Heterotic-String Lagrangean in the Bosonic Formulation

J. M. F. Labastida[(a)] and M. Pernici
The Institute for Advanced Study, Princeton, New Jersey 80540
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We consider the Becchi-Rouet-Stora-Tyutin quantization of the $N=2$ supersymmetric Siegel Lagrangean which is obtained by truncation of the $N=2$ spinning string. We show that this Lagrangean describes four chiral bosons and that it can be consistently coupled to $N=(1,0)$ supergravity. Using this system we propose a Lagrangean for the heterotic string in the bosonic formulation.

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Chiral bosons are a basic ingredient in the heterotic string.[1] A covariant Lagrangean quantization of chiral bosons in a gravity background is not known. A classical bosonic Lagrangean formulation for chiral bosons has been presented by Siegel.[2] This theory has anomalies,[3,4] which can be eliminated by the introduction of a local counterterm. In Ref. 4, it has been shown that this theory is consistent only when it describes two chiral bosons. A consistent formulation of this model in a gravity background is lacking; the difficulty in obtaining it is due to the presence of the local counterterm. In Ref. 2, $N=1$ and 2 supersymmetric extensions of the model describing chiral bosons are also considered at the classical level. An $N=(1,0)$ superspace action for a right-moving boson is given by Grisaru, Mezincescu, and Townsend,[5] and it is further analyzed by Gates, Brooks, and Muhammad.[6] These models can be obtained by consistent truncation of the $N=1$ (Brink and coworkers[7]) and $N=2$ (Brink and Schwarz[8]) supersymmetric Lagrangeans for the $N=1$ (Raman and coworkers[9]) and $N=2$ (Ademollo and co-workers[10]) spinning strings, respectively. Similarly, the truncation of the $N=4$ Lagrangean[11] for the $N=4$ spinning string[10] leads to the $N=4$ supersymmetric extension of the Lagrangean for chiral bosons.[12]

In this Letter we show that the $N=2$ model describes four chiral bosons at the quantum level, that the local counterterm is absent, and that it is possible to couple this model to background $N=(1,0)$ supergravity. Using the resulting system, we find a covariant Lagrangean quantization for the heterotic string in the bosonic formulation.

The $N=2$ supersymmetric Siegel Lagrangean describing four chiral bosons in an $N=(1,0)$ supergravity background is

$$\begin{aligned} e^{-1}\mathcal{L} = & -\mathcal{D}_+X^a\mathcal{D}_-X^a - \mathcal{D}_+Y^a\mathcal{D}_-Y^a + \tfrac{1}{2}i\bar\psi_L^{ia}\rho^+\mathcal{D}_+\psi_L^{ia} + \tfrac{1}{2}\lambda^{--}(\mathcal{D}_-X^a\mathcal{D}_-X^a + \mathcal{D}_-Y^a\mathcal{D}_-Y^a) \\ & -\tfrac{1}{4}i\lambda^{--}\bar\psi_L^{ia}\rho^+\mathcal{D}_-\psi_L^{ia} + 2\bar\chi_R^{-i}\psi_L^{ia}\mathcal{D}_-X^a + 2\bar\chi_R^{-i}\Omega^{ij}\psi_L^{ja}\mathcal{D}_-Y^a + \tfrac{1}{2}i\bar\psi_L^{ia}\rho^+\Omega^{ij}\psi_L^{ja}A^- \\ & + 2\bar\xi_R^-(\psi_L^1\mathcal{D}_-X + \psi_L^2\mathcal{D}_-Y), \end{aligned} \tag{1}$$

where $\mathcal{D}_\pm$ are gravitational covariant derivatives, and the $\pm$ indices are tangent-space indices. [In our conventions, $\mathcal{D}_\pm = (\mathcal{D}_\sigma \pm \mathcal{D}_\tau)/\sqrt{2}$, $\rho^\pm = (\rho^1 \pm \rho^0)/\sqrt{2}$, $\Omega^{12}=1$, and $\Omega^{ij} = -\Omega^{ji}$.] The matter multiplet is formed by the real scalar bosonic fields X^a and Y^a, and by the Majorana-Weyl fermions ψ_L^{ia}, $a=1,2$, $i=1,2$; and the gauge multiplet by the doubly self-dual gauge field λ^{--}, the Majorana-Weyl "Siegel gravitinos" χ_R^{-i}, and the self-dual U(1) gauge field A^-. ξ_R^- is the world-sheet gravitino.

The gauge and supergauge transformations, which we will call Siegel transformations, are the following[2,12]:

$$\begin{aligned} &\tilde\delta e^a_\pm = 0, \quad \tilde\delta\xi_R^- = 0, \quad \tilde\delta\lambda^{--} = 2\mathcal{D}_+\eta^- + \eta^-\mathcal{D}_-\lambda^{--} - \lambda^{--}\mathcal{D}_-\eta^- - 4i\bar\epsilon_R^i\rho^-\chi_R^{-i} - 4i\bar\epsilon_R^1\rho^-\xi_R^-, \\ &\tilde\delta\chi_R^{-1} = \eta^-\mathcal{D}_-\chi_R^{-1} - \tfrac{1}{2}\chi_R^{-1}\mathcal{D}_-\eta^- + \Lambda\chi_R^{-2} + \eta^-\mathcal{D}_-\xi_R^- - \tfrac{1}{2}\xi_R^-\mathcal{D}_-\eta^- + \mathcal{D}_+\epsilon_R^1 \\ &\qquad + \tfrac{1}{4}\epsilon_R^1\mathcal{D}_-\lambda^{--} - \tfrac{1}{2}\lambda^{--}\mathcal{D}_-\epsilon_R^1 + A^-\epsilon_R^2, \\ &\tilde\delta\chi_R^{-2} = \eta^-\mathcal{D}_-\chi_R^{-2} - \tfrac{1}{2}\chi_R^{-2}\mathcal{D}_-\eta^- - \Lambda\chi_R^{-1} + \mathcal{D}_+\epsilon_R^2 + \tfrac{1}{4}\epsilon_R^2\mathcal{D}_-\lambda^{--} - \tfrac{1}{2}\lambda^{--}\mathcal{D}_-\epsilon_R^2 - \Lambda\xi_R^- - A^-\epsilon_R^1, \\ &\tilde\delta A^- = \eta^-\mathcal{D}_-A^- + \tfrac{1}{2}\lambda^{--}\mathcal{D}_-\Lambda - \mathcal{D}_+\Lambda + i\bar\epsilon_R^i\Omega^{ij}\rho^-\mathcal{D}_-\chi_R^{-j} - i(\overline{\mathcal{D}_-\epsilon_R^i})\Omega^{ij}\rho^-\chi_R^{-j} \\ &\qquad - i\bar\epsilon_R^2\rho^-\mathcal{D}_-\xi_R^- + i(\overline{\mathcal{D}_-\epsilon_R^2})\rho^-\xi_R^-, \\ &\tilde\delta\psi_L^{ia} = \eta^-\mathcal{D}_-\psi_L^{ia} + \tfrac{1}{2}\psi_L^{ia}\mathcal{D}_-\eta^- + \Lambda\Omega^{ij}\psi_L^{ja} - i\rho^-\epsilon_R^i\mathcal{D}_-X^a + i\rho^-\Omega^{ij}\epsilon_R^j\mathcal{D}_-Y^a, \\ &\tilde\delta X^a = \eta^-\mathcal{D}_-X^a + \bar\epsilon_R^i\psi_L^{ia}, \quad \tilde\delta Y^a = \eta^-\mathcal{D}_-Y^a + \bar\epsilon_R^i\Omega^{ij}\psi_L^{ja}. \end{aligned} \tag{2}$$

The $N=(1,0)$ world-sheet supersymmetry transformations are given by

$$\begin{aligned}
&\delta e^a_+ = 2i\bar{\omega}_R \rho^- e^a_- \xi^-_R, \quad \delta e^a_- = 0,\\
&\delta \xi^-_R = \mathcal{D}_+ \omega_R, \quad \delta\lambda^{--} = -4i\bar{\omega}_R \rho^- \chi^{-1}_R,\\
&\delta \chi^{-1}_R = \tfrac{1}{4}\omega_R \mathcal{D}_- \lambda^{--} - \tfrac{1}{2}\lambda^{--}\mathcal{D}_- \omega_R,\\
&\delta \chi^{-2}_R = -A^- \omega_R,\\
&\delta A^- = i\bar{\omega}_R \rho^- \mathcal{D}_- \chi^{-2}_R - i(\overline{\mathcal{D}_- \omega_R})\rho^- \chi^{-2}_R,\\
&\delta \psi^1_L = -i\rho^- \omega_R \mathcal{D}_- X, \quad \delta X = \bar{\omega}_R \psi^1_L,\\
&\delta \psi^2_L = -i\rho^- \omega_R \mathcal{D}_- Y, \quad \delta Y = \bar{\omega}_R \psi^2_L.
\end{aligned} \tag{3}$$

Under Weyl transformations (δ_w), parametrized by the scalar w, the fields have the following weights: -1 for $e^a_\pm$ and A^-, and $-\frac{1}{2}$ for the gravitinos and the spinors.

The algebras of the Siegel transformations, Siegel transformations and local supersymmetry, and local supersymmetry are, respectively,

$$\begin{aligned}
&[\tilde{\delta}(\epsilon_1), \delta(\epsilon_2)]\, \tilde{\delta}(\eta^-) + \tilde{\delta}(\Lambda),\\
&[\tilde{\delta}(\epsilon), \delta(\epsilon_2)]\, \delta(\omega)] = \tilde{\delta}(\hat{\eta}^-) + \tilde{\delta}(\hat{\Lambda}),\\
&[\delta(\omega_1), \delta(\omega_2)] = \delta_{gc}(\xi^a) + \delta_l(\Omega) + \delta_w(w),
\end{aligned} \tag{4}$$

where δ_{gc} and δ_l correspond to reparametrizations and local Lorentz transformations, respectively. In the algebra (4),

$$\begin{aligned}
&\eta^- = -2i\bar{\epsilon}^i_2 \rho^- \epsilon^i_1,\\
&\Lambda = i\Omega^{kl}[(\overline{\mathcal{D}_- \epsilon^k_2})\rho^- \epsilon^l_1 - \bar{\epsilon}^k_2 \rho^- (\mathcal{D}_- \epsilon^l_1)],\\
&\hat{\eta}^- = 2i\bar{\epsilon}^1 \rho^- \omega,\\
&\hat{\Lambda} = i[(\overline{\mathcal{D}_- \epsilon^2})\rho^- \omega - \bar{\epsilon}^2 \rho^- (\mathcal{D}_- \omega)],\\
&\xi^- = -2i\bar{\omega}_2 \rho^- \omega_1, \quad \xi^+ = 0,\\
&\Omega = \tfrac{1}{2}\mathcal{D}_- \xi^-, \quad w = -\tfrac{1}{2}\mathcal{D}_- \xi^-.
\end{aligned} \tag{5}$$

Let us prove that in a flat supergravity background this model describes four chiral bosons at the quantum level. Since there are two generations of matter multiplets, the gauge anomalies cancel, as in the case of the $N=2$ spinning string,[13,14] in the critical dimensionality $D=2$.[10] We choose the Neveu-Schwarz boundary conditions for the fermions. The physical states are given by

$$Q|\Phi\rangle = 0, \tag{6}$$

where Q is the Becchi-Rouet-Stora-Tyutin charge, given by[15]

$$Q = c^i G_i + \tfrac{1}{2} f^k_{ij} c^i c^j \bar{c}_k, \tag{7}$$

and G_i are the generators of the Siegel algebra, c_i and $\bar{c}_j$ are the corresponding ghosts and antighosts, and f^k_{ij} the structure constants. Following Kato and Ogawa,[16] let us expand Q in the zero modes of the bosonic "Siegel reparametrization" ghosts:

$$Q = c_0 H + Q_B + b^0 M, \tag{8}$$

where

$$H = \tfrac{1}{2}(p^{a2}_X + p^{a2}_Y) + N - a, \tag{9}$$

is the Hamiltonian, p^a_X and p^a_Y are the left-sector momenta of the scalar fields X^a and Y^a, N is the number operator of all the left-moving matter fields, ghosts, and antighosts, and a is the intercept. One finds for the intercept,

$$a = 4\times\tfrac{1}{24} + 4\times\tfrac{1}{48} - \tfrac{1}{12} - 2\times\tfrac{1}{24} - \tfrac{1}{12} = 0, \tag{10}$$

where the contributions are given in order by the scalars, the fermions, the "Siegel reparametrization" ghosts b and c, the "superconformal" ghosts β^i and γ^i, and the ghosts of the U(1) symmetry. Since the physical states satisfy the condition

$$b_0|\Phi\rangle = 0, \tag{11}$$

it follows that

$$H|\Phi\rangle = 0. \tag{12}$$

The ground state, which is a physical state, is annihilated by all positive-frequency oscillators, apart from $\gamma^i_{1/2}$. Since $\beta^i_{-1/2}|0\rangle = 0$, H is positive semidefinite on the physical states, which have the same ghost number as the ground state; among these states, only those built by oscillators and momenta belonging to the right-moving sector satisfy (12). It follows that the model describes four right-moving scalars only. This analysis is similar to the one done in Ref. 4 for the bosonic Siegel Lagrangean. As in that case, it is crucial that the intercept be zero. As in Ref. 4, one could prove the no-ghost theorem also in the "Minkowski case," that is, when the scalars X^1 and Y^1 have the wrong sign in the kinetic term while the other two have the right one. This case is analogous to the $N=2$ spinning string,[10] in which it was proved that the only physical state is the ground state, a massless scalar. To describe the chiral bosons in the "Minkowski case" one must put the momentum of this physical scalar equal to zero (such an artificial procedure is absent in the "Euclidean case").

As in the $N=2$ spinning string,[17] another way of seeing that there are no left-moving physical states is to observe that the contribution of the left-moving sector to the partition function is 1, and therefore there is only one state, the ground state; in fact, the partition function for the left-moving sector is

$$\mathrm{Tr}\, q^H = \prod_n (1-q^n)^{-4}(1+q^n)^4(1-q^n)^2(1-q^n)^2(1+q^n)^{-4} = 1, \tag{13}$$

where the first monomial is due to the scalars X^a and Y^a, the second monomial is due to the fermions ψ_L^{ia}, the third to the "Siegel reparametrizations" ghosts b^{++} and c^-, the fourth to the ghosts b^+ and c for the U(1) symmetry, and the last one is due to the ghosts β^i and γ^i of the Siegel supergauge symmetry.

This model can be used to obtain a Lagrangean for the heterotic string in the bosonic formulation. The sixteen chiral bosons X^{aI}, $a=1,2$, $I=1,\ldots,4$, Y^{aI}, $a=1,2$, $I=1,\ldots,4$, can be grouped four by four as in (1). The bosons X^{aI} and Y^{aI} have the Fourier expansion

$$X^{aI}=X_0^{aI}+\frac{1}{2\pi}p_X^{aI}(\tau+\sigma)+\frac{1}{2\pi}\tilde{p}_X^{aI}(\tau-\sigma)+\cdots,\quad Y^{aI}=Y_0^{aI}+\frac{1}{2\pi}p_Y^{aI}(\tau+\sigma)+\frac{1}{2\pi}\tilde{p}_Y^{aI}(\tau-\sigma)+\cdots, \tag{14}$$

where p^{aI} and $\tilde{p}^{aI}$ are the left and right momenta, respectively. The physical-state conditions (6) and (12) imply that $p^{aI}=0$. The $\tilde{p}^{aI}$ are chosen to belong to an even self-dual lattice.[1] With use of the Lagrangean (1) it is simple to write the covariant Lagrangean formulation of the heterotic string *without* the need of fermionizing the sixteen internal bosonic coordinates X^{aI} and Y^{aI}. We use the condensed notation $X^A=\{X^\mu,X^{aI},Y^{aI}\}$, where $\mu=0,\ldots,9$, to denote all the 26 bosonic coordinates and similarly for the 26 left-handed Majorana spinors $\psi_L^{\tilde{A}}=\{\psi_L^\mu,\psi_L^{iaI}\}$. The Lagrangean is

$$\begin{aligned}e^{-1}\mathcal{L}_{\rm hs}=&-\mathcal{D}_+X^A\mathcal{D}_-X^A+\tfrac{1}{2}i\bar{\psi}_L^{\tilde{A}}\rho^+\mathcal{D}_+\psi_L^{\tilde{A}}+2\bar{\xi}_R^-(\psi_L^\mu\mathcal{D}_-X^\mu+\psi_L^{1aI}\mathcal{D}_-X^{aI}+\psi_L^{2aI}\mathcal{D}_-Y^{aI})\\&+\tfrac{1}{2}\lambda^{--I}(\mathcal{D}_-X^{aI}\mathcal{D}_-X^{aI}+\mathcal{D}_-Y^{aI})-\tfrac{1}{4}i\lambda^{--I}\bar{\psi}_L^{iaI}\rho^+\mathcal{D}_-\psi_L^{iaI}\\&+2\bar{\chi}_R^{-iI}\psi_L^{iaI}\mathcal{D}_-X^{aI}+2\bar{\chi}_R^{-iI}\Omega^{ij}\psi_L^{jaI}\mathcal{D}_-Y^{aI}+\tfrac{1}{2}i\bar{\psi}_L^{iaI}\rho^+\Omega^{ij}\psi_L^{jaI}A^{-I},\end{aligned} \tag{15}$$

where λ^{--I}, χ_R^{iI}, and A^{-I} are the Siegel gauge fields. The matter fields X^μ and λ^μ have the supersymmetry transformations,

$$\delta X^\mu=\bar{\omega}_R\psi_L^\mu,\quad \delta\psi_L^\mu=-i\rho^-\omega_R\partial_-X^\mu, \tag{16}$$

and are inert under the Siegel transformations (2). The transformation properties of the other fields are given in (2) and (3). This Lagrangean is invariant under $N=(1,0)$ local supersymmetry and under the Siegel transformations (2).

After we have chosen the superconformal gauge for the *vielbein* and the gravitino, and gauge fixed to zero the Siegel gauge fields λ^{--I}, χ_R^{-iI}, and A^{-I} by the gauge transformations (2), the Lagrangean (15) becomes

$$\mathcal{L}_{\rm SC}=-\partial_-X^A\partial_+X^A+\tfrac{1}{2}i\bar{\psi}_L^{\tilde{A}}\rho^-\partial_-\psi_L^{\tilde{A}}, \tag{17}$$

with the algebra of constraints formed by the world-sheet $N=(1,0)$ superconformal algebra and the four internal Siegel algebras in the left-moving sector, and by the conformal algebra in the right-moving sector. The Siegel algebras impose that of the sixteen internal scalars and fermions, and only the sixteen right-moving scalars contribute to the physical spectrum; therefore the physical states described by this theory are equivalent to those of the usual Fock-space realization of the heterotic string in the bosonic formulation.

There are several issues one could study in this theory. It should be possible to make background field computations, which up to now have been done only in the fermionic formulation; the Polyakov[18] path-integral formation could be studied, leading to a direct way of computing chiral boson determinants. In a forthcoming paper,[19] we will formulate this theory in superspace, and we will give the details of its quantization.

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[a]On leave from Instituto de Estructura de la Materia, Consejo Superior de Investigaciones Científicas, Serrano 119, Madrid 28006, Spain.

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