Minijet Production in High-Energy Nucleus-Nucleus Collisions

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Uranium-uranium collisions at the energy of the Brookhaven Relativistic Heavy-Ion Collider (100 + 100 GeV per nucleon) are predicted to produce an average of nearly 100 jets with $p_T > 3$ GeV. These jets will on average carry off 70 GeV of transverse energy E_T per unit rapidity. Central collisions produce more transverse energy than this; the E_T distribution extends up to about $(5A^{4/3}\ \mathrm{GeV}^4)/p_{T\min}^3$ per unit rapidity, which is 4 times the average. It is estimated that the minijets are likely to undergo further collisions and become thermalized.

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The UA1 Collaboration at the CERN $\bar{p}p$ collider has measured[1] the cross section for the production of jets at small values of p_T (minijets). The observed rate is in agreement with QCD and is quite large.

In this Letter, we consider the implications of this for collisions of two identical large nuclei at high energy. We find that UU collisions at the energy of the Brookhaven Relativistic Heavy-Ion Collider (100+100 GeV per nucleon) will on average produce nearly 100 jets with $p_T > 3$ GeV. We calculate the transverse-energy distribution associated with the multiple production of these minijets, and find that for central rapidities it extends out to about $(5A^{4/3}\ \mathrm{GeV}^4)/p_{T\min}^3$ per unit rapidity. Here $p_{T\min}$ is the minimum p_T chosen for the minijets; our analysis should certainly be valid down to $p_{T\min} = 3$ GeV, and even for 2 GeV the error should not be too large. Finally, we discuss the fate of the minijets; we estimate their mean free path and conclude that they are likely to undergo further collisions and so contribute to the formation of thermalized quark-gluon plasma.

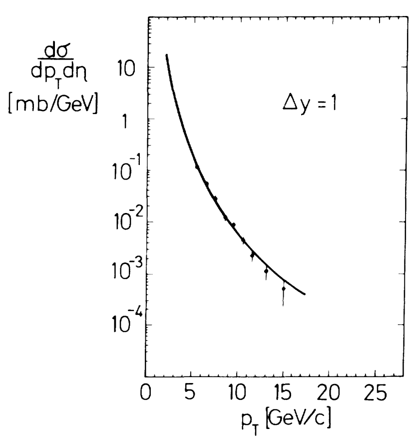

FIG. 1. The UA1 minijet data (Ref. 1) at $\sqrt{s} = 200$ GeV and the parametrization in Eq. (1).

Previous work[2] has concentrated on jets with rather large p_T, and investigated how they are broadened by rescattering in the nuclear environment. Minijets, on the other hand, will be reprocessed by the system and not emerge from it. The transverse-energy spectrum increases rapidly with both A and the incident energy, and so both of these should be as large as possible in order to generate a plasma.

Figure 1 shows the UA1 measurements of jet production in $\bar{p}p$ collisions at $\sqrt{s} = 200$ GeV. The curve corresponds to the simple parametrization

$$\frac{d\sigma_{\bar{p}p}}{dp_T} = \frac{C}{p_T^n}, \quad C = 600\ \mathrm{mb\ GeV}^4, \quad n = 5. \tag{1}$$

Here we have integrated over the unit (pseudo)rapidity interval $|\eta| < \frac{1}{2}$. According to standard theory,[3] this inclusive cross section corresponds to Fig. 2(a). If we

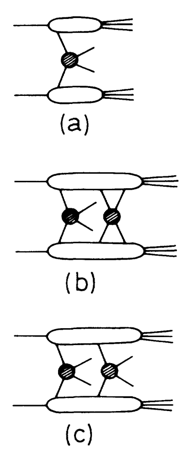

FIG. 2. (a) The standard diagram for the QCD calculation of the inclusive jet production cross section $d\sigma/dp_T$. (b) Double parton-parton collision with two hard scatterings. (c) The standard diagram of (a), modified by a final- (or initial-) state interaction.

neglect the small discrepancies known as the EMC effect and the shadowing at small x, in $A+A$ collisions the two parton distributions in Fig. 2(a) are a factor A greater. Hence $d\sigma_{AA}=A^2 d\sigma_{pp}$.

To calculate the rate per collision, we must divide by the total cross section. This is not known, but a reasonable guess is the geometrical value $4\pi R_A^2 \approx 165 A^{2/3}$ mb. Then the average number of minijets produced in $|\eta| < \frac{1}{2}$, and the average transverse energy they carry off, are

$$\bar{N}=\frac{A^2}{4\pi R_A^2}\int_{p_{T\min}} dp_T \frac{d\sigma_{pp}}{dp_T}=\frac{A^2 C}{4\pi R_A^2 (n-1) p_{T\min}^{n-1}},$$
$$\bar{E}_T=\frac{A^2}{4\pi R_A^2}\int_{p_{T\min}} dp_T\, p_T \frac{d\sigma_{pp}}{dp_T}=\frac{n-1}{n-2} p_{T\min}\bar{N}. \tag{2}$$

We insert the values of n and C from Eq. (1). (At higher energies the effective value of n will decrease and C will increase.) This gives $\bar{N}=15A^{4/3}/16p_{T\min}^4$. If we multiply by about 5 so as to integrate over η, we find for $A=238$ and $p_{T\min}=3$ GeV an average of nearly 100 minijets, carrying an average total transverse energy of about 350 GeV. These are average values for all collisions and there will be substantial fluctuations about them. We shall see that central collisions produce about 4 times the average transverse energy.

The standard diagram of Fig. 2(a) contains only one hard parton-parton scattering, and so how can $\bar{N}$ be much greater than one? The answer to this lies in a simple extension to an old theorem,[4] which says that Fig. 2(a) correctly yields the inclusive cross section $d\sigma/dp_T$, but does not, by itself, correctly describe the final state. In fact, in the present case the final state is determined mainly by multiple parton-parton collisions.

Consider, in particular, a double collision, Fig. 2(b). This yields two pairs of jets. The theorem says that, while this process certainly occurs, its contribution to $d\sigma/dp_T$ is just canceled by the interference between Fig. 2(a) and Fig. 2(c). Figure 2(c) yields the same final state as Fig. 2(a), but it includes the second hard parton-parton interaction as a virtual process. Likewise, while diagrams with more than two hard scatterings will also contribute to the final state, their contribution to $d\sigma/dp_T$ will be canceled by interference terms in which some of the hard scatterings occur as virtual processes.

In order to calculate the E_T distribution $d\sigma/dE_T$ we need to know the cross section σ_n for n hard parton-parton collisions. To evaluate this, we introduce the impact parameter $\mathbf{b}$ (see also Ametller and Treleani, and Jackson and Bøggild[5]). Consider a collision for which the impact parameter is in the neighborhood d^2b of $\mathbf{b}$. Simple geometry tells us that the single-jet inclusive cross section is

$$d\sigma_{AA}=d\sigma_{pp} T_{AA}(\mathbf{b}) d^2b. \tag{3}$$

Here T_{AA} is the standard nuclear overlap function. It is defined in terms of the nuclear density $n_A(r)$ by

$$T_{AA}(\mathbf{b})=\int d^2s\, T_A(\mathbf{s}) T_A(\mathbf{b}-\mathbf{s}),$$
$$T_A(\mathbf{s})=\int dz\, n_A((\mathbf{s}^2+z^2)^{1/2}), \tag{4}$$

and its integral over $\mathbf{b}$ is A^2. Likewise, the multijet inclusive cross sections are given by the corresponding ones for pp collisions, multiplied by the appropriate power of $T_{AA}(\mathbf{b})$.

The inclusive N-jet cross section receives contributions from all numbers of n of hard scatterings $\geq \frac{1}{2}N$. We assume[3] that, when $E_T \ll \sqrt{s}$, the multiplet scatterings are independent of one another. (Note that the partons from a nucleus need not come from different nucleons, so that the joint n-parton distribution is proportional to A^n.) Then, to obtain the correct inclusive cross sections, we have

$$\sigma_n=\int d^2b \frac{[T_{AA}(\mathbf{b})]^n}{n!} \exp\left[-\tfrac{1}{2}\int dp_T \frac{d\sigma_{pp}}{dp_T} T_{AA}(\mathbf{b})\right] \int_{p_{T\min}} dp_{T1} \cdots dp_{Tn} \frac{1}{2}\frac{d\sigma_{pp}}{dp_{T1}} \cdots \frac{1}{2}\frac{d\sigma_{pp}}{dp_{Tn}}, \tag{5}$$

where the factors $\frac{1}{2}$ occur because the inclusive cross section for producing a pair of jets is $\frac{1}{2}\, d\sigma/dp_T$.

When one jet from a hard collision emerges in the rapidity interval $|\eta| < \frac{1}{2}$, the other may or may not also. In order to overcome this ambiguity, we calculate $d\sigma/dE_T$ for half the total range of azimuth, $\Delta\phi=\pi$. We obtain this from Eq. (5) by inserting in the integral the δ function $\delta(E_T-p_{T1}-\cdots-p_{Tn})$ and then summing over $n\geq 1$. The result is, for $E_T\neq 0$,

$$\frac{d\sigma(\Delta\phi=\pi)}{dE_T}=\frac{1}{2\pi}\int_{-\infty}^{\infty} d\tau \int d^2b \exp\left[i\tau E_T+\tfrac{1}{2}\int_{p_{T\min}} dp_T \frac{d\sigma_{pp}}{dp_T} T_{AA}(\mathbf{b})(e^{-i\tau p_T}-1)\right]. \tag{6}$$

For $E_T \gg p_{T\min}$ we may expand the last bracket in powers of τ up to the τ^2 term, and integrate over τ. Thus

$$\frac{d\sigma(\Delta\phi=\pi)}{dE_T}=\pi^{-1/2}\int d^2b [\Delta^2(\mathbf{b})]^{-1/2} \exp\{-[E_T-\tfrac{1}{2}\bar{E}_T(\mathbf{b})]^2/\Delta^2(\mathbf{b})\}, \tag{7}$$

where, with the parametrization of Eq. (1),

$$\bar{E}_T(\mathbf{b}) = \int_{p_{T\min}} dp_T\, p_T \frac{d\sigma_{pp}}{dp_T} T_{AA}(\mathbf{b}) = \frac{C T_{AA}(\mathbf{b})}{(n-2)p_{T\min}^{n-2}},$$
$$\Delta^2(\mathbf{b}) = \int_{p_{T\min}} dp_T\, p_T^2 \frac{d\sigma_{pp}}{dp_T} T_{AA}(\mathbf{b}) = \frac{n-2}{n-3} p_{T\min} \bar{E}_T(\mathbf{b}). \tag{8}$$

In Fig. 3 we plot the result Eq. (7), taking the Woods-Saxon parametrization

$$n_A(r) = \frac{3A}{4\pi R_A^3} \frac{1}{1+\pi^2 d^2/R_A^2} \frac{1}{1+\exp[(r-R_A)/d]}, \tag{9}$$
$$R_A = (1.19A^{1/3} - 1.61A^{-1/3}) \text{ fm}, \quad d = 0.54 \text{ fm}.$$

The figure also indicates the result of the restriction of the b integration to more central collisions. It is evident from this, and intuitively obvious, that the central collisions provide larger values of E_T than average. It turns out that for $b \approx 0$

$$T_{AA}(\mathbf{b}) \approx A^2/\pi R_A^2 = 4A^2/\sigma_{\text{tot}} \tag{10}$$

and in consequence the E_T distribution stretches out to about 4 times the average value.

Consider finally the fate of the minijets. In a gluon gas at temperature $T \approx 200$ MeV their mean free path is[6]

$$\lambda_{\text{free}} \approx \frac{p_{T\min}}{2T^2} \frac{1}{\ln(6p_{T\min}/T)} \tag{11}$$

[if we take $\alpha_s(1 \text{ GeV}^2) \approx 0.3$]. For $p_{T\min}$ in the few-gigaelectronvolt range, this makes λ_{free} a few femtometers, so that the minijets are likely to rescatter. They may thus be thermalized. So we have a new and calculable glimpse at the fundamental issue of thermalization in high-energy collisions; for example, we obtain an initial condition on the Wigner-function evolution in color kinetic theory.[7,8]

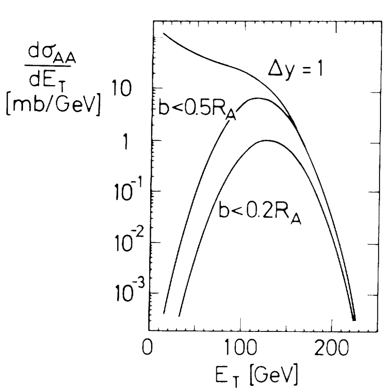

FIG. 3. E_T distribution for UU collisions at 100+100 GeV per nucleon with $p_{T\min} = 3$ GeV, integrated over the unit pseudorapidity interval $|\eta| < \frac{1}{2}$ and azimuth $\Delta\phi = \pi$. The result of restriction to central collisions is indicated in the two lower curves.

We have in this Letter considered the hard and calculable minijet contribution to $A+A$ collisions. Minijets will play an important role in nuclear collisions at very high energies since the rate scales with $A^{4/3}$ and grows rapidly with energy. This enhances the importance of one's going to as large s and A as possible when searching for quark-gluon plasma. The calculable contribution of minijets gives a lower limit to the total E_T spectrum, which also contains a soft component which is incalculable in QCD perturbation theory. The soft component can also be modeled in several different ways by building it up from pp collisions[9–11]; also a unified treatment has been constructed for pp collisions.[12] The computations presented here could easily be supplemented with some of these soft models.

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