Early-Universe Thermal Production of Not-So-Invisible Axions

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We find that for Peccei-Quinn symmetry-breaking scales $\lesssim 4\times10^8$ GeV (corresponding to axion masses $\gtrsim 0.03$ eV) thermal production of axions in the early Universe (via the Primakoff and photoproduction processes) dominates coherent production by a factor of about $1200[m_a/(1\text{ eV})]^{2.175}$. The photon luminosity from the decays of these relic axions leads to an upper limit to the axion mass of order 2–5 eV. If the axion mass saturates this bound, relic axion decays may well be detectable.

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Perhaps the only blemish on quantum chromodynamics (QCD) is the strong CP problem, namely, the fact that QCD instanton effects violate CP symmetry.[1] To date the most attractive solution appears to be the axion.[2-4] The axion is the almost massless, pseudo-Nambu-Goldstone boson associated with the spontaneously broken Peccei-Quinn (PQ) quasisymmetry. Its mass and lifetime (axion $\to 2\gamma$) are

$$m_a \simeq (1.2\text{ eV})/f_7, \tag{1}$$

$$\tau_a \simeq (6\times10^{24}\text{ sec}) m_{\text{eV}}^{-5}, \tag{2}$$

where, following the conventions of Srednicki,[5] the scale of PQ symmetry breaking is $f/N \equiv 10^7 f_7$ GeV; $m_{\text{eV}} \equiv m_a/(1\text{ eV})$ and $\hbar = k_B = c = 1$. The couplings of the axion to the other fields are model dependent; I refer the reader to Srednicki[5] and Kaplan[6] for further discussion.

The effect of axion production on stellar evolution results in an upper bound to the axion mass[7]: about 20 eV for axions which do not directly couple to electrons, and about 10^{-2} eV for axions which do couple directly to electrons. Coherent axion production (due to the initial misalignment of the vacuum angle $\theta = a/f$; a = axion field) leads to a lower limit to the axion mass based upon the axion contribution to the mass density of the Universe[8]: $m_a \gtrsim 10^{-5}$ eV (in models which undergo inflation after or during PQ symmetry breaking, this bound depends upon the initial misalignment angle, θ_i; see Turner[9]). The *axion window,* then, is between about 10^{-5} and 20 eV.

In this paper I consider the thermal production of axions in the early Universe via the Primakoff and photoproduction processes. It is shown that for axion masses greater than about 3×10^{-2} eV thermally produced axions dominate the relic axion population. Recently, Kephart and Weiler[10] have pointed out that because of their large relic abundance and lifetimes which are cosmologically well matched, coherently produced, "invisible" axions with masses in the 10–30-eV range can produce a large photon luminosity from their decays and may not be so invisible after all. By considering the photon luminosity produced by the decays of both clustered and unclustered, thermally produced, relic axions we obtain the bound $m_a \lesssim 2$–5 eV, independent of whether axions couple to electrons or not. If the axion mass saturates this bound, relic axion decays may be detectable, and observations are currently under way.[11]

First, let us review coherent axion production. When PQ symmetry breaking takes place ($T \sim f$), the vacuum angle is left undetermined because of the masslessness of the axion at high temperatures ($T \gg \Lambda_{\text{QCD}}$). At low temperatures ($T \ll \Lambda_{\text{QCD}}$) the axion develops a mass due to instanton effects and a preferred vacuum angle is picked out—the one which minimizes the vacuum energy. In general the initial vacuum angle is not aligned with this, and so the angle begins to relax. In so doing it oscillates about the preferred vacuum angle. These oscillations correspond to a very cold, nonrelativistic condensate of axions.[8] Their contribution to the energy density of the Universe today has been calculated to be[8,9]

$$\Omega_{coh} h^2 / T_{2.7}^3 \simeq 1.1\times10^{-6} m_{\text{eV}}^{-1.175}, \tag{3}$$

where $\Omega_a \equiv \rho_a/\rho_c$ is the fraction of critical density contributed by axions, $\rho_c = 1.05\times10^4 h^2$ eV cm^{-3}, h is the present value of the Hubble parameter H_0 in units of 100 km s^{-1} Mpc^{-1}, and $T_{2.7}$ is the present microwave temperature in units of 2.7 K. Equation (3) was derived under the assumption that inflation did not take place after or during PQ symmetry breaking: If it did, then the relic axion abundance depends upon the initial misalignment angle (for further discussion, see Ref. 9).

Now consider the thermal production of axions in the early Universe. For simplicity, we will only consider the interactions of the heaviest quark (Q, mass M) which couples to the axion. As we shall see, its interactions are the dominant production mechanism; the interactions of the other quarks which couple to the axion only serve to *increase* the axion production rate slightly. I am intentionally ignoring the coupling of the axion to electrons as the limit I derive is most relevant for models where the axion does not couple directly to electrons; in any case, axion production via electrons is always subdominant.

At low temperatures ($T \lesssim M$) the dominant axion production mechanism involving the heavy quark Q is the Primakoff process $q+\gamma \rightarrow q+a$; here q is any light (mass $\lesssim T$), charged fermion. At energies below M, the heavy-quark loop can be shrunk to a point and the low-energy cross section is $\langle \sigma v \rangle \simeq \alpha^3/f^2$, where the angular brackets indicate thermal averaging (for details, see Raffelt[7]). (Note that at very high temperatures, $T \gg \Lambda_{QCD}$, the photons in this diagram can be replaced by gluons and α by α_s, thereby greatly increasing the production rate.) The rate of axion production then is just given by $\Gamma_p \simeq n_q \langle \sigma v \rangle$, where n_q is the number density of light, charged particles. If we take $n_q \simeq g_* T^3/\pi^2$, i.e., for simplicity, equal to the number density of relativistic particles (g_*, as usual, counts the effective number of relativistic degrees of freedom), it follows that

$$\Gamma_p \simeq g_* \alpha^3 T^3/\pi^2 f^2. \tag{4}$$

At high temperatures ($T \gtrsim M$), the heavy-quark loop can no longer be shrunk to a point, and the dominant process is photoproduction off the thermal bath of heavy quarks: $Q+\gamma \rightarrow Q+a$. (At low temperatures this process is not important because the ambient number density of heavy quarks is suppressed by a Boltzmann factor.) The thermally averaged cross section for this process is $\langle \sigma v \rangle \simeq \alpha (M/f)^2 T^{-2}$. The axion production rate then goes like

$$\Gamma_Q \simeq \alpha T M^2/\pi^2 f^2. \tag{5}$$

Whether or not axions are in thermal equilibrium depends upon the magnitude of their production rate relative to the expansion rate H of the Universe. During its early history ($T \gtrsim 10$ eV) the Universe was radiation dominated, and $H = 1.67 g_*^{1/2} T^2/m_{\mathrm{Pl}}$. Whenever $\Gamma \gtrsim H$ axions should be in thermal equilibrium and have a number density $n_a = \zeta(3) T^3/\pi^2$, where $\zeta(3) = 1.20206\ldots$. However, whenever $\Gamma \lesssim H$ axions should be decoupled and have a constant number per comoving volume.

For high temperatures ($T \gtrsim M$) Γ/H varies as T^{-1} and for low temperatures as T, achieving its maximum at a temperature of about $T \simeq M$. The value of Γ/H at the maximum is about $(\Gamma/H)|_{T=M} \simeq 10^4 M_{30}/f_7^2$, where $M_{30} \equiv M/(30\ \mathrm{GeV})$. For $f_7 \lesssim 100 M_{30}^{1/2}$, there is a period when the axion should have been in thermal equilibrium. We know that the top quark is more massive than about 30 GeV and in some axion models[3] the heavy quark is very massive ($M \gtrsim 10^{14}$ GeV), and so it seems safe to assume that for $f_7 \lesssim 100$ axions were once in thermal equilibrium. Now let us calculate their decoupling or freezeout temperature; roughly speaking, that occurs when $\Gamma_p/H \simeq 1$: $T_d \simeq (20\ \mathrm{GeV}) f_7^2 \simeq (40\ \mathrm{GeV}) m_{\mathrm{eV}}^{-2}$. For $T \lesssim T_d$ the number density of axions just decreases as R^{-3}. Assuming that the entropy per comoving volume [$\equiv S \propto s R^3$, where the entropy density $s = 2\pi^2 g_* T^3/45$ and $R(t)$ is the Friedmann-Robertson-Walker (FRW) scale factor] remains constant (i.e., no significant entropy production), I calculate the number density of axions today by using the constancy of $n_a/s = 0.278/g_{*d}$ and $s_{\mathrm{today}} \simeq 7.04 n_\gamma \simeq 2810 T_{2.7}^3\ \mathrm{cm}^{-3}$:

$$n_a = (n_a/s) s_{today} \simeq 13 (60/g_{*d}) T_{2.7}^3\ \mathrm{cm}^{-3}, \tag{6}$$

$$\Omega_{\mathrm{thermal}} h^2/T_{2.7}^3 \simeq 1.3 \times 10^{-3} m_{\mathrm{eV}} (60/g_{*d}). \tag{7}$$

Here $g_{*d} \equiv g_*(T_d)$; for T_d of order 1–100 GeV, $g_{*d} \simeq 60$, while for $T_d \lesssim 100$ MeV (i.e., below the quark/hadron transition and $\mu^\pm$ annihilations), $g_{*d} \simeq 10$. Note that $\Omega_{\mathrm{thermal}} \simeq 1$ is achieved for $m_a \simeq 130 h^2$ eV; however, since axions more massive than about 25 eV decay in less than the age of the Universe, $\Omega_{\mathrm{thermal}} \simeq 1.0$ is likely precluded.

Comparing Eqs. (3) and (7) we see that thermal production dominates coherent production for $m_a \gtrsim 3 \times 10^{-2}$ eV (or $f_7 \lesssim 40$). The ratio of the thermally produced axions to the coherently produced axions is $\Omega_{\mathrm{thermal}}/\Omega_{\mathrm{coh}} \simeq 1200 (60 g_{*d}) m_{\mathrm{eV}}^{2.175}$.

The coherently produced axions come into existence when the vacuum angle starts to oscillate[9]: $T_{\mathrm{osc}} \simeq (6\ \mathrm{GeV}) f_7^{-0.175}$. For $f_7 \gtrsim 0.6$ axions have already decoupled, and so the thermal and coherent populations exist separately. On the other hand, for $f_7 \lesssim 0.6$, axions are still in thermal equilibrium, and the coherently produced population should thermalize, leaving a single, thermal population in the end.

Once the thermally produced axions decouple they expand freely with axion momenta undergoing red shifting $\propto R^{-1}$. As long as they are relativistic they will maintain their thermal distribution albeit with a temperature which varies as R^{-1}. When the axion temperature drops to $\simeq m_a/3$, axions become nonrelativistic and thereafter axion velocities decrease as R^{-1}. Today, then, they should be characterized by a velocity dispersion of order

$$\langle v_a^2 \rangle^{1/2} \simeq 2.7 \times 10^{-4} (60/g_{*d})^{1/3} T_{2.7} m_{\mathrm{eV}}^{-1}.$$

For axion masses less than about 25 eV, axion lifetimes are greater than the age of the Universe, so that most of the relic axions are still with us today. I now estimate the photon luminosity of relic axions. To do so some assumptions must be made about where axions might be today. I consider two plausible possibilities: (1) that they are unclustered; (2) that they cluster and account for a fraction of the *dark matter* in galaxies.

(1) *Unclustered axions.*—This is the most conservative assumption regarding their detectability. It is simple to compute the integrated photon intensity (assuming for simplicity the $\Omega = 1$ flat FRW model): $I = n_a m_a (\lambda_a/\lambda)^{7/2}/4\pi \tau_a \lambda_a H_0$, or

$$I \simeq (1.6 \times 10^{-23}\ \mathrm{erg\ cm^{-2}\ sec^{-1}\ arcsec^{-2}\ \AA^{-1}}) (\lambda_a/\lambda)^{7/2} h^{-1} (10/g_{*d}) m_{\mathrm{eV}}^7, \tag{8}$$

where $\lambda_a = (24800\ \text{Å})/m_{\text{eV}}$ is the rest-frame wavelength of a decay photon, 1 sr$\simeq 4.3\times 10^{10}$ arcsec^2, and I have ignored the very tiny velocity dispersion of the relic axions. The diffuse photon background (or night sky) in the optical (electronvolt range) has an intensity of about[12] $I_{\text{diffuse}} \simeq 10^{-18}$erg *cm* $^{-2}$ sec^{-1} *arcsec* $^{-2}$ Å^{-1}, which implies an upper limit to the axion mass of about 5 eV.

(2) *Clustered axions.*—If structure formation in the Universe proceeds via primeval density perturbations which grow through the Jeans instability (as is generally believed), then axions should participate in structure formation. One would naively expect that the ratio of axions to baryons ($\equiv r$) in a structure which has not undergone significant dissipation (such as the halo of a galaxy, or a cluster of galaxies) would be about Ω_a/Ω_b, where the fraction of critical density contributed by baryons is constrained by primordial nucleosynthesis to be[13] $0.014 \lesssim \Omega_b h^2/T_{2.7}^3 \lesssim 0.035$. Taking $\Omega_b h^2/T_{2.7}^3 \simeq 0.02$ and considering only the thermally produced axions, we find that $\Omega_a/\Omega_b \simeq 0.4(10/g_{*d})m_{\text{eV}}$. However, there are other considerations. Is the axion Jeans mass sufficiently small at the time of galaxy formation so that they can participate in the collapse which results in the formation of a galaxy (equivalently, is their velocity dispersion smaller than the gravitational velocity dispersion of a galaxy, $\langle v_{\text{gal}}^2\rangle^{1/2} \simeq 10^{-3}$)? Is there enough phase space in a galaxy for the axions? For axions in the multielectronvolt mass range and galaxy formation at red shifts $z \lesssim 10$, the answer to the first question is very likely yes. The second question is a bit more subtle. Once axions decouple, their phase-space number density is microscopically conserved. For a thermal distribution of bosons *or* fermions, the phase-space occupancy is of order unity. Thus the arguments originally applied to neutrinos apply here too.[14] Modeling bound structures as isothermal spheres (with cores radius $a \equiv a_{30}$ kpc and velocity dispersion $\sigma \equiv \sigma_{-3} 10^{-3}$) one finds that there is enough phase space for a mass in axions of about $M_a \simeq 10^7 M_\odot m_{\text{eV}}^4 a_{30}^3 \sigma_{-3}^3 g$, where g is the initial phase-space occupancy. The baryonic mass of a typical spiral galaxy is about $10^{11} M_\odot$ and for the halo $a_{30} \simeq \sigma_{-3} \simeq 1$. Thus, the maximum value of r permitted by phase-space considerations is $r_{\max} \simeq 10^{-4} m_{\text{eV}}^4$, which is less than Ω_a/Ω_b for axion masses in the electronvolt range. For spiral galaxies, then, one would expect the ratio of axions to baryons in the halo to be of order $r_{\max}$. [Because of their enormous initial phase-space occupancy ($g \gtrsim 10^{48} m_{\text{eV}}^{2.7}$—truly a Bose condensate) this argument is irrelevant for the coherent axion population.]

With use of an isothermal sphere model for the halo of our galaxy with $\rho_{\text{halo}}(r) = \rho_\odot (R^2+a^2)/(r^2+a^2)$, where r is distance from the galactic center, $R \simeq 9$ kpc is our distance from the galactic center, $\rho_\odot \simeq 5\times 10^{-25}$ g cm^{-3} is the halo density near the solar system, and a is the core radius, and with the assumption that a fraction of the halo $r_{\max}$ is axions, it is straightforward to calculate the photon intensity from the galactic halo:

$$I_{\text{halo}} = (1.6\times 10^{-23}\ \text{erg cm}^{-2}\ \text{sec}^{-1}\ \text{arcsec}^{-2}\ \text{Å}^{-1}) m_{\text{eV}}^{10} J(\theta,x), \tag{9a}$$

$$J(\theta,x) = \frac{1+x^2}{x^2+\sin^2\theta}\left[\frac{\pi}{2} + \tan^{-1}\{\cos\theta/(x^2+\sin^2\theta)^{1/2}\}\right], \tag{9b}$$

where $x \equiv a/R$ is of order unity, and θ is the angle between the direction of observation and the galactic center. The axion produced radiation is in a line of width $\Delta\lambda \simeq 10^{-3}\lambda_a \simeq (24.8\ \text{Å})/m_{\text{eV}}$ at wavelength $\lambda_a \simeq (24800\ \text{Å})/m_{\text{eV}}$. For $m_a \gtrsim 3$ eV the axion-produced line should stand out above the night sky. The predicted glow of our own axionic halo implies an upper bound to the axion mass of about 3 eV. Should the axion mass saturate this bound, the angular dependence of the axionic halo glow provides a unique signature for it, and a probe of the distribution of matter in the halo.[15]

Now consider the photon luminosity produced by axion decays in the halo of a distant galaxy whose image fills the aperture of the detector. If we again take the fraction of axions in the halo to be $r_{\max}$ and assume the baryonic mass in the halo to be of order $M_{11}\times 10^{11} M_\odot$, it is straightforward to derive the photon intensity:

$$I \simeq (2.4\times 10^{-24}\ \text{erg cm}^{-2}\ \text{sec}^{-1}\ \text{arcsec}^{-2}\ \text{Å}^{-1}) m_{\text{eV}}^{10} M_{11}, \tag{10}$$

which also results in a limit of order 3 eV.

For clusters of galaxies (mass of the order $M_{13}\times 10^{13} M_\odot$, $a_{30} \simeq 10$, and $\sigma_{-3} \simeq 10$) axions in the electronvolt mass range should contribute a fraction Ω_a/Ω_b of the cluster mass as $r_{\max}$ is greater than Ω_a/Ω_b (i.e., there is ample phase space to have $r = \Omega_a/\Omega_b$). The photon intensity from cluster axions in a detector whose aperture they fill should be about

$$I \simeq (2\times 10^{-20}\ \text{erg cm}^{-2}\ \text{sec}^{-1}\ \text{arcsec}^{-2}\ \text{Å}^{-1})(10/g_{*d}) M_{13} m_{\text{eV}}^7, \tag{11}$$

in a line of width $\Delta\lambda \simeq 10^{-2}\lambda_a \simeq (248\ \text{Å})/m_{\text{eV}}$, leading to a mass bound of order 2 eV.

In sum, I have shown that for axion masses greater than about 3×10^{-2} eV ($f_7 \lesssim 40$) the relic axion population is dominated by thermally produced axions, and for masses greater than $\simeq 2$ eV the coherent axion population itself should be thermalized. Consideration of the diffuse photon background produced by the decays of unclustered relic axions results in the mass limit $m_a \lesssim 5$ eV. A similar limit follows from axion-produced photon emission from galactic halos. A limit of $\lesssim 2$ eV follows from considering the photon luminosity due to axion decays in clusters of galaxies. Conversely, it is of interest to search for photon line emission from the decay of axions on mass 1–5 eV.[11]

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