Galaxy and Structure Formation with Hot Dark Matter and Cosmic Strings

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Galaxy and structure formation in a neutrino-dominated universe with cosmic strings is investigated. Strings survive neutrino free streaming to seed galaxies and clusters. The effective maximum Jeans mass is about $1.5\times10^{14}h_{50}^{-4}M_\odot$, lower than in the adiabatic scenario. Hence cluster formation is only marginally different from that in the cold-dark-matter and strings model, but galaxy masses are lower. The mass spectrum of galaxies is flatter than with cold dark matter, and the density profile about an individual loop is less steep, in better agreement with observations.

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The standard nucleosynthesis scenario[1] constrains the energy density in baryons to be $\Omega_B \le 0.15$, but theoretical prejudice insists that the total energy density is $\Omega = 1$. The remainder of the density of the Universe must then be nonbaryonic. It is called hot if the dark particles have relativistic peculiar velocities at the time $t_{\rm eq}$ of equal matter and radiation and cold if they do not. Massive neutrinos are the best motivated candidates of either kind, and a (hot) τ neutrino with mass $m_\nu = 30$ eV would be consistent with all existing constraints.

Models with linear adiabatic density perturbations and hot dark matter (HDM) are, however, hard to reconcile with observations. In these models perturbations on mass scales smaller than $10^{15}M_\odot$ are wiped out by the free streaming of neutrinos just before $t_{\rm eq}$.[2,3] Hence, galaxies can only form by fragmentation of larger objects.

With cosmic strings as the source of density perturbations, the situation is quite different. The strings survive neutrino free streaming and can seed small-scale structure, albeit less efficiently than with cold dark matter (CDM).[4]

The essentials of the scenario are the same as for strings and CDM. Loops with the mean separation of galaxies are to be identified with galaxies and similarly for clusters.[5,6] Thus the scale-free correlation function predicted with strings on large scales[7] is unaffected. What is different is the efficiency of accretion. Small loops accrete less and the mass spectrum of objects $n(M)$ is less steep. The density profile about an individual loop is less steep than with CDM.

Galaxy cores would be primarily baryonic, neutrinos being prevented from clustering on small scales by phase-space constraints.[8] Galaxies will have formed recently, and thus there will be significant evolution at low red shifts unlike in the CDM scenario.

Neutrino accretion.— Neutrino accretion may be understood heuristically as follows. The accretion time scale is $t_{\rm H}$, the Hubble time, during which the neutrinos move a distance $\lambda_\nu = v_\nu t_{\rm H}$, where v_ν is their velocity. On scales below λ_ν, perturbations are washed out, but on larger scales they grow in the usual way. At $t_{\rm eq}$ the rms neutrino velocity is $v_\nu \simeq 0.17$ and $t_{\rm H} \simeq 50h_{50}^{-2}$ Mpc, where h_{50} is the Hubble parameter in units of 50 km s^{-1} Mpc^{-1}. Therefore, at $t_{\rm eq}$, $\lambda_\nu(t_{\rm eq}) = \lambda_{\nu,\rm eq} \simeq 8h_{50}^{-2}$ Mpc. In the string model, perturbations on all comoving scales larger than $\lambda_{\nu,\rm eq}$ start growing at $t_{\rm eq}$, whereas scales $\lambda < \lambda_{\nu,\rm eq}$ must wait until $\lambda_{\nu,\rm eq}(a/a_{\rm eq})^{-1/2} \simeq \lambda$ before growth starts.

Now we proceed to a more precise treatment. Since the neutrinos interact very weakly, their phase-space density is conserved[3] and satisfies the Liouville equation. Brandenberger, Kaiser, and Turok[9] derived an integral equation for the Fourier-space energy-density perturbations $\delta\rho_\nu(k)$ in neutrinos about a cosmic-string loop starting from the Liouville equation and perturbing about a homogeneous initial distribution.

With the choice of $a(t_{\rm eq}) = 1$, then in terms of a new time variable $z = \frac{1}{2}[(a+1)^{1/2} - 1]$, the resulting integral equation for $\delta_\nu(k) = \delta\rho_\nu(k)/\rho_\nu$ is[10]

$$\delta_\nu(k,z) = 6\int_{z_0}^{z} dz'[\delta_\nu(k,z') + M_1/\rho_{\nu,\rm eq}]F(z,z')/[1+\alpha^2F^2(z,z')]^2, \tag{1}$$

with $F(z,z') = \ln(1+1/z') - \ln(1+1/z)$. Here $\alpha = kv_0\tau_*$ with $v_0 = T_{\nu,\rm eq}/m_\nu \simeq 0.05$ a measure of the neutrino velocity at $t_{\rm eq}$ and $\tau_* = (8\pi G\rho_{\nu,\rm eq}/3)^{-1/2} = 2^{1/2}t_{\rm H,eq}$. $\rho_{\nu,\rm eq}$ is the energy density in neutrinos at $t_{\rm eq}$.

In comoving units, $v_0\tau_* = 3.5h_{50}^{-2}$ Mpc. Modes with $k \gg (v_0\tau_*)^{-1}$ are suppressed. Equation (1) is valid right through the radiation-matter transition. z_0 is the time when accretion begins. For $k=0$, (1) yields the usual equation for the growth of perturbation in CDM.[10]

We have solved (1) numerically for $\delta_\nu(k,z)$. For large enough z, each k mode grows as $z^2 \sim a(t)$. Thus $C(k)=(\rho_{\nu,\mathrm{eq}}/M_1)\delta_\nu(k,z)z^{-2}$ tends to a constant. In Fig. 1 we plot $C(k)$ vs $\alpha(k)$ for two different values of z_0. An analytical fit, which is good for $0 \le \alpha \le 100$, is $C(k)=A/[B+\alpha^2(k)]$. For $z_0=0.01$ (i.e., initial scale factor $a_0=0.04$) the constants A and B are $A=1.1$ and $B=1.1/6$. For $z_0<0.01$, there is no significant change. For $z_0=0.1$ (i.e., $a_0=0.44$), some growth is lost and $A=1.1$ with $B=0.4$.

The mass profile of the accreted neutrinos can be calculated analytically. For $a \gg 1$, the mass inside a comoving radius x is

$$\delta M(<x)=\alpha M_1 a\{1-[1+(x/L)]e^{-x/L}\}, \tag{2}$$

where $\alpha=A/4B=\frac{3}{2}$ for $z_0 \ll 1$ and $\alpha=0.7$ for $z_0=0.1$, and $L=v_0\tau_*/\sqrt{B}\simeq 8.4h_{50}^{-2}$ Mpc for $z_0\ll 1$ and $L \simeq 5.6h_{50}^{-2}$ Mpc for $z_0=0.1$. The second term in (2) is the growth suppression due to neutrino free streaming. For $x \gg L$, there is very little suppression, but for $x \ll L$, $\delta M(<x)\simeq x^2$, quite different from CDM. Our answer for L agrees well with the naive estimate and gives the effective maximal Jeans mass M_J (mass inside a ball of radius L) quoted in the abstract.

Baryon accretion and loop decay.—Accretion of baryons begins on all scales after baryons decouple from radiation at red shift of $z_{\mathrm{rec}}\simeq 1.5\times 10^3$. This makes little difference on scales $\lambda > L$, since neutrinos are already clustering and the baryons will just track them. However, on small scales, neutrino perturbations have not started growing and baryonic clustering is important. The equation governing the fractional density enhancement in baryons $\delta=\delta\rho_B/\rho_B$ in the matter-dominated era is[10]

$$\ddot{\delta}+4\dot{\delta}/3t-2\Omega_B\delta/3t^2=4\pi G\delta\rho_s, \tag{3}$$

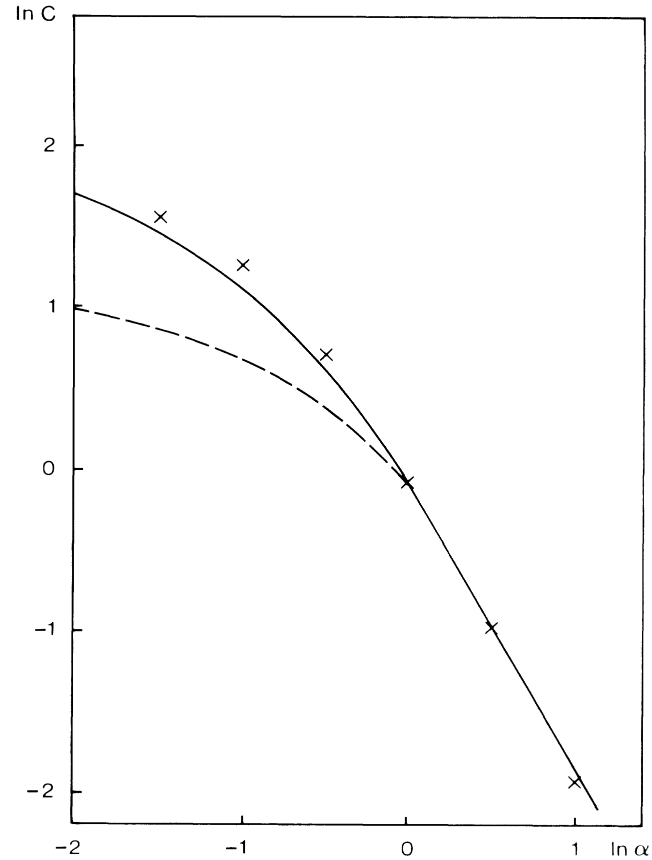

FIG. 1. The net growth of the neutrino perturbation at late times t. α is the wave number in units of $(v_0\tau_*)^{-1}$. The solid curve represents the results starting with zero perturbations at a red shift $a(t)\ll a(t_{\mathrm{eq}})$, the dashed line starting at $a(t)/a(t_{\mathrm{eq}})=0.44$.

where $\delta\rho_s$ is the source perturbation. For a point mass $\delta\rho_s=M_1(t)a^{-3}(t)\delta^3(x)$, and when we take into account the decay via gravitational radiation, $M_1(t)\simeq M_1(1-t/t_d)$, where M_1 is the initial mass and t_d is the decay time. Equation (3) can be solved to give (for $\Omega_B=\frac{1}{8}$) the accreted baryonic mass $\delta M_B(a)$:

$$\delta M_B(a)=M_1\left\{\left[\frac{3}{4}+\frac{3}{20}\left(\frac{a_{\mathrm{rec}}}{a_d}\right)^{3/2}-\frac{9}{10}\left(\frac{a_{\mathrm{rec}}}{a_d}\right)^{1/4}\right]\left(\frac{a}{a_{\mathrm{rec}}}\right)^{1/4} + \left[\frac{1}{4}-\frac{1}{12}\left(\frac{a_{\mathrm{rec}}}{a_d}\right)^{3/2}-\frac{1}{6}\left(\frac{a_d}{a_{\mathrm{rec}}}\right)^{3/4}\right]\left(\frac{a_{\mathrm{rec}}}{a}\right)^{3/2}\right\} \tag{4}$$

if $a > a_d$. Equation (4) fails once neutrinos start to cluster. On a comoving scale x, this happened when $a(t)\simeq a_J(x)=(L/x)^2$.

Loop decay also affects neutrino growth. We have integrated (1) numerically in the matter-dominated era with the source mass varying as above. The results are shown in Fig. 2 where the ratio f_ν of the growth factors with and without loop decay are plotted as functions of $z_d/z_J=(a_d/a_J)^{1/2}$.

Now we calculate the density profile taking loop decay and baryon accretion into account. We can write in a phenomenological manner

$$\delta M(<x)=M_{\mathrm{seed}}(a_J(x))[a/a_J(x)]. \tag{5}$$

The seed mass at $a_J(x)$ is the sum of the neutrino mass at $a_J(x)$ in the absence of baryons—this equals $f_\nu M_1$—and the mass in baryons at $a_J(x)$—we denote this by

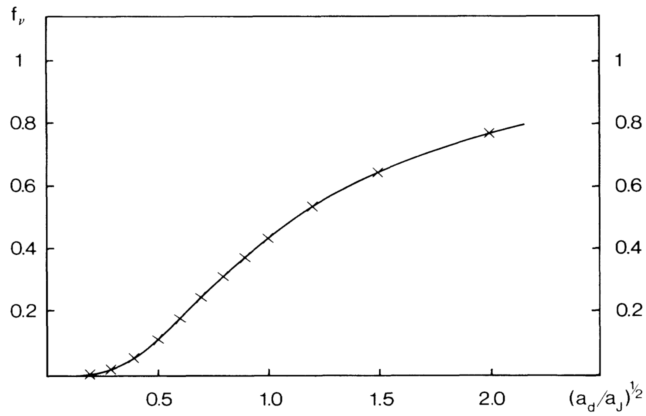

FIG. 2. Loss in growth due to loop decay for galaxy loops. f_ν is the ratio of the density perturbations with and without loop decay. a_J is the scale factor when the wavelength equals the Jeans length, a_d when the loop decays.

$f_B M_1$, where f_B is given by (4). Hence, from (2) and (5),

$$\frac{\delta M(<x)}{M} \simeq \frac{3}{4}\frac{M_1 a}{M_J}\frac{L}{x}(f_B+f_\nu)(x). \tag{6}$$

Consequences.— Now we turn to the consequences of the above calculations. We normalize $G\mu$ (μ is the mass per unit length in string, G is Newton's constant) by demanding that loops with a mean separation of Abell clusters have accreted an Abell-cluster mass around them. Since $\delta M/M \simeq 130$ inside an Abell radius,[11] the comoving scale corresponding to the Abell radius $3h_{50}^{-1}$ Mpc is $(130)^{1/3}3h_{50}^{-1}$ Mpc $\simeq 15h_{50}^{-1}$ Mpc, much larger than the maximal effective Jeans length L. Thus, only a very small growth factor is lost compared to the cosmic-string scenario with CDM.

However, galaxies are much smaller than with CDM. For galaxy loops we find $a_d/a_{\rm rec} \simeq 10$ and, hence, $f_B+f_\nu \simeq 0.75$ on scales $x \simeq 1$ Mpc corresponding to masses of $\simeq 3\times10^{11} M_\odot$. f_B+f_ν is very weakly dependent on x, although $f_B \gg f_\nu$ at small x and vice versa at large x.

The galaxy loop mass is given in terms of the cluster loop mass $M_{\text{c-loop}}$, fixed by the mass M_c of a cluster:

$$M_{\text{c-loop}} = \frac{\xi}{5}\left(\frac{\bar{\rho}}{\rho_b}\right)^{1/3}\frac{M_c}{1+z_{\rm eq}}. \tag{7}$$

Here $\bar{\rho}/\rho_b \simeq 130(\sigma_c/700)^2$ is the overdensity in a cluster today and σ_c is the one-dimensional velocity dispersion in kilometers per second. The factor 5 comes from matching the growth through the matter-radiation transition to that of a spherical collapse model,[12] and $\bar{\rho}/\rho_b$ enters because from this one can tell the red shift at turnaround.[6] ξ is a factor representing the loss in growth due to a loop being formed near $z_{\rm eq}$. If a loop is formed exactly at $z_{\rm eq}$ then $\xi \simeq 4$. The mass of a galaxy loop is $M_{\text{g-loop}} = M_{\text{c-loop}}(d_g/d_c)^2$, where d_g (d_c) is the mean separation of galaxies (clusters).

In the spherical-collapse model, a shell which collapses reaches its greatest density when $\delta\rho/\rho$ calculated in linear theory reaches 1.58. We shall use this to define, through (6), the total nonlinear mass accreted by a loop. Using $d_g/d_c = 1/11$ and $f_B+f_\nu = 0.75$, we find

$$M_{\rm galaxy} \simeq 5\times10^{10} M_\odot h_{50}^5 (\sigma_c/700)^8 (\xi/4)^3 (11 d_q/d_c)^6. \tag{8}$$

This corresponds to a rotation velocity for the shell just collapsed of

$$v_{\rm rot} = \sqrt{3}\sigma_g \simeq \sqrt{3}(M_{\rm galaxy}/M_c)^{1/3}\sigma_c$$
$$\simeq (50\ {\rm km\ s^{-1}})\left(\frac{\sigma_c}{700}\right)^2 h_{50}^2 \left(\frac{\xi}{4}\right)\left(11\frac{d_g}{d_c}\right)^2 \tag{9}$$

with $M_c = 10^{15} M_\odot h_{50}^{-1}(\sigma_c/700)^2 = 3\sigma^2 R/G$, where $R = 3h_{50}^{-1}$ Mpc is the Abell radius. If $\Omega_B = \frac{1}{8}$ and the baryons contract by a factor of 8, then the optical rotation velocity would be similar.

Our galaxies therefore look rather small, but the results include considerable observational uncertainties. If $\sigma_c = 1000$, then $M_{\rm galaxy} \simeq 10^{12} M_\odot$. Increasing ξ and h_{50} further increases the result. This increases the string tension required, since[6,12]

$$G\mu = 2\times10^{-6} h_{50}^{-1}\beta_{10}^{-1}\nu_{0.01}^{-2/3}(\xi/4)(\sigma_c/700)^{8/3}. \tag{10}$$

β_{10} and $\nu_{0.01}$ give the values of the string parameters in units of their "standard" values.[6] Increasing $G\mu$ in turn boosts the magnitude of the expected streaming velocity.[13]

With hot dark matter, galaxies look very different than they do with CDM. Phase-space arguments[8] show that 30-eV neutrinos cannot cluster on scales smaller than about 10 kpc. Hence, the inner regions of galaxies would be almost entirely baryonic. The halo would be comprised of neutrinos. The density profile for hot dark matter is $\rho(r) \simeq r^{-2}$, which gives a flat halo-rotation curve. This result follows from the analysis of Fillmore and Goldreich[14] which shows that an initial spherical perturbation with $\delta M/M \simeq r^{-\gamma}$ with $\gamma < 2$ always collapses to give flat rotation curves.

The mass function of objects expected with HDM is also different from that with CDM. From (6) we see that to a first approximation the nonlinear mass M scales as $x^3 \simeq M_1^3$, since f_B depends only very weakly on the decay time. Let $n(M)dM$ denote the number density of objects with masses in the range $[M, M+dM]$. For strings and CDM, $n(M) \simeq M^{-5/2}$ on the scale of galaxies. For HDM, we find using $n(M_1)dM_1 \simeq M_1^{-5/2}dM_1$ that $n(M) \simeq M^{-3/2}$, in better agreement with the Schechter luminosity function. This is valid for masses $M \gg M_{cu}$, where M_{cu} is given by the mass accreted by a loop which decays at $t_{\rm rec}$: $M_{cu} \simeq 4\times 10^{-4} M_{\rm galaxy}$. Objects with $M < M_{cu}$ are seeded by loops which decay at $t_d(R) \simeq t_{\rm rec}$. $t_d(R)$ is given by $t_d(R) = (\gamma G\mu)^{-1}R$, with $\gamma \simeq 5$. The mass accreted by such a loop can be determined from (4) by expansion in $R-(\gamma G\mu)t_{\rm rec}$. We find that in the limit $M \rightarrow 0$, $n(M) \sim M^{-1/2}$. For clusters ($M > M_{\rm J}$), we have the same $n(M)$ as with CDM, which has been shown to fit the data rather well.[15]

Our model does not explain the exponential decay of the luminosity function for galaxies at the bright end without invocation of the Rees-Ostriker stability arguments. The distribution of dark baryons is also an open question. Dark baryons could explain the halos of dwarf spheroidals and could also effect the ratio of mass to luminous mass. These issues deserve further attention.

We conclude that the cosmic-string theory with HDM is an interesting cosmological model which deserves further study. There are testable differences compared with a model with CDM. Flat halo-rotation curves, a characteristic mass function, and smaller galaxy masses are the main predictions. Similar conclusions have also recently been reached by Bertschinger and Watts.[16] Neutrino clustering in a more general context has been considered by Cowsik and co-workers.[17]

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