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Manifestation of Berry's Topological Phase in Neutron Spin Rotation

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Recently, Berry recognized that topological phase factors may arise when a quantum mechanical system is adiabatically transported around a closed circuit. We have measured Berry's topological phases by polarized-neutron spin rotation in a helical magnetic field. Berry's law is thus verified for fermions.

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Topological phases in simple quantum systems have been found to be of some interest recently. This interest was stimulated by a paper of Berry,¹ who derived a simple law governing these phases. Topological phases may show up whenever the system under study depends on some multiple parameter and is transported adiabatically around a closed curve in parameter space. These new phases do not depend on the interior dynamics of the system, but instead depend on its geometric history.

Topological phases are important in the context of non-Abelian gauge theories and of fractional quantization, and therefore Berry's findings have been found to be of interest in a number of recent investigations covering a variety of subjects.²⁻⁸ These phases may mimic the effect of a magnetic monopole of unit Dirac charge located at the origin of parameter space. The Aharonov-Bohm effect turns out to be a special case of the topological-phase concept. A further generalization of Berry's concept has very recently been presented by Aharonov and Anandan.⁹

Berry's law takes its simplest form when the required multiple parameter is an external magnetic field \mathbf{B} . When \mathbf{B} is varied adiabatically such that the tip of the vector \mathbf{B} describes a closed loop C [see Fig. 1(a)], then the system should, at the end of this excursion, return to its original state, according to the adiabatic theorem of

quantum mechanics. It simply will have picked up a phase factor $\exp(im\phi)$ for each spin substate $|m\rangle$, where ϕ is the usual dynamical phase

$$\phi = \kappa \int_0^T B(t) dt, \quad (1)$$

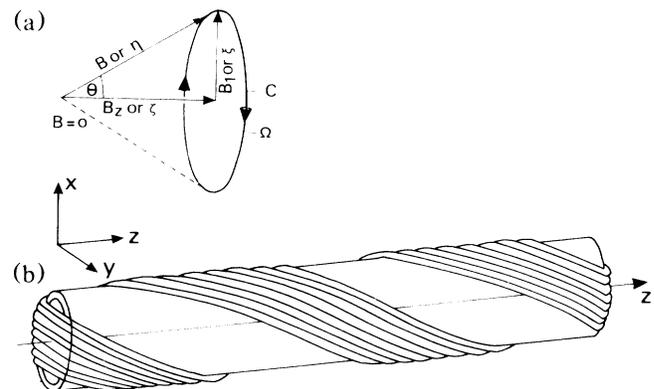


FIG. 1. (a) Adiabatic transport of the magnetic field vector \mathbf{B} around a closed loop C . Berry's phase is determined by the size of the solid angle Ω ; see Eq. (2). (b) Arrangement of the helical coil for the right-handed B_1 field. The neutron beam is along z . The solenoid for the axial B_z field is not shown.

with the gyromagnetic ratio κ . But, according to Berry, a further phase factor $\exp(im\gamma)$ will show up, with a topological phase of size¹⁰

$$\gamma = -\Omega, \quad (2)$$

where Ω is the solid angle subtended by the closed curve C as seen from the origin $\mathbf{B}=0$, at which point the system is degenerate as required in Berry's theory. The overall phase factor then is $\exp(im\Phi_t)$, with the *total phase*

$$\Phi_t = \phi + \gamma. \quad (3)$$

At first sight the experimental observation of these extra phases seems to be difficult. The adiabatic condition requires that the change in the dynamic phase ϕ be much larger than the topological phase shift γ induced by the adiabatic excursion of the system's parameters:

$$|\gamma| \ll |\phi|.$$

Therefore, the separation of γ from the measured total phase Φ_t requires very precise experimental control of the system.

Berry¹ proposed a measurement of γ on a particle interferometer. Moody, Shapere, and Wilczek⁶ proposed looking for tiny frequency shifts in nuclear magnetic resonance (NMR) signals. Both would seem to be difficult experiments (but see Note added).

We have observed Berry's phase in an experiment on neutron spin rotation in a twisted magnetic field. We believe this to be the most straightforward realization of a topological-phase experiment with fermions. In the meantime, measurements of Berry's phase for bosons¹¹ (polarized photons in a twisted optical fiber; see also Kitano, Yabuzaki, and Ogawa,¹² and Berry¹³) and for systems with orbital angular momenta¹⁴ ("pseudorotation" of atomic clusters) have also been reported.

Will a neutron spin-rotation experiment give the same information on Berry's phase γ as a neutron interferometry experiment? In neutron *interferometry*¹⁵ a phase shift Φ_t , applied to only one part of the "split" wave function, is directly read from the interference pattern

$$I \propto 1 + \cos \frac{1}{2} \Phi_t.$$

When the phase shift on the neutron is induced by a magnetic interaction, then in a *spin-rotation* experiment with polarized neutrons the phase of the neutron spin can be read from the neutron-polarization signal after the neutron-spin analyzer, which necessarily has the form

$$P = a_1 + a_2 \sin \Phi_t + a_3 \cos \Phi_t,$$

with coefficients a_1, a_2, a_3 depending on the special choice of the configuration.

Thus, when under a magnetic interaction the dynamical phase ϕ is shifted by a topological phase γ , this can

be seen in neutron spin rotation as well as in neutron interferometry. The fact that the phase shift seen in the neutron-interferometry signal is only half that seen in an ordinary spin-rotation experiment is due to the spinor character of the neutron wave function and leads to the spectacular sign change of the wave function upon a 360° rotation of the spin- $\frac{1}{2}$ system. This spinor rotation at "half speed" has been observed in several beautiful interferometry experiments¹⁶⁻¹⁸ and can be taken for granted here. The question of the spinor character of the neutron wave function, which is determined by the structure of the term $\frac{1}{2} \boldsymbol{\sigma}$ in the operator $R(\Phi_t) = \exp(\frac{1}{2} i \boldsymbol{\sigma} \cdot \Phi_t)$, is decoupled from the question of the appearance of an additional topological phase hidden in Φ_t .

In our experiment we used a slow, monochromatic beam of polarized neutrons (velocity $v \approx 500 \text{ ms}^{-1}$, polarization $P \approx 97\%$) from a neutron guide of the Institut Laue-Langevin's high-flux reactor. The neutron beam enters and leaves the field-free region of a Mumetal cylinder, which is coaxial with the beam (diameter 30 cm, length 80 cm). The direction of the polarization vector of the neutrons entering the cylinder can be chosen arbitrarily, as can be the component of polarization analyzed afterwards, and there is no loss of polarization during passage through the cylinder (for more details see Dubbers and co-workers^{19,20}).

Within the Mumetal cylinder the neutrons nonadiabatically enter and leave a static helical magnetic field, which is perpendicular to the beam, and which makes one complete right turn about the beam axis. The coil which produces the field is wound along the surface and across the openings of a hollow cylinder, 40 cm long and 8 cm in diameter, as shown in Fig. 1(b). Thus the neutrons on their flight along the axis of the cylinder see a right circularly polarized magnetic field \mathbf{B}_1 rotating by 2π over a length $L = 40 \text{ cm}$.

A second coil of the same type, wound onto the first, but twisted in the opposite direction, can be used to produce an elliptically polarized rotating field, in order to show that Eq. (2) is independent of the specific form of the contour C , as long as Ω is kept fixed.

The opening angle of the contour C is $\Omega = \pm 2\pi$ when only \mathbf{B}_1 is applied. Other values of Ω can be chosen by superimposing an axial magnetic field \mathbf{B}_z produced by a long solenoid coaxial with the neutron beam.

If $P_\alpha(0)$ is the polarization component of the neutrons entering the magnetic field region at time $t=0$, and $P_\beta(T)$ ($\alpha, \beta = x, y, z$; see Fig. 1) the polarization component analyzed after the neutrons leave the field region at time $t = T = L/v \approx 0.8 \text{ ms}$, then

$$P_\beta(T) = G_{\beta\alpha}(T) P_\alpha(0).$$

$G_{\beta\alpha}(T)$ can be calculated exactly for circularly polarized fields by going to a reference frame which rotates in phase with \mathbf{B}_1 at frequency $2\pi/T$.

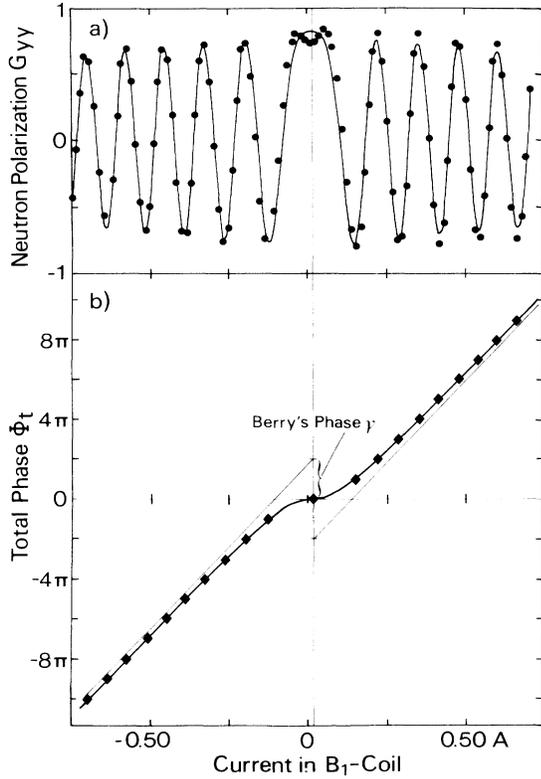


FIG. 2. (a) Neutron spin-rotation patterns of the transverse neutron spin component $P_y(T) = G_{yy}(T)P_y(0)$ in the helical B_1 field. Without Berry's phase the maxima of this curve should be equidistant. (b) Observed and calculated phase shifts Φ_t .

We write the number of neutron spin precessions about the axial, the helical, and the total magnetic fields, respectively, as

$$\zeta = \kappa B_z T / 2\pi, \quad \xi = \kappa B_1 T / 2\pi, \\ \eta = (\zeta^2 + \xi^2)^{1/2} = \kappa B T / 2\pi.$$

Then, using standard methods [e.g., Eq. (55) in the work of Dubbers²¹], we obtain, for instance,

$$G_{zz} = \frac{(\zeta \pm 1)^2 + \xi^2 \cos 2\pi[(\zeta \pm 1)^2 + \xi^2]^{1/2}}{(\zeta \pm 1)^2 + \xi^2}.$$

The \pm signs which appear in this formula refer to right- and left-handed B_1 fields. In contrast to B_1 , the B_z field as seen by the neutrons is not "switched on" nonadiabatically. Therefore, when $B_z \neq 0$, only G_{zz} can be measured unambiguously. For $B_z = 0$ the other coefficients are

$$G_{yy} = \cos 2\pi(1 + \xi^2)^{1/2}, \\ G_{zy} = -\frac{\xi}{(1 + \xi^2)^{1/2}} \sin 2\pi(1 + \xi^2)^{1/2}, \quad (4)$$

etc. If B_1 is not twisted but uniform, then $(\zeta \pm 1)^2$ is replaced by ζ^2 , and $(1 + \xi^2)$ by ξ^2 .

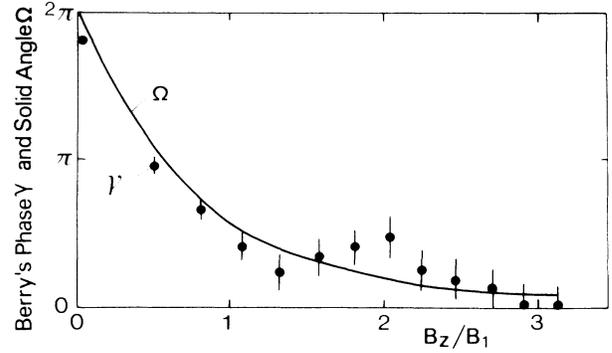


FIG. 3. Berry's phase γ at different solid angles Ω of the twisted B field.

The phase angle in these formulas is

$$\Phi_t = 2\pi[(\zeta \pm 1)^2 + \xi^2]^{1/2} - 2\pi. \quad (5)$$

The extra term -2π insures that $\Phi_t = 0$ when there is no field. In the adiabatic limit ($\eta \gg 1$, i.e., $\eta^2 \gg \eta > \zeta$) this becomes

$$\Phi_t \approx 2\pi\eta - 2\pi(1 - \zeta/\eta) = 2\pi\eta - 2\pi(1 - \cos\theta) \\ = \kappa B T - \Omega = \phi + \gamma,$$

as predicted by Berry; see Eqs. (1)-(3).

We have measured the neutron spin-rotation patterns G_{zz} as a function of both B_z and B_1 , and G_{xx} , G_{yy} , G_{zx} , G_{zy} as functions of B_1 for $B_z = 0$.

Figure 2(a) shows a measurement of G_{yy} for $B_z = 0$, and a fit by Eq. (4). Without Berry's phase the pattern in Fig. 2(a) would be a simple cosine. Figure 2(b) shows the measured phase angles as a function of B_1 [read at the maxima and minima of Fig. 2(a)], fitted with $\Phi_t = 2\pi[(1 + \xi^2)^{1/2} - 1]$ from Eq. (5). As expected, in the adiabatic limit (i.e., for large B_1) the phase is shifted by Berry's phase $\gamma = 2\pi$.

Figure 3 shows Berry's phase γ as a function of B_z/B_1 , as obtained from a measurement of $G_{zz}(B_z)$ with B_1 fixed at $(1 + \xi^2)^{1/2} = 5$. The solid curve gives the corresponding values of $\Omega = 2\pi(1 - B_z/B)$, in order to test Berry's law, Eq. (2). The first few points of Fig. 3 fall slightly below the predicted curve, because the adiabatic condition is not yet fully met. The scatter of the further points is due to imprecise reading of the larger values of Φ_t .

We draw the following conclusion from our investigation: On the one hand, Berry's phase law certainly is part of a far-reaching concept; on the other hand, in its simplest manifestation, which we believe to have realized, the appearance of a topological phase seems to be trivial: It can be generated or transformed away by going to a rotating-reference frame, which is a standard procedure²² in NMR work, and which also works in the classical case.²³ Furthermore, we are told that extra

phases due to spin rotation in helical fields are also observed in polarized-neutron scattering in the helical magnetic structure of a ferromagnet's Bloch wall.²⁴ There (as in the early experiments on twisted optical fibers, quoted in Ref. 11) the appearance of extra phases was taken to be self-evident, which indeed it is.

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Note added.—Upon first submission of this note the authors received a preprint²⁵ on the measurement of Berry's phase in the rotating frame with NMR methods.

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