Spontaneous Emission of Radiation in the Presence of a Phase-Conjugate Mirror

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The spontaneous radiation of a classical oscillator and that of a two-level atom both in the vicinity of a phase-conjugate mirror are calculated and compared. In either case the phase-conjugate mirror may be replaced by a phase-conjugate "image" of the source. We study the dependence of the decay constants and the characteristic frequencies on the phase-conjugate reflectivity and the difference between the characteristic frequency of the radiator and the pump frequency of the phase-conjugate mirror.

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The properties of the electromagnetic radiation that is emitted by an excited antenna or atom are known to be influenced by the presence of objects located within a distance of a few wavelengths of the emitter, as in the cooperative decay of atoms,[1-3] or of a single atom which is located near a mirror.[4-7] The latter problem was treated by Lyuboshitz[4,5] in terms of the coupling through an electric dipole interaction between the radiating atom and an identical image. As shown by Morawitz,[6] the six vibrational modes of the classical dipole in the presence of a mirror (the symmetric and antisymmetric modes which are polarized in the direction normal to the mirror and the two orthogonal directions parallel to the mirror) each have a precise correspondence to the symmetric and antisymmetric collective eigenstates of the Hamiltonian of two identical atoms. Because of the broken rotational symmetry of the single atom, the parallel antisymmetric and orthogonal symmetric modes exhibit individual decay rates and level shifts, which are functions of the distance of atom from the mirror.

There are several good reasons for taking an interest in the radiative decay of an atom that is located near a phase-conjugate mirror (PCM): (a) Since the PCM reverses the direction of radiation emitted by the radiating atom,[8] the possibility arises of intensified interaction between the atom and the PCM; (b) the phase of the returned wave at the location of the radiating atom is nearly independent of the distance of the atom from the PCM, up to the distance traveled by the radiation during a large fraction of the radiative decay time, so that the interaction with the PCM remains strong over much greater distance; (c) under operation with sufficient power in the pump beams of the nonlinear PCM (which will be assumed to be based on four-wave mixing in this paper), a conjugation gain in excess of unity becomes possible,[9] providing yet another mechanism by which to enhance the magnitude of the interaction; and (d) the radiating atom interacts, as discussed below, with its own phase-conjugate image, which is time reversed at one frequency.

My model for the PCM is based on the four-wave mixing process,[8] according to which the PCM may create two photons of wave vectors $\mathbf{k}$ and $\mathbf{k}'$ at the loss of two pump photons of wave vectors $\pm\mathbf{k}_m$. The condition of energy conservation then yields a relation between the angular frequencies

$$\omega_k + \omega_{k'} = 2\omega_m. \qquad (1)$$

The difference $\omega_{k'} - \omega_k$ constitutes a frequency shift between conjugate and incident radiation, which was not included in previous treatments[10,11] of a radiating dipole in the presence of a PCM. My inclusion of it has led to the bulk of the new results.

Classical linear dipole radiator.—I consider a classical dipole oscillator placed at a distance D from a PCM. The dipole is described by the conventional damped isotropic oscillator equation:

$$\ddot{\boldsymbol{\mu}} + \omega_0^2 \boldsymbol{\mu} + \alpha_{\rm nr}\dot{\boldsymbol{\mu}} = \mathbf{E}_s(t) + \mathbf{E}_c(t), \qquad (2)$$

where $\boldsymbol{\mu}(t)$ is the dipole-moment vector, ω_0 is the natural angular oscillation frequency, $\alpha_{\rm nr}$ is a decay constant describing nonradiative decay, $\mathbf{E}_s(t)$ is the self-field,[12] accounting for the radiative decay, and $\mathbf{E}_c(t)$ is the field returned by the PCM to the dipole's location. The self-field is the reaction of the radiated field upon the oscillating charges. The time-varying quantities in Eq. (2) are defined to be the complex positive-frequency parts of the physical (real) variables.

The PCM is assumed to shift each frequency component of incident radiation according to Eq. (1). This effect is nonnegligible, even for a resonantly interacting dipole-PCM system, where $\omega_0 = \omega_m$, because of the nonzero bandwidth. The conjugate field is then obtained as

$$E_c(t) = rE_s^*(t-\tau)\exp[-i\omega_m(2t-\tau)], \qquad (3)$$

where r is the amplitude reflectivity of the PCM, $\tau = 2D/C$ equals the round-trip delay time in terms of the distance D from the PCM plane, and the asterisk denotes complex conjugate. The presence of spatial distortion may affect the accuracy of Eq. (3). Such effects can nevertheless be absorbed into the constant r, provided that two conditions be satisfied. The first is that the radiation be time harmonic, and the second is that τ does not depend too strongly on the direction of propagation. The latter requires

$$(2D\Omega/C)(\sec\theta - 1) \ll 1, \tag{4}$$

where Ω is the maximum frequency shift and θ represents the largest angle at which a ray may be conjugated by the PCM. Then the spatial distortion which arises from the presence of fields which are not conjugated, such as that due to finite-size effects of the PCM, and the evanescent fields discussed in Ref. 10, do not affect the time dependence in Eq. (3), although r becomes spatially dependent. Since our interest is in quasiharmonic fields, these two conditions can be satisfied to sufficient accuracy. Another situation to which Eq. (3) applies is the case of a radiator inside a spherical PCM cavity.

The final assumption determines the self-field[12]

$$\mathbf{E}_s(t) = -\alpha_r \dot{\boldsymbol{\mu}}(t) \tag{5}$$

in terms of the radiative decay rate α_r.

From combination of Eqs. (2), (3), and (5), the differential equation for the dipole is obtained without the inclusion of the term for the exciting source:

$$\ddot{\boldsymbol{\mu}}(t) + (\alpha_r + \alpha_{\mathrm{nr}})\dot{\boldsymbol{\mu}}(t) + \omega_0^2 \boldsymbol{\mu}(t) = -r\alpha_r \dot{\boldsymbol{\mu}}^*(t-\tau)\exp[-i\omega_m(2t-\tau)]. \tag{6}$$

This equation may be solved by our assuming a solution of the form

$$\mu(t) = [A\exp(-i\nu t) + B^*\exp(i\nu^* t)]\exp(-i\omega_m t), \tag{7}$$

where $\nu = \Delta\omega - i\Gamma$. Assuming that $\delta = \omega_0 - \omega_m \ll \omega_m$, and eliminating two solutions that correspond to $\nu \simeq \pm\omega_m$, we obtain a characteristic equation

$$(1 - \alpha_r^2 |r|^2 e^{2i\nu\tau}/4\omega_0^2)\nu^2 + i(\alpha_r + \alpha_{\mathrm{nr}})\nu + \tfrac{1}{4}\alpha_r^2 |r|^2 e^{2i\nu\tau} - \delta^2 - \tfrac{1}{4}\alpha^2 = 0. \tag{8}$$

The two solutions of Eq. (8) are given by

$$\nu_\pm = -\tfrac{1}{2}(\alpha_r + \alpha_{\mathrm{nr}})i \pm (\delta^2 - \tfrac{1}{4}\alpha_r^2 |r|^2 e^{2i\nu_\pm \tau})^{1/2}. \tag{9}$$

I now restrict the discussion to the case $\nu_\pm \tau \to 0$.

When $|r|^2 < |r_{C1}|^2$, where

$$|r_{C1}|^2 = 4\delta^2/\alpha_r^2, \tag{10}$$

we find two solutions for $\Delta\omega$ and just one for Γ. Above this critical reflectivity, the real part $\Delta\omega$ vanishes, and two solutions for the rate constant appear. Thus, at sufficiently high pump power, the radiation frequency locks to the pump frequency even when $\delta \neq 0$. This was not predicted in the earlier work.[10,11] Above a second

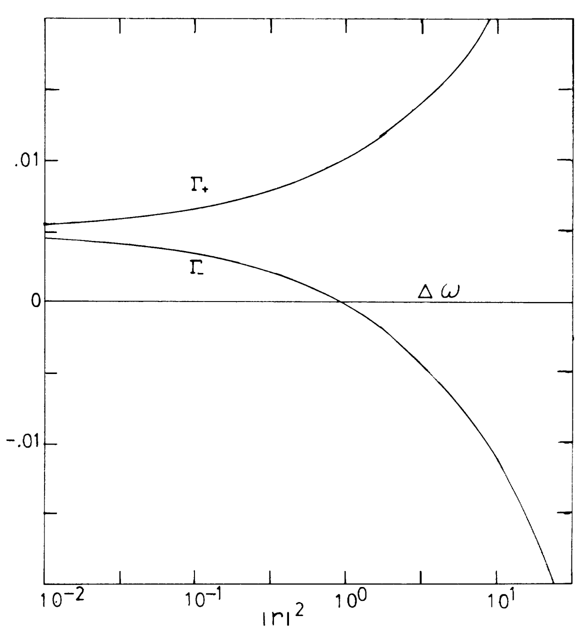

FIG. 1. Frequency $\Delta\omega$ and decay constants Γ_+ and Γ_- as functions of mirror reflectivity $|r|^2$ for the classical linear dipole interacting resonantly ($\omega_0/\omega_m = 1$) with the PCM. $\alpha_{\mathrm{nr}} = 0$, $\alpha_r = 0.01\omega_m$.

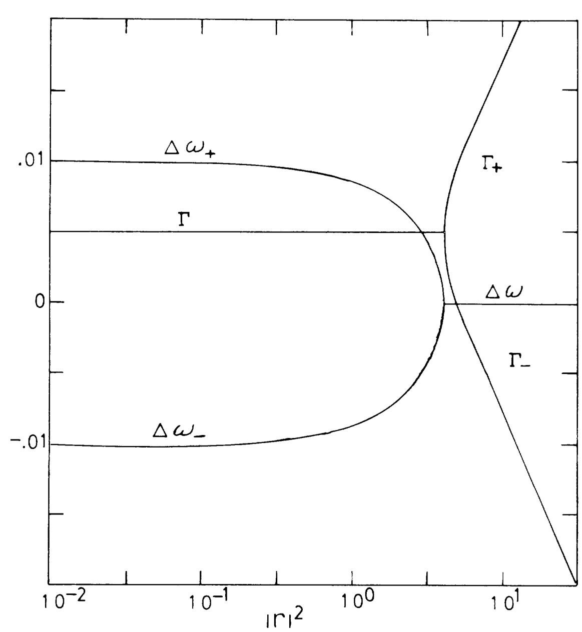

FIG. 2. Side-band frequencies $\Delta\omega_- = -\Delta\omega_+$ and decay constants Γ_+ and Γ_- as functions of mirror reflectivity $|r|^2$ for the classical linear dipole interacting off resonance ($\omega_0/\omega_m = 1.01$) with the PCM. $\alpha_r = 0.01\omega_m$, $\alpha_{\mathrm{nr}} = 0$.

critical value,

$$|r_{C2}|^2 = [(\alpha_r + \alpha_{nr})/\alpha_r]^2 + |r_{C1}|^2, \tag{11}$$

the PCM transfers more energy to the radiator than this radiates, resulting in exponential growth. Note that $|r_{C2}|^2 = 1$ in the case $\alpha_{nr} = \delta = 0$, as one would expect. Near-unit values of r_{C2} are possible only when the nonradiative loss is small and $\delta < \alpha_r$.

Evaluation of $B_\pm/A_\pm$ showed that this ratio is near unity when $\Delta\omega = 0$, but exceeds 1 for $\Delta\omega < 0$ and is less than 1 when $\Delta\omega > 0$.

These results are illustrated in Figs. 1 and 2. Figure 1 shows the case $\delta = 0$. The onset of instability at $|r|_{C2}$ is noted in this figure. In Fig. 2, in which $\delta = 0.01\omega_m$, both critical points are indicated.

Note that the solutions are functions of $|r|^2$ only and do not depend upon the phase of r. This result should be reasonable on grounds that the Poynting vector of the phase-conjugate field, which, when integrated over the surface of a sphere surrounding the radiator, yields the total energy radiated back to the dipole, is proportional to $|r|^2$. Moreover, a contrary result, as obtained in Refs. 10 and 11, would imply that energy could be transmitted from the PCM to the oscillator merely by a change in the phase of the pump waves. This is in contradiction to the fact that the change in phase of a wave is accomplished without expenditure of energy.

Two-level system.— Let us consider briefly a two-level atom radiating near a PCM. Let the atom be located at the origin of the coordinates, interacting with the PCM via the quantized radiation field. The interaction Hamiltonian consists of the usual dipole term $H_{I2} = -\boldsymbol{\mu}\cdot\mathbf{E}$ of the atom-field interaction, plus the field-PCM interaction

$$H_{I2} = \tfrac{1}{2}\sum_{\mathbf{k}\sigma}\sum_{\mathbf{k}'\sigma'} r(\mathbf{k}\sigma,\mathbf{k}'\sigma')a^\dagger_{\mathbf{k}'\sigma}a^\dagger_{\mathbf{k}'\sigma'}\exp(-2i\omega_m t) + \text{H.c.},$$

where $k_x + k_x' = k_y + k_y' = 0$, and $a^\dagger_{k\sigma}$ is the creation operator of a photon of wave vector k and polarization σ. The coefficients $r(\ldots)$ are functions of the PCM. The state of the system is described by a set of complex amplitudes, C_1 for the upper level of the radiating atom and $C_2(\mathbf{k},\sigma)$ for the lower level plus the emitted photon. In the spirit of a semiclassical approximation, Eq. (1) is assumed to apply, so that the double sum in H_{I2} becomes a single sum. In a continuance of reasoning along classical lines, the two photons in the incident mode are ignored after interaction with the PCM. The following equations are then obtained:

$$i\hbar\, dC_1/dt = \sum_{\mathbf{k},\sigma}\mu_{12}f_k\exp[-i(\omega_k-\omega_0)t]C_2(\mathbf{k},\sigma) + \sum_{\mathbf{k},\sigma}\mu_{12}f_k\exp[-i(\omega_{k'}-\omega_0)t]\hat{R}_{\mathbf{k}',\sigma'}C_2(\mathbf{k},\sigma), \tag{12a}$$

$$i\hbar\, dC_2(\mathbf{k},\sigma)/dt = \mu_{12}f_k\exp[i(\omega_k-\omega_0)t]C_1 + \mu_{12}f_k\hat{R}_{\mathbf{k},\sigma}\{\exp[i(\omega_{k'}-\omega_0)t]C_1\}, \tag{12b}$$

where μ_{12} is the matrix element of μ, and $f_k = (\hbar\omega_k/2\epsilon_0 V)^{1/2}$.

The second sum on the right-hand side of Eq. (12a) represents the action of the PCM. Equation (12a) was obtained by splitting off (from the first term on the right) those states directed away from the PCM. Each $C_2(\mathbf{k},\sigma)$ in this group is related to $C_2(k',\sigma')$ of the corresponding incident photon according to

$$C_2(k',\sigma') = \frac{1}{i\hbar}r_{k',\sigma'}\int\exp[-i(2\omega_m-\omega_k-\omega_{k'})t]C_2(k\sigma)\,dt \equiv \hat{R}_{k'\sigma'}C_2(k\sigma).$$

Likewise, in Eq. (12b), a second term from H_{I2} is obtained in the form

$$r_k\exp[-i(2\omega_m-\omega_k-\omega_{k'})t]C_2(\mathbf{k}'\sigma') = r_kC_2(k',\sigma')$$
$$= \frac{1}{i\hbar}r_k\mu_{12}f_k\int\exp[i(\omega_k-\omega_0)t]C_1\,dt = \mu_{12}f_k\hat{R}_{k\sigma}\{\exp[i(\omega_k-\omega_0)t]C_1\}.$$

On inspection of Eqs. (12), we note that as far as the interaction with the PCM is concerned, a similarity to Eq. (6) exists. In both systems, terms are present which describe the interaction of the radiator with a reciprocally related transformed radiating "image" system, by the intervention of a PCM-reflectivity operator. In the classical case, the image of the radiating isotropic dipole is conjugated according to the same rule as the classical radiation field. On the other hand, the image wave functions of Eqs. (12) are not time reversed.

Equations (12) are different to solve, except in the degenerate case, $\omega_0 = \omega_m$. Then we can try the *Ansatz*

$$C_1(t) = C_1(0)e^{-i\Delta\omega t}e^{-\Gamma t}. \tag{13}$$

In the general case, more than one or even two frequencies and decay constants are needed in the solution. In view of the nonlinearity of the two-level system, this is not surprising. One would expect a large number of such harmonics to arise when (a) the bandwidth of the PCM is more than a few multiples of δ; (b) the combined radiative and nonradiative lifetime of the transition equals at least a number of photon round trips from atom to PCM; and (c) $C(0) \cong 1$, or the PCM reflectivity exceeds unity.

Thus, in the resonant case, we obtain

$$\Gamma = \Gamma\sum_{\mathbf{k},\sigma}\frac{\mu_{12}^2|f_k|^2(1-|R_{\mathbf{k}\sigma}|^2)}{(\omega_k-\omega_m-\Delta\omega)^2+\Gamma^2} \tag{14}$$

and

$$\Delta\omega = \sum_{\mathbf{k},\sigma} \frac{\mu_{12}^2 |f_k|^2 (1 - |R_{\mathbf{k}\sigma}|^2)(\omega_k - \omega_m - \Delta\omega)}{(\omega_k - \omega_m - \Delta\omega)^2 + \Gamma^2} \tag{15}$$

for the damping constant and the frequency shift, respectively. I have made use of the conjugation symmetry relation of the reflectivity:

$$R_{k'\sigma'} = -R_{k\sigma}^*, \tag{16}$$

and since the linewidth of the atomic transition is assumed to be small, I approximated

$$f_{k'} = f_k. \tag{17}$$

Equations (14) and (15) are easily solved in the situation where the refectivity $R_{k\sigma}$ can be taken to be the same constant (r) for all modes (corresponding to a phase-conjugate cavity geometry). The solution for Γ from Eq. (14) is

$$\Gamma = \begin{cases} \Gamma_0(1-|r|^2), & \text{for } |r| < 1, \\ 0, & \text{for } |r| \geq 1, \end{cases} \tag{18}$$

and for $\Delta\omega$, from Eq. (15),

$$\Delta\omega = 0, \quad \text{for all } |r|. \tag{19}$$

All results appear to agree well with physical considerations.

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