Determination of the $pp\pi^0$ Coupling Constant and Breaking of Charge Independence

J. R. Bergervoet, P. C. van Campen, T. A. Rijken, and J. J. de Swart
Department of Theoretical Physics, University of Nijmegen, Nijmegen, The Netherlands
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In a phase-shift analysis of all pp scattering data below $T_{\rm lab}=350$ MeV, where $\chi^2/N_{\rm DF}=1.07$ ($N_{\rm DF}$ is number of degrees of freedom) is reached, the long- and intermediate-range pp interaction has been studied. Using an intermediate-range interaction the Nijmegen potential, we find for the $pp\pi^0$ coupling constant $f_0^2=(72.5\pm0.6)\times10^{-3}$ or $g_0^2=13.1\pm0.1$ and for the π^0 mass 134.7 ± 2.1 MeV. Even when we take account of the model dependence due to the potential tail, this value of f_0^2 is significantly lower than the value of the charged coupling constant in πN scattering, indicating a large breaking of charge independence.

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The coupling of the neutral pion field ϕ to the proton field ψ is described by either the pseudoscalar (PS) or the pseudovector (PV) phenomenological vertex functions $\mathcal{L}_{\rm PS}$ or $\mathcal{L}_{\rm PV}$, where $\mathcal{L}_{\rm PS}=g_0(4\pi)^{1/2}(\bar\psi i\gamma_5\psi)\phi$ and $\mathcal{L}_{\rm PV}=(f_0/m_+)(4\pi)^{1/2}(\bar\psi i\gamma_\mu\gamma_5\psi)\partial^\mu\phi$. To make the PV coupling constant f_0 dimensionless it is customary[1] to introduce in $\mathcal{L}_{\rm PV}$ the charged-pion mass m_+. These different vertex functions give rise to the same one-pion-exchange (OPE) potential between protons, provided that one has $g_0^2=(2M/m_+)^2f_0^2=180.8f_0^2$, where M is the proton mass.

Differences between these two phenomenological vertex functions show up when one looks at the vertex $p\bar p\to\pi^0$. When one considers the underlying quark picture,[2] then it is unbelievable that the vertices $p\to p\pi^0$ and $p\bar p\to\pi^0$ can both be described by these simple vertex functions with the same g_0 or f_0, even if these couplings are modified by form factors describing the spatial extension of the hadrons. Especially $\mathcal{L}_{\rm PS}$ is unbelievable, because it predicts a very strong $p\bar p\to\pi^0$ vertex. That $\mathcal{L}_{\rm PS}$ is unbelievable does not mean that $\mathcal{L}_{\rm PV}$ is correct. We think that both expressions are only valid approximations in a very restricted kinematic domain.

The coupling of the charged pions to the nucleons is described by the charged coupling constant f_c, where $f(pn\pi^+)f(np\pi^-)=2f_c^2$. When one assumes charge independence for the pion-nucleon interaction, then one has $f_0^2=f_c^2$. However, charge independence of the strong interactions is only an approximate symmetry, because it is broken by the presence of the electroweak interactions and by the mass difference between the up and down quarks. In the past it was believed that this breaking of charge independence was small, because it was assumed to be mainly of electromagnetic origin.[3] A recent calculation,[4] where one tries to include also the quark mass difference, gives f_0^2 smaller than f_c^2 by 7% to 10%.

The charged coupling constant f_c is determined rather precisely in πN scattering, where one seems to agree on $f_c^2=(79\pm1)\times10^{-3}$.[1] The best place to determine the neutral coupling constant f_0 is probably in pp scattering. In Table I different determinations of f_0^2 are listed.

The tensor character is an important feature of the OPE potential. In the phase shifts, the long-range OPE tensor potential can best be seen from the tensor combination of the triplet odd waves ($^3P,{}^3F,\ldots$). The first indication for a very low value of the neutral coupling constant $g_0^2\approx13$ or $f_0^2\approx0.072$ we[11] got from a single-energy phase-shift analysis of pp analyzing power (polarization) data at 10 MeV.[12] These data give a much smaller tensor combination Δ_T of ^3P phase shifts than the present best NN potentials, because all these potentials have too large a value of f_0^2. Later studies[13] showed that within a potential model it is impossible to obtain such a small Δ_T with reasonable values for the ρ- and ω-coupling constants and f_0^2 in the neighborhood of 0.079. Later, a much more precise pp analyzing-power experiment[14] at 9.85 MeV confirmed the low value of Δ_T. This experiment gives $\Delta_T=-0.933\pm0.007$, while the Nijmegen soft-core potential[15] gives $\Delta_T=-0.98$ and the parametrized Paris potential[16] gives $\Delta_T=-1.01$.

The value of f_0^2 presented here is a result of our phase-shift analysis of all pp data scattering data with $T_{\rm lab}<350$ MeV.[17] It is a continuation of the lower-energy analyses performed by our group.[10,18] We use a method which is sensitive to the long and intermediate

TABLE I. The neutral coupling constant $10^3\times f_0^2$. (a) and (b) indicate different data sets.

Bugg[a]		75.2 ± 3.9
MacGregor, Arndt, and Wright[b]		81.4 ± 4.6
Breit *et al.*[c]		73.1 – 81.8
Bugg *et al.*[d]		77.8 ± 3.6
Kroll[e]		80.3 ± 2.2
Bergervoet *et al.*[f]	(a)	80.2 ± 6.6
	(b)	74.1 ± 5.5
Present work		72.5 ± 0.6

[a]Ref. 5. [d]Ref. 8.
[b]Ref. 6. [e]Ref. 9.
[c]Ref. 7. [f]Ref. 10.

range ($r > b$) of the pp interaction. This allows us to check in a quantitative way the long- and intermediate-range parts of any pp potential. We used it to check the OPE potential, to determine the neutral-pion coupling constant, and to compare the long- and intermediate-range parts ($r > 1.4$ fm) of the soft-core Nijmegen potential[15] and the parametrized Paris potential.[16]

The data set for $T_{\text{lab}} < 30$ MeV is extensively discussed in Ref. 10; that for $T_{\text{lab}} > 30$ MeV is roughly speaking, a combination of the data sets used in the analyses of Arndt and co-workers[19] and the data lists published by Bystricky and Lehar,[20] whereof the data with too high[10] χ^2 values are rejected. This leaves us with 1234 scattering observables. Of all groups of data, 26 have an experimentally undetermined normalization, so for a correct model without any adjustable parameters one expects the χ^2 value: $\langle\chi^2\rangle = 1208 \pm 49$.

The method of analysis is about the same as in our 0–30-MeV analysis.[10] The lower partial waves (with $J \leq 4$) are parametrized by means of an energy-dependent P matrix at $r=b$ and for $r > b$ a potential tail $V_L = V_{\text{em}} + V_{\text{nuc}}$. Here V_{em} is the electromagnetic potential, consisting of the modified relativistic Coulomb potential[21] and the vacuum polarization potential.[22] The longest-range part of V_{nuc} is the OPE potential

$$V_{\text{OPE}} = \tfrac{1}{3} f_0^2 \frac{M}{E}\left(\frac{m}{m_+}\right)^2 \frac{e^{-mr}}{r}\left[\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + S_{12}\left(1+\frac{3}{mr}+\frac{3}{(mr)^2}\right)\right], \quad (1)$$

where m is the π^0 mass and $E = (M^2+k^2)^{1/2}$ with k the c.m. relative momentum.

In the highest partial waves ($J \geq 10$), which are very insensitive to the short-range interaction, we use the phase shifts due to V_{em} and V_{OPE}, computed in Coulomb-distorted–wave Born approximation. The main part of the phase shifts with intermediate values of J ($5 \leq J \leq 9$) is due to V_{em} and V_{OPE}. A correction is determined from the lower partial waves by optimal mapping techniques.[23]

Since the parametrization of the short-range interaction ($r < b$) is purely phenomenological, the number of P-matrix parameters is determined by the criterion that the description of the data does not improve significantly if one parameter is added. Counting also the pion coupling constant, we need 28 parameters, which is not too different from the number of parameters used in other multienergy phase-shift analyses[19,24] in this energy range.

The long-range interaction depends on f_0^2. In fact, it is this dependence which allows us to determine f_0^2. All realistic models for the pp interaction include the OPE potential as the longest-range part, but they differ in the description of the shorter-range forces which are due to heavier and/or higher-order boson exchange (HBE). Therefore we have included in V_{nuc} the HBE of some modern potential models. Our method of analysis is especially suited to measurement of the quality of potential tails ($r > 1.4$ fm) via the attained minimal χ^2 in the analysis.

As possible choices for V_{nuc} we have considered the following: (i) $V_{\text{nuc}} = V_{\text{OPE}}$. The change in V_{OPE} due to a form factor as in the Nijmegen soft-core potential[15] is of no influence, since its effect is of short range. (ii) $V_{\text{nuc}} = V_{\text{OPE}} + V_{\text{HBE}}^{\text{N}}$, where $V_{\text{HBE}}^{\text{N}}$ is the non-OPE part of the Nijmegen soft-core potential.[15] (iii) $V_{\text{nuc}} = V_{\text{OPE}}^{s} + V_{\text{HBE}}^{\text{N}}$, where V_{OPE}^{s} is the static OPE potential [leaving out the factor M/E in Eq. (1)]. (iv) $V_{\text{nuc}} = V_{\text{OPE}} + V_{\text{HBE}}^{\text{P}}$, where $V_{\text{HBE}}^{\text{P}}$ is the non-OPE part of the parametrized Paris potential.[16]

For each potential tail, the P-matrix parameters and f_0^2, that affects all partial waves, have been adjusted in a least-squares fit to the data. The results for χ^2_{min} and f_0^2 are given in Table II. For the cases (ii), (iii), and (iv) we used $b = 1.4$ fm. Taking only the OPE potential tail in V_{nuc} appeared not to be reasonable for $b = 1.4$ fm. This indicates that HBE forces are not negligible outside 1.4 fm. Therefore we used $b = 1.8$ fm in case (i) and also the number of P-matrix parameters was increased by one to get a more reasonable fit to the data. Even then the description is the least good: The χ^2_{min} remains about 20 higher than with the other tails. The tail of the Nijmegen potential is seen to be somewhat ($\Delta\chi^2 = 6.6$) better than the tail of the Paris potential. The value of f_0^2 found with the Paris potential deviates also (by about 3 standard deviations) from the others, which are very consistent.

The model dependence due to the chosen potential tails gives an estimate for the systematic error in the determination of f_0^2. The energy at which the results for f_0^2 in cases (ii) and (iii) imply the same OPE potential is about 9 MeV, indicating the importance of the analyzing-power data around 10 MeV[12,14] in this determination. This importance can more clearly be seen from a fit to all data minus these analyzing-power data. This raises f_0^2 by about 1.4×10^{-3} and enlarges the error in the determination of f_0^2 by about 50%.

In order to show that we really look at the OPE potential, characterized by its exchanged mass and its specific spin dependence, we have checked the consistency be-

TABLE II. Results for the different potential tails.

V_{nuc}	χ^2_{min}	$10^3\times f_0^2$
V_{OPE}	1288.9	71.9 ± 0.8
$V_{\text{OPE}} + V_{\text{HBE}}^{\text{N}}$	1266.7	72.6 ± 0.6
$V_{\text{OPE}}^{s} + V_{\text{HBE}}^{\text{N}}$	1265.9	72.5 ± 0.6
$V_{\text{OPE}} + V_{\text{HBE}}^{\text{P}}$	1273.3	74.6 ± 0.6

tween different subsets of all partial waves in the determination of f_0^2 and also determined the π^0 mass in the same way as we determined f_0^2.

To save computer time, the tests on the consistency between the partial waves have been done with a matrix representation of the data. The results are for $V_{\text{nuc}} = V_{\text{OPE}} + V_{\text{HBE}}^{\text{N}}$. Introducing different coupling constants for the spin-triplets f_T and for the spin-singlets f_S, we obtain $f_T^2 = (72.5 \pm 0.6) \times 10^{-3}$ and $f_S^2 = (74 \pm 2) \times 10^{-3}$. This result indicates the importance of the spin-triplet waves in the determination. When we next introduce different coupling constants for the ^3P waves $f(^3P)$ and all other partial waves $f(\text{rest})$, we find $f^2(^2P) = (72.2 \pm 0.6) \times 10^{-3}$ and $f^2(\text{rest}) = (73.8 \pm 0.9) \times 10^{-3}$. Also, for the other potential tails, the values from the different subsets of partial waves are rather consistent. We see that the ^3P waves are very important in the determination of f_0^2.

In our judgment the determination of the π^0 mass from the pp scattering data is a crucial test. The mass as well as the coupling constant can be determined from the potential tail, but only the mass is accurately known. We find $m = 134.7 \pm 2.1$ MeV, in complete agreement with the more accurate value $m_0 = 134.9642 \pm 0.0038$ MeV.[25] In Fig. 1 we sketch the χ^2 surface as a function of m and f_0^2. A strong correlation between f_0^2 and m is seen. Because of the correlation, the correct value found for m supports the value found for f_0^2.

Let us summarize and discuss our results. In our study of the long and intermediate range of the pp interaction we find that the data, which are described with

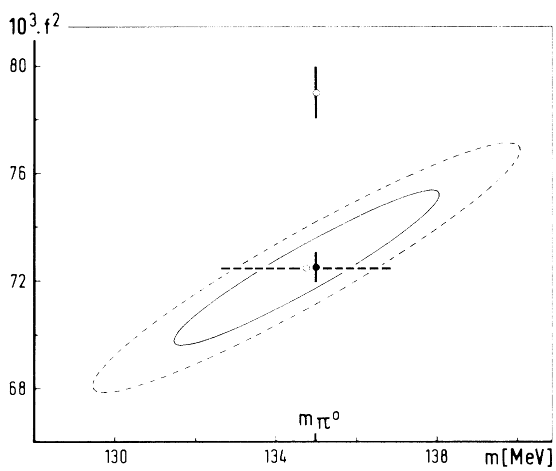

FIG. 1. Ellipses of constant χ^2 in the (m, f^2) plane with optimal adjustment of the P-matrix parameters. Solid ellipse: 69% confidence region ($\Delta\chi^2 = 2.4$). Dashed ellipse: 95.5% confidence region ($\Delta\chi^2 = 6.2$). Filled circle with vertical bar: value and error bar for f_0^2 (with m fixed). Open circle with horizontal bar: value and error bar for m (with free f_0^2). Open circle with vertical bar: value and error bar for f_c^2 from πN scattering.

a $\chi^2/N_{\text{DF}} \approx 1.07$, favor the tail of the soft-core Nijmegen potential[15] over the tail of the parametrized Paris potential[16] by 2.5 standard deviations. Using the tail of this Nijmegen potential for the description of the forces with intermediate range, we find for the neutral pion-proton coupling constant $f_0^2 = (72.5 \pm 0.6) \times 10^{-3}$ or $g_0^2 = 13.1 \pm 0.1$. We quote here the value for the fit with the lowest χ^2_{min}. The error given is purely statistical. From Table II we get an impression of the model dependence of our result, which gives then an estimate of a possible systematic error. No other systematic errors have been found in this analysis, because the results with subsets of all partial waves are consistent and also the mass of the exchanged π^0 is in excellent agreement with its rest mass. Our result for f_0^2 is in fair agreement (see Table I) with earlier determinations, except with the value quoted by Kroll,[9] who used forward dispersion relations. Our value of f_0^2 is smaller and much more precise than these earlier determinations. It deviates significantly[26] from the value $f_c^2 = (79 \pm 1) \times 10^{-3}$ or $g_c^2 = 14.3 \pm 0.2$ for the charged coupling constant. This indicates a large breaking of charge independence or SU(2)-isospin symmetry. This breaking is of the same order of magnitude as a very recent estimate in a simple quark model, where it is due to the mass difference between the up and down quarks.[4] This large SU(2) symmetry breaking of the pion-nucleon coupling constants leads to the expectation of even larger SU(3)-flavor symmetry[27] breaking of the meson-baryon coupling constants.

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