Unitary Symmetry and the Stability of Σ Hypernuclei

Carl B. Dover
Brookhaven National Laboratory, Upton, New York 11973

and

Herman Feshbach
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 10 July 1987)

In the limit of unbroken SU(3) symmetry, we exhibit a selection rule which forbids the decay of certain hypernuclear states involving a coherent admixture of Λ and Σ hyperons. This may provide an explanation of the narrow widths of some hypernuclear excitations observed in the Σ continuum.

PACS numbers: 21.80.+a

In several (K^-,π^+) experiments[1] using nuclear targets, relatively long-lived Σ-hypernuclear states were seen, whose decay width $\Gamma \lesssim 5$ MeV is less than the typical values $\Gamma \approx 10-20$ MeV obtained from optical-model estimates.[2] Although the existence of such narrow structures remains controversial, particularly for ${}^{12}_{\Sigma}\mathrm{C}$ and ${}^{16}_{\Sigma}\mathrm{O}$, one is invited to speculate on the nature of dynamical mechanisms which could lead to such a width suppression. Several possibilities have been considered, namely, spin selectivity[2] in the $\Sigma N \rightarrow \Lambda N$ conversion process, as well as Pauli blocking and dispersive/binding effects in the nuclear medium.[3] Here we explore another possibility, namely that the observed narrow Σ widths point to the existence of an approximate selection rule based on broken SU(3) symmetry.

The basic idea is the following: We assume that the hypernuclear state in question is an eigenfunction of the quadratic Casimir operator C of SU(3), namely

$$C=\sum_{i<j} C_{ij},\quad C_{ij}=\sum_{a=1}^{8} F_a(i)F_a(j)=\mathbf{T}_i\cdot\mathbf{T}_j+\mathbf{U}_i\cdot\mathbf{U}_j+\mathbf{V}_i\cdot\mathbf{V}_j-\tfrac{1}{3}(T_{i3}T_{j3}+U_{i3}U_{j3}+V_{i3}V_{j3}), \tag{1}$$

where the SU(3) generators F_a and the T-, U-, and V-spin operators are defined as in Gasiorowicz.[4] Further, we hypothesize that the transition operator t_{12} for baryon-baryon scattering and reactions is of the form

$$t_{12}=a+bC_{12} \tag{2}$$

where, for the moment, a and b are spin- and flavor-independent amplitudes. A quantitative treatment of the two-body problem requires the introduction of a term cG^3 in Eq. (2) proportional to the third-order Casimir operator[5] G^3, but we omit this here to simplify the discussion. In the two-body case, the $\Sigma N \rightarrow \Lambda N$ conversion process is mediated by the C_{12} terms, while the a term enters only for elastic scattering. We now observe that for states of differing C_{12} eigenvalues $\kappa \neq \kappa'$,

$$\langle \psi(\kappa) | \sum_{i<j} t_{ij} | \psi(\kappa') \rangle = 0. \tag{3}$$

Thus, in the limit of unbroken SU(3) symmetry, one obtains a *selection rule* which forbids transitions between eigenstates $\psi(\kappa)$ of the hypernucleus. Note that $\psi(\kappa)$ involves a *coherent mixture* of Σ and Λ, coupled to a nuclear core, unlike the weak-coupling limit, where eigenstates consist of pure Σ or Λ configurations. In this paper we assume that such a selection rule remains approximately valid in the realistic case where SU(3) symmetry is broken via explicit hypercharge dependence of $\{a,b,c\}$. The manifestations of SU(3) symmetry breaking include the sizable mass splittings of strange and nonstrange mesons and baryons, and the existence of an np, but not a hyperon-nucleon, bound state. We anticipate an analogy with the case of isospin T in nuclear physics. There, even in the presence of a strong Coulomb potential [analogous to a mass difference in SU(3)], T is essentially a good quantum number because of the action of the $\mathbf{T}_i\cdot\mathbf{T}_j$ symmetry potential [analogous to C_{12} in SU(3)]. Symmetry breaking occurs largely in the *diagonal* elements of the mass matrix; the analogy in SU(3) is provided by the Gell-Mann–Okubo mass formula.[6]

We find that the form (2), if a term cG^3 is included,[5] is sufficiently flexible to reproduce the observed low-energy cross sections[7] in the hypercharge $Y_1+Y_2=1$ sector, namely those for $\Sigma^+p \rightarrow \Sigma^+p$, $\Lambda p \rightarrow \Lambda p$, and $\Sigma^-p \rightarrow \Sigma^-p$, $\Sigma^0 n, \Lambda n$. These are spin-averaged quantities, and reveal little about the spin dependence of $\{a,b,c\}$. In a one-gluon-plus-quark–exchange approximation, appropriate only for the very short-range part of the baryon-baryon interaction, we obtain the form of Eq. (2), with $c=0$ and a mild spin dependence for a. This approximation does not account for the data. Realistic one-boson-exchange potentials,[8] on the other hand, display a strong spin-isospin dependence for $\Sigma N \to \Sigma N$, ΛN. For $\Sigma N \to \Lambda N$, for instance, the spin-triplet amplitudes dominate. We defer the discussion of spin dependence to a later article.[9]

Denoting by $\{B_1 B_2\}$ and $[B_1 B_2]$ the symmetric (S) ${}^1S_0$ and antisymmetric (A) ${}^3S_1$ baryon-baryon couplings, respectively, we find

$$C_{12}(\{pp\},\{pn\},[pn],\{\Sigma^+ p\},[\Sigma^+ p]) = [\{pp\},\{pn\},0,\{\Sigma^+ p\},0], \quad (4)$$

whereas for $Y_1 + Y_2 = 1$, charge 1, we find the SU(3) eigenfunctions $\psi(S,T,\kappa)$ of spin S, isospin T, and C_{12} to be

$$\begin{aligned}
\psi(1,\tfrac{3}{2},0) &= [\Sigma N]_{T=3/2},\\
\psi(1,\tfrac{1}{2},0) &= -\tfrac{1}{2}\sqrt{2}[\Lambda p] + \tfrac{1}{2}\sqrt{2}[\Sigma N]_{T=1/2},\\
\psi(1,\tfrac{1}{2},-\tfrac{3}{2}) &= \tfrac{1}{2}\sqrt{2}[\Lambda p] + \tfrac{1}{2}\sqrt{2}[\Sigma N]_{T=1/2},\\
\psi(0,\tfrac{3}{2},1) &= \{\Sigma N\}_{T=3/2},\\
\psi(0,\tfrac{1}{2},1) &= \tfrac{3}{10}\sqrt{10}\{\Lambda p\} + \tfrac{1}{10}\sqrt{10}\{\Sigma N\}_{T=1/2},\\
\psi(0,\tfrac{1}{2},-\tfrac{3}{2}) &= \tfrac{1}{10}\sqrt{10}\{\Lambda p\} - \tfrac{3}{10}\sqrt{10}\{\Sigma N\}_{T=1/2},
\end{aligned} \quad (5)$$

where the $T=\frac{1}{2},\frac{3}{2}$ combinations are given by $(\frac{2}{3})^{1/2}\Sigma^+ n - (\frac{1}{3})^{1/2}\Sigma^0 p$ and $(\frac{1}{3})^{1/2}\Sigma^+ n + (\frac{2}{3})^{1/2}\Sigma^0 p$, respectively.[10] The ψ's can be constructed simply with the SU(3) Clebsch-Gordan coefficients derived by de Swart.[5] Note that $\{\Lambda p\}$ is the dominant component in $\psi(0,\frac{1}{2},1)$, and $\{\Sigma N\}$ is dominant in $\psi(0,\frac{1}{2},-\frac{3}{2})$, whereas $[\Lambda p]$ and $[\Sigma N]$ are equally weighted in $S=1$, $T=\frac{1}{2}$ configurations.

In a straightforward manner, one may extend the analysis to the three-body YNN system. For $S_3=\frac{1}{2}$, $T_3=0$ (charge $+1$), we must consider linear combinations of the eight basis states $\psi_k = \Lambda_\uparrow\{pn\}$, $\Lambda_\uparrow[pn]_0$, $\Lambda_\downarrow[pn]_1$, $\Sigma_\uparrow^0\{pn\}$, $\Sigma_\uparrow^0[pn]_0$, $\Sigma_\uparrow^0[pn]_1$, $\Sigma_\uparrow^-\{pp\}$, and $\Sigma_\uparrow^+\{nn\}$ [in fact, symmetrized combinations

$$(\Lambda^\uparrow\{pn\} - \{pn\}^{(13)}\Lambda_\uparrow^{(2)} + \{pn\}\Lambda_\uparrow)/\sqrt{3},$$

where the superscripts label the ordering where the Λ appears as particle 2 and the arrows indicate the z component of spin]. Here,

$$\{pn\} = (p_\uparrow n_\downarrow - p_\downarrow n_\uparrow + n_\uparrow p_\downarrow - n_\downarrow p_\uparrow)/2,$$

$$[pn]_0 = (p_\uparrow n_\downarrow + p_\downarrow n_\uparrow - n_\uparrow p_\downarrow - n_\downarrow p_\uparrow)/2,$$

and

$$[pn]_1 = (p_\uparrow n_\uparrow - n_\uparrow p_\uparrow)/\sqrt{2}.$$

The eigenfunctions $\psi^{(3)}(S,T,\kappa)$ for the three-body case are given by

$$\psi^{(3)}(S,T,\kappa) = \sum_{k=1}^{8} \alpha_k(S,T,\kappa)\psi_k, \quad (6)$$

where $\mathbf{S} = \frac{1}{2}\sum_{i=1}^{3}\boldsymbol{\sigma}_i$, $\mathbf{T} = \frac{1}{2}\sum_{i=1}^{3}\boldsymbol{\tau}_i$, and $\kappa = \langle \sum_{i<j} C_{ij}\rangle$. The coefficients $\alpha_k(S,T,\kappa)$ are tabulated in Table I.

Now consider the (K^-,π) reaction on a nuclear target at momentum transfer $q=0$. In this case, the baryon part of the operator for the nuclear transition is $\sum_i U_i^-$ for the (K^-,π^-) reaction, $\sum_i V_i^-$ for (K^-,π^0), and $\sum_i V_i^- T_i^-$ for (K^-,π^+). Here, (U_i^-, V_i^-, T_i^-) are the usual U-spin, V-spin, and isospin lowering operators.[4] Since

$$\left[\sum_i U_i^-, \sum_{i<j} C_{ij}\right] = \left[\sum_i V_i^-, \sum_{i<j} C_{ij}\right] = 0, \quad (7)$$

the corresponding (K^-,π^-) or (K^-,π^0) processes do not change the eigenvalue $\kappa = \langle \sum_{i<j} C_{ij}\rangle$ of the target. Note that for $q \neq 0$, the transition operators U_i^-, etc., are weighted with relative phases $\exp(i\mathbf{q}\cdot\mathbf{r}_i)$, and κ is no longer exactly conserved. However, for small q, we expect hypernuclear production to be dominated by SU(3)-conserving transitions. From Eq. (4), we see that $\sum_{i<j} C_{ij}$ is proportional to the number of $T=1$ pairs in the target nucleus. As an example, ^{3}He and ^{3}H have $\kappa = \frac{3}{2}$, so the reaction ${}^3\mathrm{H}(K^-,\pi^-){}^3_Y\mathrm{H}$ would produce only the components $\psi^{(3)}(S,T,\kappa)$ of Eq. (6) with $S=\frac{1}{2}$ (spin-non-flip) and $\kappa=\frac{3}{2}$. For the (K^-,π^+) reaction, on the other hand, there is no selection rule which con-

TABLE I. Coefficients $\alpha_k(S,T,\kappa)$ for YNN eigenstates $\psi^{(3)}(S,T,\kappa)$.

S	T	κ	$\alpha_k(S,T,\kappa)$, $k=1-8$
$\frac{1}{2}$	0	$\frac{3}{2}$	$0, \frac{1}{2}, -1/\sqrt{2}, 1/2\sqrt{3}, 0, 0, -1/2\sqrt{3}, -1/2\sqrt{3}$
$\frac{1}{2}$	0	$-\frac{3}{2}$	$0, 1/2\sqrt{3}, -1/\sqrt{6}, -\frac{1}{2}, 0, 0, \frac{1}{2}, \frac{1}{2}$
$\frac{1}{2}$	1	$\frac{3}{2}$	$\sqrt{3}/2\sqrt{2}, 0, 0, 0, 1/2\sqrt{2}, -\frac{1}{2}, 1/2\sqrt{2}\ -1/2\sqrt{2}$
$\frac{1}{2}$	1	$-\frac{1}{2}$	$1/\sqrt{2}, 0, 0, 0, -1/\sqrt{6}, 1/\sqrt{3}, 0, 0$
$\frac{1}{2}$	1	$-\frac{1}{2}$	$1/2\sqrt{2}, 0, 0, 0, 1/2\sqrt{6}, -1/2\sqrt{3}, -\sqrt{3}/2\sqrt{2}, \sqrt{3}/2\sqrt{2}$
$\frac{1}{2}$	2	$\frac{3}{2}$	$0, 0, 0, \sqrt{2}/\sqrt{3}, 0, 0, 1/\sqrt{6}, 1/\sqrt{6}$
$\frac{3}{2}$	0	$-\frac{3}{2}$	$0, \sqrt{2}/\sqrt{3}, 1/\sqrt{3}, 0, 0, 0, 0, 0$
$\frac{3}{2}$	1	$-\frac{1}{2}$	$0, 0, 0, 0, \sqrt{2}/\sqrt{3}, 1/\sqrt{3}, 0, 0$

serves κ, as for (K^-,π^-) or (K^-,π^0). However, for the case of two valence protons, $\sum_i V_i^- T_i^-$ generates $\{\Sigma^- p\}$, while for the neutron-proton system, C_{12} is conserved.

Under these circumstances, the finite width of the Σ hypernuclear states is a consequence of symmetry breaking. It can be anticipated that this width will be smaller than that calculated from the experimentally observed $\Sigma^- p$ transition cross section, since a substantial part of that cross section will be generated by symmetry-conserving interactions given by Eq. (2). In principle, one should be able to determine the strength and nature of the symmetry breaking by demanding a simultaneous fit to NN and YN cross sections. The detailed analysis, and its impact on the width of a many-body system, will be presented elsewhere.[9] Note that a substantial portion of the symmetry breaking may be accommodated by hypercharge dependence of the parameter a, and this does not contribute to the decay widths. Of course, more YN data would be extremely helpful. The symmetry breaking that one can deduce from the two-body data must, in the long run, be consistent with what one knows about the symmetry breaking in the baryon-baryon interaction, such as that induced by the differing masses of the baryons and bosons, the exchange of the latter giving rise to the interaction.[8] We leave the issue of the origin of symmetry breaking for later consideration.[9]

Let us now consider the introduction of symmetry breaking through the parameter b. If different values b^N and b^Y are taken for hypercharge $Y=2$ (NN) and $Y=1$ (YN), respectively, then the effective transition operator for the many-body system is of the form

$$\sum_{i<j} t_{ij} e^{i\mathbf{k}\cdot\mathbf{r}_j} = (\langle b\rangle - 3\Delta b/2)\sum_{i<j} C_{ij} e^{i\mathbf{k}\cdot\mathbf{r}_j} + \Delta b \sum_{i<j} (Y_i+Y_j) C_{ij} e^{i\mathbf{k}\cdot\mathbf{r}_j}, \tag{8}$$

where $\langle b\rangle = (b^N+b^Y)/2$, $\Delta b = b^N - b^Y$, and $\mathbf{k}$ is the momentum transfer imparted to the nucleon at $\mathbf{r}_j$ in the $\Sigma N \to \Lambda N$ process. Note that $\mathbf{k}=0$ in the SU(3) limit, but $|\mathbf{k}| \approx 280$ MeV/c when free-space masses are used for the Λ and Σ.

In the weak-coupling limit, the eigenstates of the system consist of pure Λ+core and Σ+core configurations (with, of course, various Y single-particle states and excited-core states admixed). The width of Σ states in this case has been considered by Auerbach.[11] For $\sum_{i<j} C_{ij}$ eigenstates, the decay matrix element obtained from Eq (8) will be quite different than for the weak-coupling limit. This approach, which corresponds to a strong-coupling limit, is appropriate for the description of the short-range properties of the YN system, whereas the weak-coupling picture[11] applies to the long-range YN interaction mediated by one-pion exchange. Quantitative calculations are required before we can claim that coherent Σ-Λ admixtures (i.e., the tendency of the system to form $\sum_{i<j} C_{ij}$ eigenstates) exert a strong influence on Σ-hypernuclear decay widths. These calculations are in progress.[9] Here, we have focused on the possibility of width suppression for two-body and three-body systems of strangeness -1, but our arguments also apply to such clusters circulating around an inert nuclear core, since the surface localization of the hyperon wave function can lead to a suppression of $\Sigma \to \Lambda$ conversion on the core.

For the strangeness -2 two-body case, the SU(3)-flavor-singlet state with $S=0$, $T=0$, $\kappa=-3$, which is a linear combination of $\Lambda\Lambda$, ΞN, and $\Sigma\Sigma$ components, corresponds to the H dibaryon proposed by Jaffe.[12] Note that the production of the H via the reaction $K^- + \{pp\} \to K^+ + H$ involves the change of C_{12} by four units and may be suppressed. One can also construct strangeness -2 three-body eigenstates of $\sum_{i<j} C_{ij}$. There is the intriguing possibility that one or more of these may exist as bound states stable with respect to decay into $\Lambda\Lambda N$. Similar possibilities exist for the four-body system.

The energetics of hypernuclear ground states suggest a weak-coupling limit in which the mass lies close to $m_\Lambda + (A-1)m_N$. If small Σ admixtures are present, they could show up as modifications of magnetic moments or weak-decay branching ratios. Our emphasis is on excited states, where Σ-Λ mixing would be revealed in decay widths. The approach taken here is distinct from the discussion[13] of strangeness analog resonances based on the Sakata model, where one considers coherent admixtures of particle-hole configurations, but neglects Σ-Λ mixing. In Ref. 2, it is shown how the conversion width of a Σ can be suppressed for some states because of the spin dependence of the $\Sigma N \to \Lambda N$ amplitude.

This work has been authored under Contract No. DE-AC02-76CH00016 with the U.S. Department of Energy.

[1]R. Bertini *et al.*, Phys. Lett. **90B**, 375 (1980), and **158B**, 19 (1985); H. Piekarz *et al.*, Phys. Lett. **110B**, 428 (1982).

[2]A. Gal and C. B. Dover, Phys. Rev. Lett. **44**, 379 (1980).

[3]J. Dabrowski and J. Rozynek, Phys. Rev. C **23**, 1706 (1981); J. A. Johnstone and A. W. Thomas, Nucl. Phys. **A392**, 409 (1983); R. Brockmann and E. Oset, Phys. Lett. **118B**, 33 (1982); see also the review by A. Gal, in *Proceedings of the 1986 Institute for Nuclear Study International Symposium on Hypernuclear Physics*, edited by H. Bando, O. Hasimoto, and K. Ogawa (Institute for Nuclear Study, University of Tokyo, Tokyo, Japan, 1986), p. 31.

[4]S. Gasiorowicz, *Elementary Particle Physics* (Wiley, New York, 1966), pp. 261–274.

[5]J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963), and in

Proceedings of the CERN School of Physics, CERN Report No. 66-29, 1966 (unpublished), Vol. II, pp. 1–84.

[6]S. Okubo, Prog. Theor. Phys. **27**, 949 (1962); see also pp. 287–288 of Ref. 4.

[7]G. Alexander *et al.*, Phys. Rev. **173**, 1452 (1968); B. Sechi-Zorn *et al.*, Phys. Rev. **175**, 1735 (1968); R. Engelman *et al.*, Phys. Lett. **21**, 587 (1966); F. Eisele *et al.*, Phys. Lett. **37B**, 204 (1971).

[8]M. M. Nagels, T. A. Rijken, and J. J. deSwart, Phys. Rev. D **15**, 2547 (1977), and **20**, 1633 (1979).

[9]C. B. Dover and H. Feshbach, to be published.

[10]We use phase conventions for the Σ field consistent with those of S. Meshkov, C. A. Levinson, and H. J. Lipkin, Phys. Rev. Lett. **10**, 361 (1963).

[11]N. Auerbach, Phys. Rev. C **35**, 1798 (1987).

[12]R. L. Jaffe, Phys. Rev. Lett. **38**, 195 (1977), and **38**, 617(E) (1977).

[13]A. K. Kerman and H. J. Lipkin, Ann. Phys. (N.Y.) **66**, 738 (1971); H. Feshbach and A. K. Kerman, in *Preludes in Theoretical Physics,* edited by A. de Shalit, H. Feshbach, and L. van Hove (North-Holland, Amsterdam, 1966), p. 260; H. J. Lipkin, Phys. Rev. Lett. **14**, 18 (1965).