Anthropic Bound on the Cosmological Constant

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In recent cosmological models, there is an "anthropic" upper bound on the cosmological constant Λ. It is argued here that in universes that do not recollapse, the only such bound on Λ is that it should not be so large as to prevent the formation of gravitationally bound states. It turns out that the bound is quite large. A cosmological constant that is within 1 or 2 orders of magnitude of its upper bound would help with the missing-mass and age problems, but may be ruled out by galaxy number counts. If so, we may conclude that anthropic considerations do not explain the smallness of the cosmological constant.

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Our knowledge of the present expansion rate of the Universe indicates that the effective value Λ of the cosmological constant is vastly less than what would be produced by quantum fluctuations[1] in any known realistic theory of elementary particles. In view of the continued failure to find a microscopic explanation of the smallness of the cosmological constant, it seems worthwhile to look for a solution in other, "anthropic," directions.[2] Perhaps Λ must be small enough to allow the Universe to evolve to its present nearly empty and flat state, because otherwise there would be no scientists to worry about it. Without having a definite framework for such reasoning, one can at least point to four lines of current cosmological speculation, in which anthropic considerations could set bounds on the value we observe for the effective cosmological constant:

(a) The effective cosmological constant may evolve very slowly, perhaps because of slow changes in the value of some scalar field, as in the model of Banks.[3] In this case, it would be natural to expect that for some very long epoch the cosmological constant would remain near zero. The question then is, why do we find ourselves in such an epoch? As remarked by Banks, the answer may be anthropic: Perhaps only in such epochs is life possible.

(b) The Universe may evolve through a very large number of first-order phase transitions, in which bubbles form within bubbles within bubbles..., each bubble having within it a smaller value of the vacuum energy, and hence of the effective cosmological constant. If the steps in vacuum energy are very small, then it would be natural to expect that there would be some phase in which the effective cosmological constant is correspondingly small. Abbott[4] has suggested a scalar-field theory with a potential that has an infinite number of closely spaced local minima; bubbles form within bubbles as the scalar-field value jumps from one minimum to the next. Recently Brown and Teitelboim[5] have proposed a model in which a similar sequence of phase transitions occurs, but in which the bubble walls are elementary membranes coupled to a three-form gauge field, with the difference in cosmological constants between the inside and outside of each membrane caused by the differences in the values of the four-form field strength.[6] In models of the type discussed in Refs. 4 and 5 it may not be strictly necessary to invoke the anthropic principle because gravitational effects can stop the process of bubble formation when the vacuum energy is about to become negative.[7] However, it takes an enormously long time to reach this final stage, and anthropic arguments may be needed to explain why we are not still in an earlier stage, with large effective cosmological constant.

(c) Fluctuations in scalar fields can trigger cosmic inflation in regions of the Universe where the fields happen to be large. Except near the edges, the inflationary region would appear to its inhabitants as a separate subuniverse. In this region further fluctuations can produce new inflations, and so on. This has been studied by Linde,[8] who remarks that the physical constants of the subuniverse in which we live may be in part constrained by the requirement that life could arise in such a subuniverse.

(d) Quantum fluctuations in the very early Universe may cause incoherence between different terms in the state vector of the Universe; each term would then in effect represent a separate universe. Such a picture has been considered by Hawking.[9] Our own Universe could correspond to any one of the terms in the state vector, subject only to the anthropic condition, that it be a universe in which life could develop.

Without committing ourselves to any one of these cosmological models, it seems appropriate at least to ask, just what limit does the anthropic principle place on the effective cosmological constant Λ?

Fortunately, at least for $\Lambda > 0$, the anthropic principle provides a rather sharp upper bound on Λ. This is because in a continually expanding universe, the cosmological constant (unlike charges, masses, etc.) can affect the evolution of life in only one way. Without undue anthropocentrism, it seems safe to assume that in order for any sort of life to arise in an initially homogeneous and isotropic universe, it is necessary for sufficiently large gravitationally bound systems to form first. (By "sufficiently large" is meant large enough to form stars, and large enough also to contain the heavy elements produced by early generations of stars so that planets can form around subsequent generations of stars. Galaxies and probably also the larger globular clusters are sufficiently large in this sense.[10]) However, once a sufficiently large gravitationally bound system has formed, a cosmological constant would have no further effect on its dynamics, or on the eventual evolution of life. In particular, it makes no difference if the e-folding time of the cosmic expansion is much shorter than the time required for the evolution of intelligent life.[11] (Note that I am here *not* requiring that the cosmological constant have a value consistent with the astronomical knowledge, but only that it have a value consistent with the appearance of beings that could measure it.) Thus, irrespective of what we think are the possible forms of intelligent life, the necessary and sufficient anthropic condition on the cosmological constant is that it should not be so large as to prevent the appearance of gravitationally bound states.

I evaluate this bound here in the context of conventional big-bang cosmologies, described by a Robertson-Walker metric with initially small perturbations. For definiteness I will concentrate here on the case of positive cosmological constant and zero spatial curvature, $k=0$, and with the energy density of the Universe dominated since recombination by nonrelativistic matter. However, it would not be hard to adapt the arguments to any other case in which the Universe does not recollapse.

Let us consider then the fate of a density perturbation in a Universe with $\Lambda > 0$. Such a perturbation can be modeled[12] as a sphere within which there is a uniform excess density $\Delta\rho(t)$, and a gravitational field described by a Robertson-Walker (RW) metric with positive curvature constant $\Delta k > 0$, and with RW scale factor $a(t)$. The evolution of the perturbation is governed by a Friedmann equation

$$(da/dt)^2 + \Delta k = \tfrac{8}{3}\pi G a^2(\rho + \Delta\rho + \rho_V), \qquad (1)$$

where $\rho(t)$ is the unperturbed cosmic mass density, ρ_V is the constant vacuum energy density

$$\rho_V \equiv \Lambda/8\pi G, \qquad (2)$$

and $\Delta\rho(t)$ is the perturbation, satisfying the equation of mass conservation

$$a^3(\rho+\Delta\rho) = \text{const.} \qquad (3)$$

I am *not* assuming here that $\Delta\rho$ or Δk is small, but there is a branch of the solutions of Eq. (1) for which, as $t \to 0$, $\Delta\rho \propto t^{-4/3}$ while $\rho \propto t^{-2}$, so that in this limit $\rho_V \ll \Delta\rho \ll \rho$. I assume that all perturbations are on this branch, so that the universe looks smooth for $t \to 0$. The strength of such a perturbation can then be characterized by the parameter

$$\tilde{\rho} \equiv \lim_{t\to 0}\{[\Delta\rho(t)]^3/\rho^2(t)\}. \qquad (4)$$

Treating Δk and $\Delta\rho$ to first order for $t \to 0$ in Eq. (1), we find for the perturbed curvature constant

$$\Delta k = \tfrac{40}{9}\pi G a^2(\rho+\Delta\rho)^{2/3}\tilde{\rho}^{1/3}. \qquad (5)$$

Equation (1) shows that the perturbed scale factor $a(t)$ will increase to a maximum and then collapse back to $a=0$, provided that there is a value of $a(t)$ where the right-hand side of Eq. (1) is equal to Δk. The right-hand side of Eq. (1) reaches a minimum at a value of $a(t)$ such that $\rho+\Delta\rho = 2\rho_V$, and so the condition for a given perturbation to undergo gravitational condensation is that at this minimum, the right-hand side of Eq. (1) should be less than Δk. This condition can be written

$$8\pi G \rho_V^{1/3}[\tfrac{1}{2}(\rho+\Delta\rho)]^{2/3}a^2 < \Delta k. \qquad (6)$$

With use of Eq. (5) to express the perturbed curvature constant in terms of the parameter $\tilde{\rho}$, this becomes

$$\rho_V < \tfrac{500}{729}\tilde{\rho}. \qquad (7)$$

If there is an upper bound on the parameter $\tilde{\rho}$, and if this upper bound is independent of Λ (because it refers to very early times, when ρ_V is negligible), then Eq. (7) provides our anthropic bound on the vacuum energy density.

[It is instructive to compare this with the result of a linear analysis, in which Δk and $\Delta\rho$ are treated throughout as first-order perturbations. The solution of the second-order differential equation[13] for $\Delta\rho/\rho$ takes the form

$$\Delta\rho/\rho \propto (\sinh\tau)^{-2/3} Q_{1/3}^{2/3}(\coth\tau),$$

where

$$\tau \equiv (6\pi G\rho_V)^{1/2} t,$$

and $Q_\nu^\mu(z)$ is the associated Legendre function of the second kind.[14] This behaves for $\tau \ll 1$ like $\tau^{2/3}$, and then rises monotonically[15] to a constant limit as $\tau \to \infty$. By comparing the asymptotic behavior of $\Delta\rho/\rho$ for $\tau \to 0$ and $\tau \to \infty$, we can see that if we normalize so that $\Delta\rho/\rho \to \epsilon\tau^{2/3}$ for $\tau \to 0$, then for $\tau \to \infty$

$$\Delta\rho/\rho \to (2/\sqrt{\pi})\Gamma(\tfrac{11}{6})\Gamma(\tfrac{2}{3})\epsilon = 1.437\epsilon.$$

With this normalization, the parameter $\tilde{\rho}$ is just $\epsilon^3\rho_V$, so that $\Delta\rho/\rho \to 1.437(\tilde{\rho}/\rho_V)^{1/3}$. One might guess that the necessary and sufficient condition for gravitational condensation is that the linear analysis should give an asymptotic value of $\Delta\rho/\rho$ at least of order unity. If this were correct, then the upper bound on ρ_V for gravitational condensation to occur would be

$$\rho_V < (1.437)^3\tilde{\rho} = 2.97\tilde{\rho}.$$

This is quite different from the result (7), showing the inadequacy here of linear methods.]

Now, what is the distribution of $\tilde{\rho}$ values for actual perturbations? Eventually, the theory of the very early Universe may develop to the point that the distribution of $\tilde{\rho}$ will be known in terms of fundamental constants. Alternatively, it may some day become possible to read off the distribution of values of $\tilde{\rho}$ from observations of small-angle fluctuations in the cosmic microwave background temperature.

For the present, it seems that the best we can do is to use the existence of quasars at high red shifts to set an empirical lower bound on any maximum value for $\tilde{\rho}$, and hence a lower bound on the anthropic upper bound (6) on ρ_V. This is not as pointless as it may seem. If it turns out that the empirical lower bound on the maximum value of the right-hand side of Eq. (6) is much larger than empirically allowed values for ρ_V, then we would have to conclude that the anthropic principle does not explain why the cosmological constant is as small as it is.

In the model we have been considering, the time required for the scale factor $a(t)$ of a perturbation to reach its maximum and then collapse back to $a=0$ is an increasing function of ρ_V, so that it is bounded below by its value when $\rho_V=0$:

$$t_c > \tfrac{9}{2}\pi(250\pi G\tilde{\rho})^{-1/2}. \tag{8}$$

Actually, we do not measure the age of early gravitational condensations, but we do observe their red shifts. The time corresponding to a red shift z_c is bounded *above* by its value for $\rho_V=0$:

$$t_c < \tfrac{2}{3}(3/8\pi G\rho_0)^{1/2}(1+z_c)^{-3/2}, \tag{9}$$

where ρ_0 is the present cosmic mass density. Thus, putting together (8) and (9), we see that the observation of gravitational condensations (e.g., quasars) of red shift z_c or greater sets an empirical lower bound on the maximum value of the parameter $\tilde{\rho}$:

$$\tfrac{500}{729}\tilde{\rho}_{\max} > \tfrac{1}{3}\pi^2\rho_0(1+z_c)^3. \tag{10}$$

For instance, we know that quasars exist with red shifts up to about $z=4.4$, so taking $z_c=4.5$ in (10) gives a lower bound of $550\rho_0$ on the anthropic upper bound on ρ_V.

Now, if the intrinsic distribution of ρ_V values is smooth and featureless below the anthropic bound (as would be expected in the models of Refs. 3–5, 8, and 9 if the natural scale for ρ_V is set by the Planck mass), then it seems likely that ρ_V would be within 1 or 2 orders of magnitude of its upper bound. (This can be made more precise by calculations like those of Carter in Ref. 2.) With a lower bound of $550\rho_0$ on the anthropic upper bound on ρ_V, we would then conclude that ρ_V must be much greater than the present mass density ρ_0. Is this plausible? The answer unfortunately depends on data whose interpretation is far from settled.

On one hand, a vacuum energy density much larger than ρ_0 would resolve a problem that has been posed by measurements of ρ_0 for those who believe, on grounds of either inflation or aesthetics, that (as assumed here) the Universe has a vanishing unperturbed curvature constant k. This implies that the total energy density $\rho_V+\rho_0$ is equal to its critical value $3H_0^2/8\pi G$; that is $\Omega_V+\Omega_0=1$, where $\Omega_V=8\pi G\rho_V/3H_0^2$ and $\Omega_0=8\pi G\rho_0/3H_0^2$. The deuterium abundance indicates[16] that the contribution of baryons to Ω_0 is no greater than about 0.03 (for $H_0=100$ km/sec Mpc), while observations of galaxies suggest[17] that they contribute about 0.02 to Ω_0. Even allowing for nonbaryonic extra-galactic matter, the dynamics of clusters of galaxies lead to estimated values[17] of Ω_0 only about 0.1 to 0.2. It would be difficult to satisfy the condition $\Omega_0+\Omega_V=1$ if Ω_V were much smaller than Ω_0. However, if for instance we suppose that ρ_V is at least 1% of its anthropic upper bound (10), then Ω_V/Ω_0 must be at least 5.5, so that we would not need Ω_0 to be greater than 0.15.

The assumption of a vacuum energy density roughly comparable with the anthropic upper bound would also help[18] with the problem of cosmic ages that arises if the Hubble constant H_0 is as large as 100 km/sec Mpc. In this case, if $\Lambda=0$ and $k=0$, then the age of the Universe is about $\frac{2}{3}H_0^{-1}=6.5\times10^9$ yr, less than the (10–20) $\times10^9$ yr usually given[19] for the globular clusters. However, if at present ρ_V is much greater than ρ_0, then over most of the history of the Universe $R(t)$ behaves as $\exp(Ht)$ rather than $t^{2/3}$, and the age of the Universe is greater than $\frac{2}{3}H_0^{-1}$. Specifically, the present age of a gravitational condensation which forms at a red shift z_c is

$$t_0-t_c=\tfrac{2}{3}(1+\rho_0/\rho_V)^{1/2}H_0^{-1}\{\operatorname{invsinh}(\rho_V/\rho_0)^{1/2}-\operatorname{invsinh}[(\rho_V/\rho_0)^{1/2}(1+z_c)^{-3/2}]\}. \tag{11}$$

If for instance $z_c=4$ and $\rho_V/\rho_0=9$ (i.e., $\Omega_0=0.1$), then the age (11) is $1.1H_0^{-1}$, which even for H_0 close to 100 km/sec Mpc leaves adequate time for the evolution of globular clusters.

On the other hand, counts of galaxies as a function of red shift indicate[20] (for $k=0$) that $\Lambda/3H_0^2=0.1^{+0.2}_{-0.4}$ or in other words $\rho_V/\rho_0=0.1\pm0.3$. According to (10), this is at least 3 orders of magnitude less than the anthropic upper bound (7).

A similar conclusion would be reached if it turns out that gravitational condensations occur at red shifts larger than 4.5. For instance, if gravitational condensations occur at a red shift $z_c=30$, then according to Eq. (10), the anthropic upper bound on ρ_V is at least $10^5\rho_0$. But Ω_0 cannot be less than about 0.01, indicating (for $k=0$)

a vacuum energy density ρ_V no greater than $100\rho_0$, which is again at least 3 orders of magnitude less than the anthropic upper bound.

Thus if the interpretation of galaxy number counts in Ref. 20 holds up, or if gravitational condensations are found at red shifts $z \gg 4$, we will be able to conclude that the cosmological constant is so small that even the anthropic principle could not explain its smallness.

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[1]See, e.g., Ya. B. Zeldovich, Pis'ma Zh. Eksp. Teor. Fiz. **6**, 883 (1967) [JETP Lett. **6**, 316 (1967)].

[2]For a comprehensive survey of the anthropic principle, see J. D. Barrow and F. J. Tipler, *The Anthropic Cosmological Principle* (Clarendon, Oxford, 1986). Also see P. C. W. Davies, *The Accidental Universe* (Cambridge Univ. Press, Cambridge, 1982), Chap. 5; B. Carter, Philos. Trans. Roy. Soc. London A **310** 347 (1983).

[3]T. Banks, Nucl. Phys. **B249**, 332 (1985).

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[6]The introduction of a three-form gauge field has the effect of making the effective cosmological constant a constant of integration, as shown by E. Witten, in *Shelter Island II: Proceedings of the 1983 Shelter Island Conference on Quantum Field Theory and the Fundamental Problems of Physics, Shelter Island, New York, 1983,* edited by R. Jackiw *et al.* (M.I.T. Press, Cambridge, MA, 1985); M. Henneaux and C. Teitelboim, Phys. Lett. **143B**, 415 (1984).

[7]S. Coleman and F. De Luccia, Phys. Rev. D **21**, 3305 (1980).

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[9]S. Hawking, as quoted by M. Gell-Mann, Phys. Scr. **T15**, 202 (1987).

[10]This remark about globular clusters is due to H. Smith. He points out that the observed low gas abundance in globular clusters presumably results from their inevitable frequent passages through the plane of our galaxy with consequent sweeping out of the gas from the clusters during these passages. If globular clusters formed first, and then did not associate with galaxies, there could still be heavy elements formed by the evolving stars in a cluster, condensing to the plane of rotation of the cluster, perhaps reaching densities high enough to allow the formation of second-generation stars with planets. This could work for large enough globular clusters which thus effectively become small galaxies. A small globular cluster would probably not have sufficient gravitational attraction to hold the gas against the blast effects of supernovae stripping it from the cluster.

[11]In this respect, I differ from Barrow and Tipler (Ref. 2) and consequently obtain a different anthropic bound on Λ. They require that $|\Lambda|^{-1/2}$ (roughly the vacuum Hubble time) should be at least as large as the main-sequence stellar lifetime t_*. This seems to me correct for $\Lambda < 0$ [where the Universe recollapses in a time $\pi(3|\Lambda|)^{-1/2}$], but not for $\Lambda > 0$, the case under consideration here. It is true that for $\Lambda > 0$, gravitational condensation must occur within a time of order $\Lambda^{-1/2}$, because after that time the matter density drops below the vacuum density, and gravitational condensation becomes impossible. However, once gravitationally bound systems form, their subsequent evolution is unaffected by the cosmological constant, and it makes no difference how long it takes for stars to evolve on the main sequence.

[12]P. J. E. Peebles, Astrophys. J. **147**, 859 (1967).

[13]See, e.g., S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), Eq. (15.10.57).

[14]I use the notation of the *Handbook of Mathematical Functions,* edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1965), Sec. 8.

[15]I am very grateful to P. Candelas for a numerical integration of the differential equation for $\Delta\rho/\rho$.

[16]A. M. Boesgaard and G. Steigman, Annu. Rev. Astron. Astrophys. **23**, 310 (1985).

[17]*Dark Matter in the Universe,* edited by G. R. Knapp and J. Kormendy, Proceedings of the International Astronomical Union Symposium No. 117 (Reidel, Dordrecht, 1987).

[18]G. de Vaucouleurs has long advocated the introduction of a cosmological constant to reconcile the apparent inconsistency of his measurement of the Hubble constant with globular cluster ages; see Astrophys. J. **268**, 468 (1983), Appendix B, and Nature (London) **299**, 303 (1982), etc. Also see P. J. E. Peebles, Astrophys. J. **284**, 439 (1984).

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