Light-Neutrino Masses and the Strong *CP* Problem

Michael Shin
Department of Physics, University of California, Irvine, Irvine, California 92717
(Received 30 April 1987)

A minimal extension of the standard model is proposed to incorporate the light-neutrino masses and the resolution of the strong *CP* problem. A unique light pseudo-Goldstone boson, whose coupling to the electron is 1 or 2 orders of magnitude greater than that of the conventional hadronic axion, arises from the model. The scale of Peccei-Quinn symmetry breaking is shown to be greater than 10^{10} GeV through its connection to the light-neutrino masses, and is further constrained if the (hypothetical) fourth family of quarks and leptons exists.

PACS numbers: 12.15.Ff, 12.10.Dm, 12.15.Cc, 14.80.Gt

One of the fundamental questions in particle physics of today is whether the neutrinos have finite nonvanishing masses. Indeed, the recently proposed Mikheyev-Smirnov-Wolfenstein[1] (MSW) mechanism as the solution to the solar-neutrino puzzle suggests that this may be so. In an attempt to understand the tiny masses of the light neutrinos in the context of more general flavor problem of quarks and leptons, we recently proposed[2] that the light-neutrino masses and the strong *CP* problem of quarks are intimately related to each other. To be more precise, it was reported[2] that the scale of the seesaw mechanism[3] which drives the smallness of the light-neutrino masses is to be identified with the scale of Peccei-Quinn[4] (PQ) symmetry breaking, 10^8-10^{12} GeV,[5] if the strong *CP* problem is resolved by the standard PQ mechanism with the invisible[6,7] axion. Moreover, this theoretical proposal seems to be in agreement[2] with the mass ranges[8] of light neutrinos required to resolve the solar neutrino puzzle via the MSW mechanism.

Although our discussion on the above theoretical feature was made within the context of the SO(10) gauge group in Ref. 2, it is easy to see that the same feature should also be present in a more general class of theories whenever the symmetry principles present in the leptonic Yukawa sector are related to $U(1)_{PQ}$ of the quark Yukawa sector. In this article, I would like to discuss how the same feature can be realized in a minimal way within the gauge group of the standard model $[SU(3)_C \otimes SU(2)_L \otimes U(1)_Y]$, and its phenomenological implications.

The simplest way to realize the tiny masses of light neutrinos via the seesaw[3] mechanism in $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is to introduce the right-handed neutrinos and their large Majorana masses in a manner suggested by Chikashige, Mohapatra, and Peccei[9] (CMP), by the introduction[10] of a lepton-number–carrying complex singlet scalar (which I call σ). Spontaneous symmetry breaking (SSB) induces a large vacuum expectation value (VEV) of this scalar, and the right-handed neutrinos acquire large Majorana masses through this VEV. If this scalar also carries the $U(1)_{PQ}$ quantum number, and if this is the only scalar with superlarge VEV, then the scale of the seesaw mechanism is indeed the PQ-symmetry–breaking scale. The next question is then how to resolve the strong *CP* problem via the PQ mechanism with the invisible axion. For this there are two orthogonal ways in the literature. The first is to introduce an extra Higgs-doublet scalar into the theory, creating the Dine-Fischler-Srednicki[6] (DFS) type of invisible axion. The second way is to introduce a color-carrying superheavy quark into the theory (with one Higgs-doublet scalar only), in a manner suggested by Kim, Shifman, Vainshtein, and Zakharov[7] (KSVZ). In this Letter, I choose the second option[11] for the following reasons: (i) The introduction of more than one Higgs doublet usually requires more than one fine tuning (gauge hierarchy problem) in the scalar potential. Moreover, in such cases, the $U(1)_{em}$ gauge group is not usually automatically unbroken, and thus it is less probable that the photon becomes a massless gauge boson. (ii) The DFS axion model has the domain-wall problem,[12] while the KSVZ axion model does not. (iii) The introduction of a heavy quark in the KSVZ manner is as much less *ad hoc* as the introduction of heavy right-handed neutrinos. They are fermions of beyond the standard model anyway, and there exists no reason why only heavy leptons (the right-handed neutrinos) should be present beyond the standard model. By our requiring them to acquire their large masses through the same VEV of σ, and thus treating these heavy fermions on the same footing, the theory becomes aesthetically more appealing. Having made these points, I now present my model.

The particle content of the model consists of n (≥ 3) generations of standard quarks and leptons, one Higgs doublet ϕ ($Y_\phi = \frac{1}{2}$, $\tilde{\phi} \equiv i\tau_2\phi^*$), gauge bosons of the standard model, n generations of right-handed neutrinos N_{0jR} ($j = 1,2,\ldots,n$), one superheavy quark $Q = Q_L + Q_R$, and one $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$-singlet complex scalar σ. The only fields which carry nonzero PQ charges (with their PQ charges in parentheses) are σ

(-1), Q_R $(\frac{1}{2})$, Q_L $(-\frac{1}{2})$, N_{0jR} $(\frac{1}{2})$, and the standard-model leptons $(\frac{1}{2})$. To begin with, we set up the following notations for the weak eigenstates of quarks and leptons: For quarks, $q_{0jL} \equiv (u_{0jL}, d_{0jL})^{\mathrm{T}}$, u_{0jR}, and d_{0jR}. The subscript 0 stands for the weak-eigenstate field. For leptons, $\psi_{0jL} \equiv (\nu_{0jL}, l_{0jL}^-)^{\mathrm{T}}$ and l_{0jR}^- are the standard-model leptons, while N_{0jR} are right-handed neutrinos. The charge conjugates of the neutrinos will be denoted by $\nu_{0jR}^c \equiv C(\bar{\nu}_{0jL})^{\mathrm{T}}$ and $N_{0jL}^c \equiv C(\bar{N}_{0jR})^{\mathrm{T}}$, where C is the charge-conjugation matrix. The scalar potential of the model is then simply given by

$$V(\phi,\sigma) = \lambda_1(\phi^\dagger\phi - v^2/2)^2 + \lambda_2(\sigma^*\sigma - V_{\mathrm{PQ}}^2/2)^2 + \lambda_{12}(\phi^\dagger\phi - v^2/2)(\sigma^*\sigma - V_{\mathrm{PQ}}^2/2),$$

with VEV's $\langle\phi\rangle = (0, v/\sqrt{2})^{\mathrm{T}}$, $\langle\sigma\rangle = \exp(i\theta_0)V_{\mathrm{PQ}}/\sqrt{2}$, and $V_{\mathrm{PQ}} \gg v = 246$ GeV, where θ_0 is the phase[13] of the VEV of σ. The Yukawa couplings, consistent with the PQ charges mentioned above, are

$$L_Y = -\{y_{ij}^{(u)}\bar{q}_{0iL}\tilde{\phi}u_{0jR} + y_{ij}^{(d)}q_{0iL}\phi d_{0jR} + y_{ij}^{(l)}\bar{\psi}_{0iL}\phi l_{0jR}^- + y_{ij}^{(\nu)}\bar{\psi}_{0iL}\tilde{\phi}N_{0jR} + y_{ij}^{(N)}\bar{N}_{0iL}^c\sigma N_{0jR} + y^{(Q)}\bar{Q}_L\sigma Q_R\} + \text{H.c.}$$

These Yukawa couplings and VEV's generate the mass terms of neutrinos,

$$(\bar{\nu}_{0L}, \bar{N}_{0L}^c)\begin{pmatrix} 0 & m_\nu \\ m_\nu^T & M_N \end{pmatrix}\begin{pmatrix} \nu_{0R}^c \\ N_{0R} \end{pmatrix} + \text{H.c.},$$

where m_ν and M_N are $n\times n$ (for n generations) matrices, with[14] $(m_\nu)_{ij} \equiv (v/\sqrt{2})y_{ij}^{(\nu)}$ and $(M_N)_{ij} \equiv (V_{\mathrm{PQ}}/\sqrt{2}) \times\exp(i\theta_0)y_{ij}^{(N)}$, while the mass terms for charged leptons are $\bar{l}_{0L}^-(m_l)l_{0R}^- + \text{H.c.}$, with $(m_l)_{ij} = vy_{ij}^{(l)}/\sqrt{2}$. To discuss the couplings of the gauge bosons and the scalar σ to the mass eigenstates of leptons, we need the following definitions of the unitary matrices V ($2n\times 2n$ matrix), U_L, and U_R ($n\times n$ matrices):

$$V^{\mathrm{T}}\begin{pmatrix} 0 & m_\nu \\ m_\nu^{\mathrm{T}} & M_N \end{pmatrix}V \equiv \mathrm{diag}(m_1, \ldots, m_n, M_1, \ldots, M_n)$$

and $U_L^\dagger(m_l)U_R \equiv \mathrm{diag}(m_e, m_\mu, \ldots)$, where m_j's (M_j's) are the light (superheavy) neutrino-mass eigenvalues with

$$0 < m_1 \le m_2 \le \cdots \le m_n \ll M_1 \le M_2 \le \cdots \le M_n.$$

Then the mass eigenstates (without the subscript 0) are

$$\begin{pmatrix} \nu_L \\ N_L^c \end{pmatrix} = V^{\mathrm{T}}\begin{pmatrix} \nu_{0L} \\ N_{0L}^c \end{pmatrix}, \quad \begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix} = V^\dagger\begin{pmatrix} \nu_{0R}^c \\ N_{0R} \end{pmatrix}$$

for neutrinos, while $l_L^- = U_L^\dagger l_{0L}^-$ and $l_R^- = U_R^\dagger l_{0R}^-$, for charged leptons. If we denote the $2n\times 2n$ unitary matrix V and its complex conjugate U by $n\times n$ submatrices,

$$V \equiv \begin{pmatrix} V_a & V_b \\ V_c & V_d \end{pmatrix} \equiv U^* \equiv \begin{pmatrix} U_a^* & U_b^* \\ U_c^* & U_d^* \end{pmatrix}, \tag{1}$$

and define the four-component spinor fields for neutrinos, $\nu_i \equiv \nu_{iL} + \nu_{iR}^c$ and $N_i \equiv N_{iR} + N_{iL}^c$, the neutrino-$Z^0$ couplings are found to be

$$L_{Z^0\nu N} = -(1/\sqrt{2})(G_{\mathrm{F}}M_Z^2/\sqrt{2})^{1/2}Z_\mu^0\{\bar{\nu}\gamma^\mu(U_a^\dagger U_a)(1-\gamma_5)\nu + \bar{N}\gamma^\mu(U_b^\dagger U_a)(1-\gamma_5)\nu + \bar{\nu}\gamma^\mu(U_a^\dagger U_b)(1-\gamma_5)N + \bar{N}\gamma^\mu(U_b^\dagger U_b)(1-\gamma_5)N\},$$

while neutrino-$W^\pm$ couplings are

$$L_{W^\pm\nu N} = -(G_{\mathrm{F}}M_W^2/\sqrt{2})^{1/2}\{W_\mu^-[\bar{l}^-\gamma^\mu(U_L^\dagger U_a)(1-\gamma_5)\nu + \bar{l}^-\gamma^\mu(U_L^\dagger U_b)(1-\gamma_5)N] + \text{H.c.}\}.$$

Note that in Eq. (1), although U and V are unitary ($U^\dagger U = 1 = V^\dagger V$), the corresponding submatrices[15] are not unitary $[V_a^\dagger V_a = 1 + O(\epsilon^2)$, $V_d^\dagger V_d = 1 + O(\epsilon^2)$, $(V_b)_{ij} = O(\epsilon)$, $(V_c)_{ij} = O(\epsilon)$, $\epsilon \equiv v/V_{\mathrm{PQ}}]$. If we represent the two independent degrees of the scalar σ by

$$\sigma \equiv e^{i\theta_0}(V_{\mathrm{PQ}} + \rho + i\chi)/\sqrt{2},$$

the (tree-level) coupling of χ (the pseudo-Goldstone boson which obtains a tiny mass through the color anomaly) to neutrinos is found to be

$$L_{\chi\nu N} = -i\chi[\bar{N}(V_d^{\mathrm{T}}M_NV_d)\gamma_5N + \bar{N}(V_d^{\mathrm{T}}M_NV_c)\gamma_5\nu + \bar{\nu}(V_c^{\mathrm{T}}M_NV_d)\gamma_5N + \bar{\nu}(V_c^{\mathrm{T}}M_NV_c)\gamma_5\nu]/V_{\mathrm{PQ}}.$$

With this preliminary discussion, let me now discuss the physics of the model. Except for the very fact that the light neutrinos acquire tiny masses through the seesaw mechanism and their possible consequences on the neutrino oscillations, which was well discussed in Ref. 2, the most interesting physics of the model is in the properties of the light scalar χ (the scalar ρ obtains superlarge mass $\sim V_{\mathrm{PQ}}$). In the limit where the color charge of the superheavy quark Q vanishes (or Q does not exist at all), χ becomes an exactly massless Goldstone, the majoron of the CMP model.[9] Its primary coupling would be to the right-handed neutrinos. On the other hand, in the limit where the coupling of σ to the right-handed neutrinos N_{0jR} vanishes (or the right-handed neutrinos do not exist in the theory), χ becomes the invisible

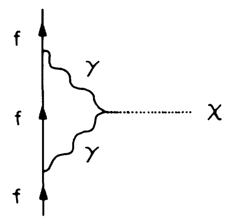

FIG. 1. Diagram for $\bar{f}i\gamma_5 f\chi$ coupling through the photon exchange.

axion of the KSVZ model,[7] acquiring a tiny mass through the color anomaly. Since it has both properties of these models, it can be called "majoraxion." From the PQ charge content of the model, the mass of the χ is easily found[16] to be

$$m_\chi = \frac{m_\pi (f_\pi/f_\chi)\sqrt{z}}{1+z}, \quad f_\chi = V_{\rm PQ},$$

where $f_\pi = 94$ MeV, $z \equiv m_u/m_d \simeq 0.56$. With the assumption that (for the sake of simplicity) Q is a color triplet, with electric charge $Q_{\rm em}$ [$Q_{\rm em}$ is zero if Q is singlet under $SU(2)_L \otimes U(1)_Y$], the coupling of χ to photons is the same as that of the KSVZ axion,

$$L_{\chi\gamma\gamma} = \frac{\alpha_{\rm em}}{8\pi}\frac{1}{V_{\rm PQ}}\left[6Q_{\rm em}^2 - \frac{2}{3}\frac{(4+z)}{(1+z)}\right]\chi F_{\mu\nu}^{\rm em}\tilde{F}_{\rm em}^{\mu\nu}.$$

The coupling of χ to the ordinary quarks and the charged leptons vanishes at the classical Lagrangean level, but they are generated through the color anomaly and one-loop diagrams given in Figs. 1–3. The color anomaly generates the familiar coupling of χ to the light quarks (u,d),

$$L_{qq\chi}^{\rm (CA)} = -\frac{1}{V_{\rm PQ}}\frac{m_u m_d}{m_u+m_d}(\bar{u}i\gamma_5 u + \bar{d}i\gamma_5 d)\chi, \tag{2}$$

while Fig. 1 generates[17] $\bar{f}i\gamma_5 f\chi$ coupling ($f = q_i$ or l_i^-),

$$L_{ff\chi}^{\rm (Fig.\,1)} = \frac{3\alpha_{\rm em}^2 q_f^2}{2\pi}\left(6Q_{\rm em}^2 \ln\frac{V_{\rm PQ}}{m_f} - \frac{2}{3}\frac{4+z}{1+z}\ln\frac{\Lambda}{m_f}\right)\frac{m_f}{V_{\rm PQ}}\bar{f}i\gamma_5 f\chi, \tag{3}$$

where q_f is the electric charge (in units of that of e^+) of f, and $\Lambda \simeq 4\pi f_\pi \simeq 1$ GeV. These couplings are the same as those of the KSVZ axion. Now we come to the crucial part of the model, which distinguishes itself from the KSVZ axion model, namely the contributions from Figs. 2 and 3 [these arise from the fact that $U(1)_{\rm PQ}$ is basically the lepton-number symmetry in this model, and the tree-level coupling of σ to N_{0jR}]. To estimate the contributions from these diagrams, we need to know the details of intergenerational mixing angles present in $L_{Z^0\nu N}$, $L_{W^\pm \nu N}$, and $L_{\chi\nu N}$ (i.e., the mixing matrices V_a, V_b, etc.), which in turn depend on the details of the structure of mass matrices M_N, m_ν, and m_l. Since the details of these structures are not known to us, we shall make the simplifying assumption that the off-diagonal couplings and thus the intergenerational mixing angles are rather small,[18] to make rough estimates of these diagrams. With this assumption, they are calculated to be

$$L_{ff\chi}^{\rm Fig.\,2} \simeq \frac{1}{32\pi^2}\frac{m_f}{V_{\rm PQ}}\left(\sum_{k=1}^{n} |y_{kk}^{(\nu)}|^2\right)\tau_3 \bar{f}i\gamma_5 f\chi, \tag{4}$$

$$L_{l_j^- l_j^- \chi}^{\rm Fig.\,3} \simeq \frac{1}{16\pi^2}\frac{m_{lj}}{V_{\rm PQ}}\left(\sum_{k=1}^{n}\{|(U_L)_{kj}|^2 |y_{kk}^{(\nu)}|^2\}\right)\bar{l}_j^- i\gamma_5 l_j^- \chi, \tag{5}$$

where $\tau_3 = 1$ for $f = u,c,t,\ldots$, and $\tau_3 = -1$ for $f = d, s,b,\ldots,e^-,\mu^-,\tau^-,\ldots$, in Eq. (4). For the coupling of χ to the light quarks (u,d) and therefore to the nucleons, (2) is the dominating contribution since the contributions from (3) and (4) are much smaller [at most $\sim 1/32\pi^2$ of (2)]. Thus the hadronic property of χ is essentially the same as the KSVZ axion. However, the leptonic property is in general much different from that of the KSVZ axion since Eq. (4) receives the largest contribution from the heaviest light-neutrino Yukawa coupling $y_{nn}^{(\nu)}$ ($n \geq 3$). For instance, the Dirac-mass Yu-

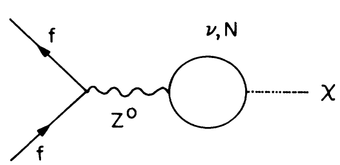

FIG. 2. Diagram for $\bar{f}i\gamma_5 f\chi$ coupling through Z^0 exchange.

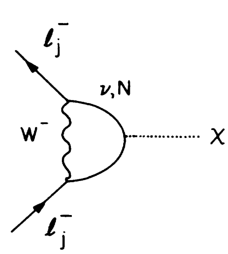

FIG. 3. Diagram for $\bar{l}_j^- i\gamma_5 l_j^- \chi$ coupling through $W^\pm$ exchange.

kawa coupling $y_{33}^{(\nu)}$ is expected[19] to be

$$y_{33}^{(\nu)} \simeq y_{33}^{(u)} = \sqrt{2}m_t/v \simeq 0.3[m_t/(50\text{ GeV})],$$

while $y_{44}^{(\nu)}$ is expected to be ~ 1 if the hypothetical fourth quark family (t') with $m_{t'} \simeq 150$–200 GeV exists. Therefore, for the coupling of χ to electrons, the "effective chiral PQ charge" generated[20] by Fig. 2 can be as large as $\sim 1/32\pi^2 \sim 10^{-2}$–$10^{-3}$. This is to be compared with the "effective chiral PQ charge" of electron in the KSVZ model [generated by Eq. (3) alone], which is[21] $\sim 10^{-4}$. This implies that χ is more accessible to laboratory experiments through its larger (by 1 or 2 orders of magnitude) coupling to electrons than the KSVZ axion, and is to be judged by future experiments.

Finally, let us consider the constraints on the PQ symmetry breaking scale V_{PQ} implied by its connection to the light-neutrino masses. The heaviest light-neutrino mass m_{ν_n} ($\equiv m_n$) is expected to be

$$m_{\nu_n} \simeq \frac{|y_{nn}^{(\nu)}|^2 (v/\sqrt{2})^2}{y_{nn}^{(N)} V_{\text{PQ}}/\sqrt{2}} \simeq \left| \frac{(y_{nn}^{(\nu)})^2}{y_{nn}^{(N)}} \right| (42\text{ eV}) \left(\frac{10^{12}\text{ GeV}}{V_{\text{PQ}}} \right).$$

With the assumption $y_{nn}^{(N)} \simeq y_{nn}^{(\nu)}$, the cosmological mass-density bound[22] $m_{\nu_n} \leq 100$ eV gives $V_{\text{PQ}} \gtrsim |y_{nn}^{(\nu)}| (4\times 10^{11}\text{ GeV})$. Since $y_{nn}^{(\nu)}$ is expected to be $|y_{nn}^{(\nu)}| \geq |y_{33}^{(\nu)}| \simeq 0.3[m_t/(50\text{ GeV})]$, we are led to the bound $V_{\text{PQ}} \gtrsim (1.2\times 10^{11}\text{ GeV})[m_t/(50\text{ GeV})]$. Allowing an order of magnitude of uncertainty in this estimate as a safety factor, we have $V_{\text{PQ}} \gtrsim 10^{10}$ GeV. Thus it is very likely that the PQ-symmetry–breaking scale is in the upper half ($10^{10}\text{ GeV} \lesssim V_{\text{PQ}} \lesssim 10^{12}$ GeV) of the present allowed range[5] (10^8–10^{12} GeV), and this window becomes even narrower if the fourth generation of quarks and leptons exists.

[1]S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978).

[2]M. Shin, in *Proceedings of the Twenty-Third International Conference on High Energy Physics, Berkeley, California, 1986,* edited by S. Loken (World Scientific, Singapore, 1987); K. Kang and M. Shin, Mod. Phys. Lett. **A1**, 585 (1986), and Phys. Lett. B **185**, 163 (1987), and Nucl. Phys. **B287**, 687 (1987).

[3]M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity,* edited by P. Van Nieuwenhuizen and D. Z. Freidman (North-Holland, Amsterdam, 1979); T. Yanagida, in *Proceedings of the Workshop on Unified Theory and Baryon Number of Universe, KEK, Tsukuba, Japan, 1979,* edited by A. Sawada and A. Sugamoto (KEK, Tsukuba-gun, Ibaraki-ken, Japan, 1979).

[4]R. Peccei and H. Quinn, Phys. Rev. Lett. **38**, 1440 (1977).

[5]J. Preskill *et al.*, Phys. Lett. **120B**, 127 (1983); L. F. Abbott *et al.*, Phys. Lett. **120B**, 133 (1983); M. Dine *et al.*, Phys. Lett. **120B**, 137 (1983); D. A. Dicus *et al.*, Phys. Rev. D **18**, 1829 (1978); M. Fukugita *et al.*, Phys. Rev. D **26**, 1840 (1982); A. Pantziris and K. Kang, Phys. Rev. D **33**, 3509 (1986).

[6]M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. **B104**, 199 (1981).

[7]J. E. Kim, Phys. Rev. Lett. **43**, 103 (1979); M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B166**, 493 (1980).

[8]S. P. Rosen and J. M. Gelb, Phys. Rev. D **34**, 969 (1986).

[9]Y. Chikashige, R. N. Mohapatra, and R. Peccei, Phys. Lett. **98B**, 265 (1981).

[10]Here I am assuming that all fermions acquire their masses through VEV's of scalars, without any bare-mass terms.

[11]The first option was recently considered by P. Langacker, R. Peccei, and T. Yanagida, Mod. Phys. Lett. **A1**, 541 (1986). These authors cite Ref. 2 in their discussion.

[12]P. Sikivie, Phys. Rev. Lett. **48**, 1156 (1982). In the DFS model, the number of domain walls is the number of quark flavors.

[13]This phase θ_0 will be fixed by the PQ mechanism, via $\bar{\theta} \equiv \theta_{\text{QCD}} + \arg\det[\text{quark-mass matrices}] = 0$.

[14]$(M_N)_{ij} = (M_N)_{ji}$ since it is a Majorana term.

[15]For the detailed discussion on $V_a, \ldots, V_d$, see J. Schechter and J. Valle, Phys. Rev. D **25**, 774 (1982).

[16]For the discussions on the properties of the most general axion, see the recent reviews by H. Y. Cheng, Indiana University Report No. IUHET-125, 1986 (to be published), and J. E. Kim, Seoul National University Report No. SNUHE 86/09, 1986 (to be published). See also D. Kaplan, Nucl. Phys. **B260**, 215 (1985).

[17]M. Srednicki, Nucl. Phys. **B260**, 689 (1985).

[18]Since the observed Kobayashi-Maskawa angles are all small [see F. Gilman and K. Kleinknecht, Phys. Lett. **170B**, 74 (1986)], this may be a reasonable assumption.

[19]In grand unified theories [e.g., in SO(10)], the Dirac mass terms for neutrinos are related to those of charge $\frac{2}{3}$ quarks, while the charged-lepton mass terms are related to those of charge $-\frac{1}{3}$ quarks.

[20]For the electron, the contribution from Eq. (5) is much smaller than that of Eq. (4), since it is further suppressed by the small mixing-angle factor $|(U_L)_{1n}|^2$.

[21]For $Q_{\text{em}} = 0, -\frac{1}{3}, \frac{2}{3}$, the corresponding electron-χ coupling in units of m_e/V_{PQ} (i.e., the "effective chiral PQ charge") in Eq. (3) is 3.7×10^{-4}, 6.7×10^{-5}, and 1.4×10^{-3}, respectively. Although we do not know the value of Q_{em} at this time, the possibility of $Q_{\text{em}} = 0$, which corresponds to the case when Q is an $SU(2)_L \otimes U(1)_Y$ singlet, is rather interesting since the other heavy fermions (i.e., the right-handed neutrinos) are also all $SU(2)_L \otimes U(1)_Y$ singlets.

[22]R. Cowsik and J. McClelland, Phys. Rev. Lett. **29**, 669 (1972).