Unitarity Bound on the Scale of Fermion Mass Generation

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Partial-wave unitarity implies an upper limit on the energy scale of the mechanism that generates quark and lepton masses.

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The origin of quark and lepton masses is one of the deepest puzzles left open by the standard model. We are ignorant not only of the structure of the mass-generating mechanism but even of the energy scale at which it operates. One possibility is elementary Higgs scalars,[1] perhaps supersymmetric, which are most plausible below 1 TeV. Dynamical mass generation, as for instance in "extended technicolor,"[2] suggests a much higher energy scale, of order 100 TeV, to explain u and d quark masses of order 10 MeV. Here we present a model-independent upper bound on the energy scale of fermion mass generation. We show that $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)$ gauge interactions and partial-wave unitarity imply that the energy scale E associated with the generation of fermion mass m_f is bounded by

$$E \leq E_f = \frac{16\pi}{\sqrt{2} G_\mathrm{F} m_f} \cong \frac{\pi}{m_f}, \tag{1}$$

where G_F is the Fermi constant and the final expression yields E_f in teraelectronvolts for m_f supplied in teraelectronvolts. For example, for $m_t \gtrsim 50$ GeV we have $E_t \lesssim 60$ TeV. If there is a fourth generation of quarks with masses of order 1 TeV, Eq. (1) suggests[3] that the physics of their mass generation lies at or below a few teraelectronvolts.

Partial-wave unitarity is a powerful tool for fixing the domain of validity of an effective field theory and, therefore, the energy scale below which new physics must emerge. For instance, the Fermi theory of weak interactions implies fermion-fermion scattering amplitudes growing like G_F's, which would violate partial-wave unitarity at $E \cong 300$ GeV; hence the prediction that new physics must emerge at some lower energy.[4] Faith in the laws of probability was vindicated in 1983 by the discovery of the W and Z bosons at 80 and 90 GeV.

Our bound on the mass-generation scale E_f is similar to this classical weak-interaction unitarity limit. We observe that the effective theory obtained by introduction of bare (gauge-symmetry–violating) fermion masses m_f into the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)$ gauge theory contains scattering amplitudes $f\bar{f} \to W_L W_L$ (W_L denotes longitudinal polarization) which grow like $G_\mathrm{F} m_f \sqrt{s}$ and saturate unitarity at the scale $\sqrt{s} = E_f$ in Eq. (1). The true gauge-invariant mass-generating mechanism must therefore become manifest at a scale $E \lesssim E_f$ to avoid a violation of unitarity.

This use of unitarity differs from another application of unitarity to spontaneously broken gauge theories in past years.[5,6] At very high energy, $s \gg m_\mathrm{H}$ or $s \gg m_f$ where m_H and m_f denote Higgs and heavy-fermion masses, respectively, contributions to tree-level scattering amplitudes occur proportional to $G_\mathrm{F} m_\mathrm{H}^2$ or $G_\mathrm{F} m_f^2$ that would violate partial-wave unitarity for large enough m_H or m_f. These critical values ($\cong 1$ TeV in each case) do not restrict the allowed range of Higgs-boson and fermion masses but rather define the domain beyond which perturbation theory fails and strong coupling begins.[7] A closer analogy to the present analysis is the use of unitarity and low-energy theorems for $W_L W_L$ scattering to deduce a growing amplitude proportional to $G_\mathrm{F} s$ that would, if unmodified, saturate unitarity at $\sqrt{s} = 1.8$ TeV, thus setting an upper limit on the energy scale of the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)$-breaking mechanism that gives the W and Z their masses.[8]

To derive the upper bound on the scale of fermion mass generation we consider the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)$ gauge theory with W and Z masses arising from some unspecified form of spontaneous symmetry breaking. We imagine, however, that fermion masses are not generated spontaneously but rather introduce bare fermion masses which break the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)$ gauge invariance. In this way we construct an effective nonrenormalizable field theory that approximates the true gauge-invariant and renormalizable theory at energies below the scale of fermion mass generation. As an example, we could consider the standard isodoublet Higgs model, in which the W and Z acquire mass from the vacuum condensate of the Higgs field, but with the usual fermion–Higgs-boson Yukawa interactions omitted and replaced by bare fermion mass terms.

We now consider fermion-antifermion scattering to longitudinally polarized gauge-boson pairs, since these amplitudes in tree approximation exhibit the bad high-energy behavior that causes the theory to be nonrenormalizable at one loop. (The amplitudes are given in Appendix B of Ref. 6.) The worst behaved Feynman diagrams, proportional to $G_F s$, occur in helicity- (or chirality-) conserving channels such as $\bar{f}_+ f_- \to W_L^+ W_L^-$ (subscripts on f denote helicity). For instance, if f is a heavy lepton of negative weak isospin, $T_3 = -\frac{1}{2}$ and $Q = -1$, then there are contributions from s-channel photon exchange,

$$\mathcal{M}^{\gamma}_{+-} = -2\sqrt{2} G_F s \beta_W \sin^2\theta_W \sin\theta (1 + 2M_W^2/s) \tag{2}$$

(β_W and θ are the W velocity and scattering angle in the center of mass), from s-channel Z exchange,

$$\mathcal{M}^{Z}_{+-} = -(G_F/\sqrt{2})(1 - \beta_f - 4\sin^2\theta_W)[s^2 \sin\theta/(s - M_Z^2)]\beta(1 + 2M_W^2/s) \tag{3}$$

(β_f is the fermion velocity in the center of mass), and from t-channel $(I = +\frac{1}{2})$ fermion exchange,

$$\mathcal{M}^{f'}_{+-} = [\sqrt{2} G_F m_f^2 s \sin\theta/(1 + \beta_f)(t - m_{f'}^2)][\beta_f \beta_W + \beta_f \cos\theta - (2M_W^2/s)\beta_w] \tag{4}$$

(for simplicity Kobayashi-Muskawa angles are neglected).

Adding Eqs. (2), (3), and (4) we find that the bad high-energy behavior is canceled and that the sum is proportional to a constant as $s \to \infty$. For $m_f \gg M_{W,Z}$ this constant is proportional to $G_F m_f^2$ and was used in Ref. 6 to study the critical values of m_f that mark the onset of strong coupling. In the present context the lesson is that the nonrenormalizability of our effective theory, being due to the fermion mass terms, is not manifested in the chirality-conserving sector. We must look instead to chirality-nonconserving channels where scattering occurs only by virtue of the fermion masses.

For the chirality-nonconserving channel $\bar{f}_+ f_+ \to W_L^+ W_L^-$ we have the three Feynman amplitudes[6] as in the chirality-conserving case, Eqs. (2)–(4):

$$\mathcal{M}^{\gamma}_{++} = -4\sqrt{2} G_F m_f \sin^2\theta_W \sqrt{s} \cos\theta \beta_W (1 + 2M_W^2/s), \tag{5}$$

$$\mathcal{M}^{Z}_{++} = -\sqrt{2} G_F m_f (1 - 4\sin^2\theta_W)[s\sqrt{s}\cos\theta/(s - M_Z^2)]\beta_W (1 + 2M_W^2/s), \tag{6}$$

$$\mathcal{M}^{f'}_{++} = \sqrt{2} G_F m_f \sqrt{s}(1 + \cos\theta)\beta_W - \frac{\sqrt{2} G_F m_f s \sqrt{s}}{4(t - m_{f'}^2)} \left[-\beta_W (1 + \cos\theta) \left((\beta_f - \beta_W)^2 + 4\frac{m_{f'}^2}{s} \right) + \frac{4M_W^2}{s}(\beta_W \cos\theta - \beta_f \cos 2\theta) \right]. \tag{7}$$

Each amplitude has bad high-energy behavior, proportional to $G_F m_f \sqrt{s}$, which does not cancel in the sum. With neglect of the masses of f, f', W, and Z relative to s, the sum of Eqs. (5)–(7) at large s is

$$\mathcal{M}^{\gamma + Z + f'}_{++} \cong \sqrt{2} G_F m_f \sqrt{s}. \tag{8}$$

Equation (8) exhibits the bad high-energy behavior of our effective theory. In the standard isodoublet Higgs model, the bad high-energy behavior would be canceled by the s-channel Higgs-exchange contribution

$$\mathcal{M}^{H}_{++} = -\sqrt{2} G_F m_f \frac{s\sqrt{s}}{s - m_H^2} \beta_f \left(1 - \frac{2M_W^2}{s} \right). \tag{9}$$

With our normalization the $J = 0$ partial-wave amplitude is

$$a_0 = \frac{1}{32\pi} \int_{-1}^{1} d\cos\theta \, \mathcal{M}. \tag{10}$$

From Eq. (8) we then find for the effective nonrenormalizable theory at large s that

$$a_0(\bar{f}_+ f_+ \to W_L^+ W_L^-) = \sqrt{2} G_F m_f \sqrt{s}/16\pi. \tag{11}$$

Partial-wave unitarity requires

$$|a_J| \le 1, \tag{12}$$

so that Eq. (11) saturates unitarity at $\sqrt{s} = E_f$ given by Eq. (1). The true theory must incorporate the physics of fermion mass generation at an energy scale less than or of the order of E_f.

The scale of mass-generating physics could be much less than E_f, as, for instance, in the standard Higgs model,[1] Eq. (9), where m_H as light as 10 GeV is not impossible. Much higher scales are expected in dynamical models of mass generation. For instance, the conventional extended technicolor (ETC) estimate [Fig. 1(a)] is[2]

$$m_f \cong g_{\rm ETC}^2 (v^3/m_{\rm ETC}^2), \tag{13}$$

where $v = (\sqrt{2} G_F)^{-1/2} \cong 0.25$ TeV. For $g_{\rm ETC}^2/4\pi \sim 1$, Eq. (13) implies $m_{\rm ETC} \cong 2$ TeV for $m_t \cong 50$ GeV, and $m_{\rm ETC} \cong 200$ TeV for $m_u \cong 5$ MeV. In particular, $m_{\rm ETC} < E_f$ holds if $m_f < 256\pi^2 v/g_{\rm ETC}^2$, a condition that is assured for the plausible range of m_f and $g_{\rm ETC}$. It is important to stress, however, that in a technicolor theory, the process $f_+ \bar{f}_+ \to W_L^+ W_L^-$ will not directly constrain

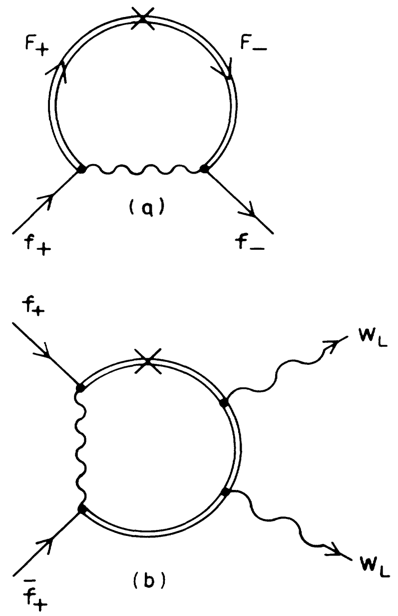

FIG. 1. (a) Mass-generation mechanism in extended technicolor. The internal wavy line represents the exchange of an ETC boson. The double line is the technifermion propagator and the cross indicates the insertion of the dynamical technifermion mass. (b) A technicolor contribution to the process $f_+\bar{f}_+ \to W_L W_L$.

the ETC scale. The growth of the partial-wave amplitude Eq. (11), for example, will not continue to $\sqrt{s} \cong m_{\rm ETC}$. Instead, it is not difficult to see by estimating Fig. 1(b) and its partners that this amplitude will be damped like $1/\sqrt{s}$ once $\sqrt{s}$ gets beyond $O(2\pi v) \sim 1.5$ TeV. More damping will set in at $m_{\rm ETC}$ when the $1/p^2$ behavior of the ETC gauge boson propagator becomes operative.

More generally, the ETC mass-generating mechanism could be replaced by any four-fermion interaction (with appropriate Lorentz structure) coupling ordinary quarks and leptons to techniquarks, as might for instance arise in composite models.[9] Then Eq. (13) would still hold with $g^2/m_{\rm ETC}^2$ replaced by the effective four-fermion coupling G. The expected damping of this vertex at $p > G^{-1/2}$ would in turn lead to the further damping of $f_+\bar{f}_+ \to W_L^+ W_L^-$ beyond $G^{-1/2}$.

In both the standard Higgs theory and the technicolor theory described here, the fermion mass-generation mechanism operates well below our upper bound E_f. An interesting question is whether there exists a natural mechanism that saturates the bound.

Returning to the top quark, its mass is bounded below by the failure to observe it at Tristan,[10] $m_t > 25$ GeV, and at the CERN SPS collider,[11] $m_t > 43$ GeV. Unless its contribution to the W and Z vacuum polarization is canceled by some presently unknown physics, there is also an upper limit[6,12] on m_t, set at 180 GeV in recent fits[13] to deep inelastic neutrino scattering and the W and Z masses. These bounds imply that the unitarity scale E_t is bracketed by $\cong 65$ TeV and $\cong 15$ TeV. The heavy-fermion contributions to vacuum polarization may also constrain, but do not exclude, the possibility of a fourth fermion generation. For instance, if m_t is not much above the present lower limit, a plausible fourth generation could be accommodated with $m_\nu \ll 1$ GeV, $m_L \lesssim 50$ GeV, and $m_U \cong m_D \cong 1$ TeV provided that $|m_U - m_D| \lesssim 200$ MeV. Since we would then have $E_U \cong E_D \cong 3$ TeV, such a heavy fourth generation could offer the best opportunity to study directly the physics responsible for fermion mass generation.

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