V_0=120$ V; $V_S=3$ kV; $V_{S1}=115$ V; $V_{S3}=0$; $f=50.0$ MHz.

lowing considerations.[15] From Fig. 2 we see that for sufficiently small η_p, saturation is associated with a local flattening of the distribution function. The velocity width of this flattening is roughly the trapping width $2\Delta v_T$, where $\Delta v_T^2=4eE_{sat}/mk$. On the assumption that the slope of the velocity distribution function of the beam is constant over Δv_T and that $k\Delta v_T/\omega \ll 1$, the power required to form the plateau is approximately $\Delta P_{beam} \simeq nm\Delta v_T^3(\omega/k)^2 \partial f/\partial v$, where n is the density of electrons per unit length. Equating this with the power in the wave, $P_{wave} \simeq E_{sat}^2/2k^2R$, yields the scaling $k_B^2=\beta k_i^2$, where $k_B^2=(k/\omega)^2 ekE_{sat}/m$. In obtaining this scaling we have assumed that the wave-power saturation corresponds to a very small positive slope of the local plateau in the distribution function which just balances the dissipative damping (k_{i0}) due to the Ohmic losses in the helix. The constant β depends sensitively on the exact size and shape of the final distribution function and is too crudely calculated for a meaningful comparison with the experiment. One can show[16] more generally that if one assumes that the slope of the unperturbed velocity distribution function is a constant, then the Vlasov-Poisson equations scale according to the trapping scaling. We also note that if $\partial f/\partial v$ were not constant over $2\Delta v_T$ the scaling would be different.[15] In Fig. 4 we show the scaling of the saturation electric field with the growth rate. The solid line is a line of slope 2 drawn through the points indicating the trapping scaling. A least-squares fit of the points gives $k_B^2 \propto k_i^\alpha$ with $\alpha=1.94 \pm 0.20$, which is in excellent agreement with the trapping scaling. Both the growth rate and the saturation electric field are measured. The growth rate is varied by our varying the total beam current and keeping the beam voltage fixed. That is, the beam density is varied. We have arranged the phase velocity of the wave to be far from the peak of the distribution function. The shape of the distribution function is such that $\partial f/\partial v$ is constant over an increasingly wider velocity range as the velocity decreases from that of the peak of the distribution.

To summarize, we have observed the growth and saturation of a single wave on a warm-beam, slow-wave-structure system. The nature of the distribution function at saturation depends on $\eta_p=k_i\omega/k^2\Delta v$. When η_p is sufficiently small, the distribution develops a localized plateau and the saturation electric field scales as the square of the growth rate in agreement with the trapping scaling. When η_p is larger the entire distribution function can become distorted.

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