Dimensionality Crossover in Superconducting Wire Networks

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We have fabricated wire networks in the form of triangular arrays of third-order Sierpinski gaskets and percolation arrays on square lattices. For the gaskets the superconducting phase boundary, $T_c(H)$, exhibits a clear crossover between the homogeneous and inhomogeneous (fractal) regimes. This is formally equivalent to the phonon-fracton crossover. Surprisingly, the random, square-lattice percolation networks, which are expected to exhibit a similar crossover, seem to behave in a manner characteristic of two-dimensional systems, independent of length scale.

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The study of disorder in superconductors has been pursued in a number of ways in recent years. Strongly disordered superconductors have been modeled as weakly coupled grains of clean material with the disorder parametrized by the strength of this coupling.[1,2] This picture has been used to explain the sudden disappearance of superconductivity in ultrathin films at a universal value of the sheet resistance, $R=\hbar/4e^2=6.1$ kΩ.[1] A second approach introduces disorder microscopically[3] and is useful when one is describing homogeneous samples such as amorphous alloys.[4] Although T_c is depressed by the disorder, no universal behavior controlled by R is expected. In a third model of disordered systems, *geometric* inhomogeneities occur over a range of length scales. Systems treated with this model include insoluble metal-insulator mixtures such as Al-Ge,[5,6] and most thin films, which grow percolatively in two dimensions.[6,7]

A wide variety of disordered systems, in addition to superconducting films, are expected to exhibit overall percolative structure. Examples include glasses and possibly all amorphous systems.[5,6,8,9] In the last few years a great deal of progress has been made in the understanding of geometric disorder, due in part to the description of percolation in terms of fractal geometry.[9] Wire networks are particularly useful models for observing the effects of inhomogeneity, since superconducting properties are quite sensitive to dimensionality.[8,10–14]

To date, studies of wire networks have concentrated on pure fractal geometries,[8,10] and periodic [12,13] and quasiperiodic[14] lattices. Here we study triangular arrays of Sierpinski gaskets (SG) which, like naturally occurring examples of percolation, exhibit both fractal and 2D regimes. We find that these arrays exhibit a crossover as the appropriate probe length, ξ_s, is varied from $\xi_s<\xi_p$ to $\xi_s>\xi_p$ (ξ_p is the correlation length). We contrast the behavior of the gasket arrays with measurements on random, percolation networks, which are also fractal at short length scales.[9] Surprisingly, these seem to exhibit two-dimensional behavior well into the inhomogeneous regime.

The samples were prepared by evaporation of pure Al films onto oxidized Si substrates. The films, 50 nm thick, were deposited directly onto the substrates through a liftoff mask written in two-layer electron-beam resist by a Cambridge model EBMF-2-150 electron beam microfabricator. The gaskets are of third order and in all other respects are identical to those reported upon earlier.[8,10] Each gasket sits on a site of a 100×100 triangular lattice (Fig. 1). The percolation networks are formed from an 800×800 square lattice of wire bonds which are present with probability p (missing with probability $1-p$) (Fig. 2). For both sample types the wire widths are approximately 0.3 μm and the lattice size is $a(\mathrm{SG})=a(\mathrm{perc})=1.7$ μm. The normal-state resistance of the samples was between 1 and 100 Ω, while the superconducting coherence length in all samples was $\xi_s(0)=0.22$ μm, determined by the large-field ($Ha^2=\phi>\phi_0$), 1D phase boundary.[8,10]

We begin an analysis of the superconducting phase boundary by considering the two-dimensional (2D), or homogeneous, case. To emphasize its definition as a diffusion length, the superconducting coherence length may be written

$$\xi_s^2(T)=D\tau_{\mathrm{GL}}(T), \tag{1}$$

where $\tau_{\mathrm{GL}}(T)=(\pi\hbar/8k_{\mathrm{B}})(T_c-T)^{-1}$ is the Ginzburg-Landau lifetime for the Cooper pairs and D is the diffusivity. Substitution of (1) into the definition of the perpendicular critical field, $H_{c2}=\phi_0/2\pi\xi_s^2$, immediately yields $H_{c2}(T)=(4k_{\mathrm{B}}c/\pi eD)(T_c-T)$ which is the usual

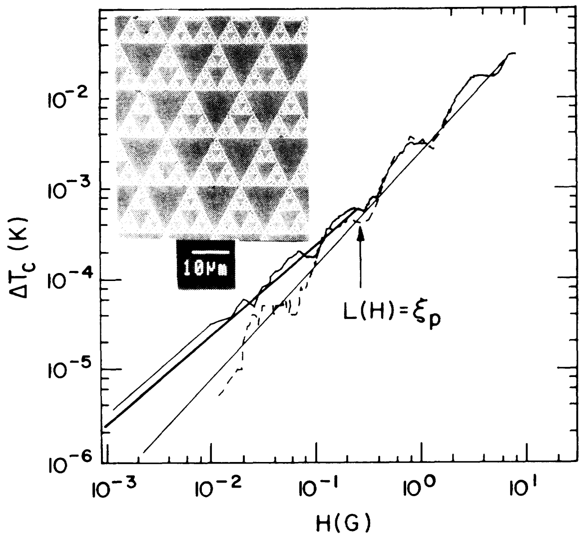

FIG. 1. Phase boundaries of the SG array (solid curve) and the pure SG (dashed curve). The straight lines have slope $m=1$ for the gasket array and $m=1.161$ for the SG. A section of the array is shown in the inset. An arrow marks the point where $L(H)=\xi_p$.

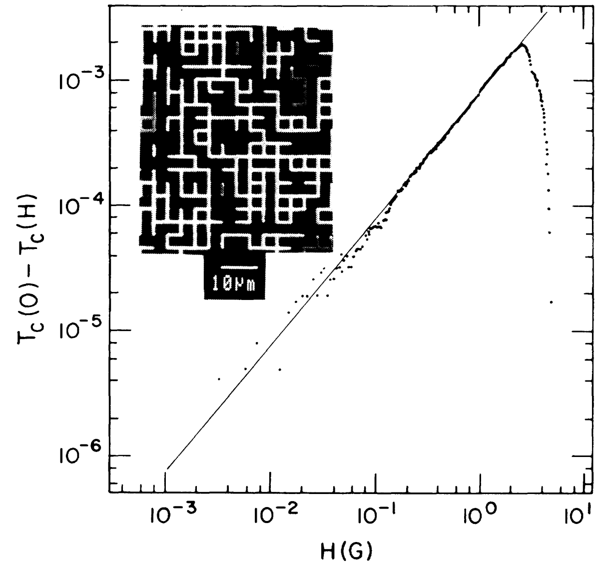

FIG. 2. Phase boundary of the $p=0.56$ percolation network. The points are data and the solid line is a regression fit with slope of 1.00. The network is pictured in the inset.

result for the phase boundary of 2D films near T_c. In the one-dimensional case, where both the width and thickness of the superconductor are less than ξ_s (i. e., where $W,d<\xi_s<L$), this result must be modified by the exchanging of $\xi_s(T)W/\sqrt{12}$ for $\xi_s^2(T)$ in the definition of H_{c2}.[15] From the expressions for $T_c(H)$ in one and two dimensions, we note that the power-law exponent relating T_c and H is dependent upon dimension:

$$T_c-T=\Delta T_c \sim H^{2/d} \quad (d=1,2). \tag{2}$$

In order to extend (2) to fractal dimensions it is necessary to discuss the effect of fractal geometry on the superconducting coherence length. The homogeneous case is defined by the inequality $\xi_s(T)>\xi_p$, where ξ_p is the correlation length or the size of the fractal regions on the network. In this regime the superconducting order parameter varies only over distances greater than ξ_p and diffusion properties follow the usual Euclidean laws. Specifically, $\xi_s^2(T)\sim D\tau_{\rm GL}(T)\sim(T-T_c)^{-1}$. In the case of the SG array, the correlation length is identified with the size of the third-order gaskets, or $L_3=2^3a=8a$, while for the percolation model the correlation length is less intuitive and is found to be $\xi_p=a(p-p_c)^{-\nu}$,[7] where $\nu=\frac{4}{3}$ is the correlation-length exponent.[16]

Below T_c the coherence length decreases as $\Delta T_c^{-1/2}$ and at a sufficiently low temperature one enters the inhomogeneous region where $a<\xi_s(T)<\xi_p$. Both simulations[17] and analytical theories[18] of diffusion on fractals indicate that normal diffusive behavior is replaced by an anomalous process obeying the following power law:

$$\langle x^2\rangle \sim t^{D_f/\bar{\bar{d}}} \simeq t^{1/(1+\Theta/2)}, \tag{3}$$

where D_f is the fractal dimension and $\bar{\bar{d}}$ is the fracton (or spectral) dimension. The second expression in (3) makes use of the relationship between D_f and $\bar{\bar{d}}$, defining the anomalous diffusion exponent, Θ, where $D\sim L^{\Theta}$.[16] The effect of (3) is to decrease the coherence length via a "slower" diffusion mechanism, $\xi_s^2\sim\tau_{\rm GL}^{1/(1+\Theta/2)}$, and to alter the relationship between H and ΔT_c in (2). The phase boundary now has the form $\Delta T_c\sim H^{1+\Theta/2}$.

At very small values of ΔT_c (equivalently, small fields) we expect a linear relationship between ΔT_c and H. A crossover to the inhomogeneous regime is expected at the point where $\xi_s(T)\simeq L(H)\simeq\xi_p$.[11] For the SG, $\Theta=0.322$,[10] while on the infinite cluster $\Theta\simeq0.8$.[16]

A related consequence of the fractal "disorder" at short length scales is the enhancement of H_{c2} as p approaches p_c. (In our percolation networks it was possible to vary p and thus ξ_p.) Using (1) and (3) one can calculate the enhancement in $H_{c2}(p<1)$ over H_{c2} in the periodic network at the temperature where $\xi_s=\xi_p$. It follows that (in the homogeneous regime) the slope of the critical field will depend upon $\Delta p=p-p_c$ as

$$[dH_{c2}/dT]_{T_c}\sim\Delta p^{-k}, \tag{4}$$

where $k=\nu\Theta$. Taking $\nu=\frac{4}{3}$ and $\Theta(\text{perc})=0.8$ we find $k=1.06$, in good agreement with recent numerical simulations by Simonen and Lopez[19] who found $k=0.93$. Of course, when $\xi_p>\xi_s$ we should expect no dependence of H_{c2} on p.

For purposes of investigating large-scale dimensionality, the most significant feature of $\Delta T_c(H)$ is its form at low fields (i.e., $\phi/\phi_0\ll1$). We have shown above that

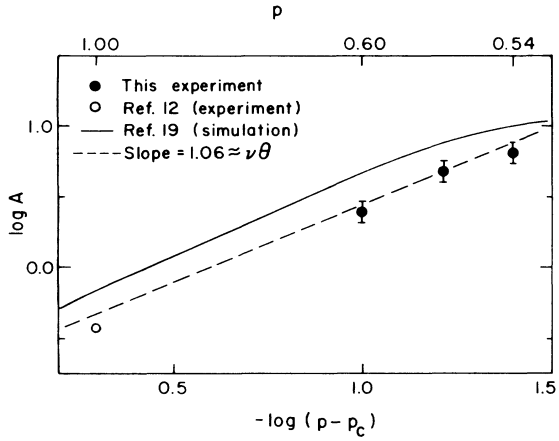

FIG. 3. Plot of $A \sim dH_{c2}/dT$ vs $p-p_c$.

in fractal networks—or in the inhomogeneous regime—one expects power-law behavior of the form $\Delta T_c \sim H^m$ where $m = 1+\Theta/2 > 1$. Here, m is an exponent characteristic of diffusion on the fractal. This power-law behavior has been confirmed in experiments on pure fractals (SG with $\Theta = 0.322$)[10] and periodic networks.[12,13]

Figure 1 shows the phase boundary of the gasket array over a range of fields defined by the inequality 10^{-3} G $< H <$ 10 G. Using the length scale $L^2(H) = (4/\sqrt{3}) \times (\phi_0/H)$, we find that L spans the crossover region, extending from $L(10^{-3}\ \mathrm{G}) \simeq 100a > \xi_p = 8a$ to the inhomogeneous regime where $\xi_s(0) < L(10\ \mathrm{G}) \simeq a < \xi_p$ [$L(H)$ is the length of the side of a triangle which contains one quantum of flux, ϕ_0]. In the inhomogeneous regime the phase boundary is identical to that of a pure SG[10] (Fig. 1). At still smaller fields, specifically for a probe length, $L(H)$, greater than ξ_p, $\Delta T_c(H)$ is larger in the array than in the pure fractal. This is due to a crossover[20] to the homogeneous regime where diffusion is "faster" and where $\xi_s(T)$ grows linearly with τ_{GL}.

It is possible to *estimate* the behavior of the critical field of the $n=3$ order gasket array (in the homogeneous regime) by noting that the transition temperature will vary linearly in the field, $\Delta T_c \simeq H$. Further, as we have noted earlier, the crossover should occur at about $L(H) = \xi_p$. These criteria define the straight line marked in Fig. 1. The magnitude and slope of the data are consistent with this prediction, although the homogeneous region covers less than a decade in temperature.

It is natural to expect comparable behavior for the percolation networks. In fact, for our samples the correlation length is quite long, bounded below by $\xi_p > \xi_{0.6} = a(0.6-p_c)^{-\nu} = 37\ \mu$m. Thus, over most of our range of field and temperature we expect to observe inhomogeneous behavior (i.e., $m = 1+\Theta/2$). It is surprising, then, that we find *only* values of $m = 1.00 \pm 0.03$ for all our values of p (0.54, 0.56, 0.60, and 1.0) and $H \ll \phi_0/a^2$. An example is shown in Fig. 2.[21]

Another test of the "dimensionality" of the percolation network is the magnitude of the critical field. Toward these ends it is convenient to discuss the quantity dH_{c2}/dT. In order to compare our data with the simulation of Ref. 19, we define a parameter A which is proportional to the derivative dH_{c2}/dT:

$$H_{c2}(T) = A\{\phi_0/\xi^2(0)\}(\Delta T_c/T_c), \tag{5}$$

and thus

$$A = \{\xi^2(0)T_c/\phi_0\}[dH_{c2}/dT]_{T_c}. \tag{6}$$

In Fig. 3 we have plotted A as a function of Δp, on logarithmic axes. A appears to vary with p for even our smallest values of Δp. This is consistent with our earlier observations suggesting that the samples are in the homogeneous regime, where A is expected to fall off as Δp^{-k}. The straight line in Fig. 3 has a slope of $k = \nu\Theta = 1.06$, indicating that these data are in good agreement with our expectation, (4).

We have fabricated two different networks which are models for percolative disorder, relevant to the growth of real, superconducting films. Our networks focus on the inhomogeneous nature of percolating films and measurements of the phase boundary in a magnetic field are expected to reflect this. The two sample types are logically intermediate between pure fractal networks and periodic networks, both of which have been studied previously. The SG network combines a well-understood fractal at short length scales with a periodic structure over large distances. The percolation networks add randomness and dead-end bonds in an attempt at a more sophisticated representation of real, percolating systems.

For the SG array, $\Delta T_c(H)$ data show a distinct crossover from fractal to 2D behavior. The critical field (in the homogeneous region) is smaller in the ($n=3$) gasket array than in the pure fractal, consistent with our concept of the effects of disorder on H_{c2}.

The superconducting properties of random, percolating networks are much more difficult to predict, despite geometric similarities with the SG array. The phase boundary, $\Delta T_c \sim H^1$, and the dependence of A on p are strong evidence that the percolation networks are, in some sense, homogeneous at all measured p and H. This is in contradiction with expectations based on the calculated values of ξ_p. This may be due to the complicated structure of the percolation fractal as opposed to the SG. On the one hand, the percolation networks are random, washing out structure in $\Delta T_c(H)$ at small fields (compare Fig. 1 with Fig. 2). In addition, the presence of dangling bonds has been shown to affect the transition temperature of single loops in a magnetic field.[21] Finally, in the presence of a finite magnetic field it may be necessary to treat this as a problem in quantum (not classical) percolation.[19,22,23] Although much is understood about the geometry of inhomogeneous films, it is clear that the superconducting properties are not yet solved.

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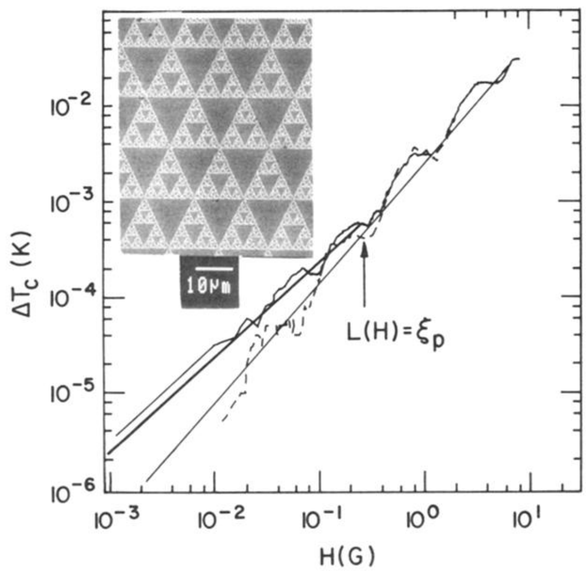

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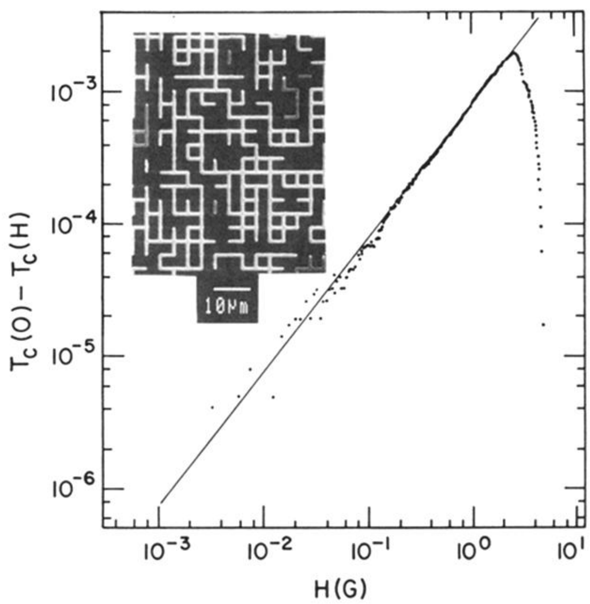

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