## Comment on "Synchrotron X-Ray Study of a Fibonacci Superlattice"

Todd *et al.*[1] have presented and discussed the diffraction pattern of a quasiperiodic superlattice made by the deposition of layers of GaAs and AlAs of two thicknesses in a Fibonacci sequence. Despite deviations from the ideal Fibonacci structure due to fluctuations in the number of atomic planes in the layers, diffraction peaks characteristic of the ideal structure were observed. This was conjectured to be due to some unusual property associated with the quasiperiodic lattice. We show below that an effect which occurs in periodic superlattices[2] occurs also in Fibonacci superlattices and is responsible for the persistence of the Fibonacci peaks in the diffraction pattern.

Consider a Fibonacci superlattice in which the two layers differ in thickness but have the same charge density $\rho(z)$ in the direction of layering. The total charge density is $\rho(z)h(z)$, where $h(z)$ is the layer distribution function of the Fibonacci superlattice. The Fibonacci superlattice peaks in the diffraction pattern occur through $|D(Q)|^2$, the Fourier transform of the Patterson function or self-convolution of $h(z)$. A generalization of Eq. (6) of Ref. 2 leads to

$$|D(Q)|^2=\sum_{n_1=1}^{N}\sum_{n_2=1}^{N}\mathcal{N}_n\cos[(z_{n_1}-z_{n_2})Q]\times\left[1+2\sum_{m=1}^{\infty}\exp\left(-\frac{m^2d^2}{n\sigma^2}\right)\cos(mdQ)\right]$$

for a Fibonacci superlattice of $N$ layers when there are discrete fluctuations which are multiples of an atomic plane spacing $d$. $z_l$ is the position of the $l$th layer of the perfect Fibonacci lattice, and $n=|n_1-n_2|$.

In Fig. 1 are plots of the above transform for a lattice of thirteen layers of thicknesses 37 and 59.867 Å. The thickness ratio is the golden mean, $\tau=(1+\sqrt{5})/2$. In a manner entirely analogous to the periodic case,[2] Fibonacci peaks near multiples of $2\pi/d$ are unaffected by discrete fluctuations while away from these regions the peaks are damped and broadened.

We contend that the effect manifest in Fig. 1(b) is responsible for the persistence of Fibonacci peaks in the diffraction pattern of Ref. 1. We have chosen the layer thicknesses and the atomic plane spacing to correspond with the actual GaAs-AlAs Fibonacci lattice of Ref. 1. Consequently the regions of persistent peaks at 2.2 and 4.4 $Å^{-1}$ correspond to the regions labeled 002 and 004 in Fig. 2 of Ref. 1. These regions are where persistent Fibonacci peaks were observed.

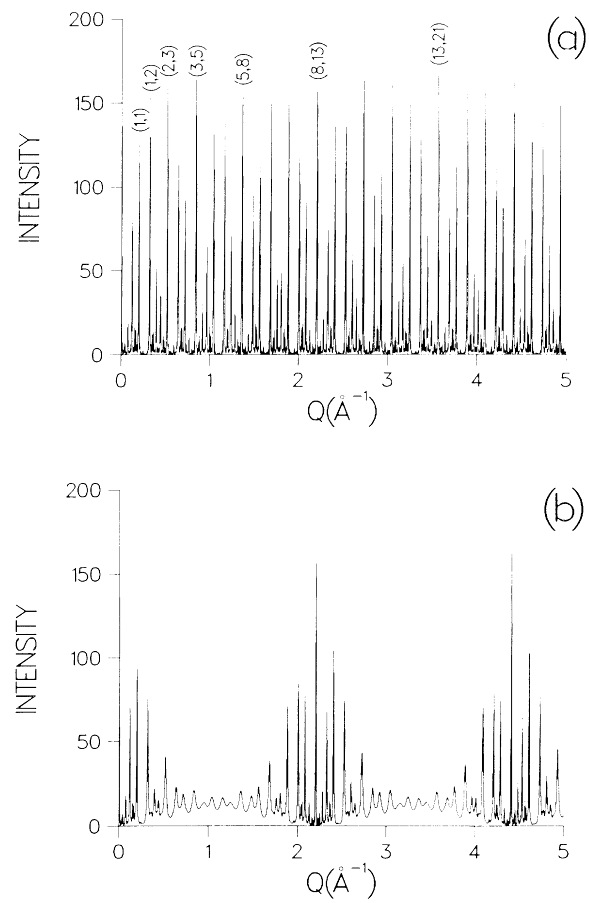


FIG. 1. (a) Transform of the Patterson function of a perfect Fibonacci lattice ($\sigma=0$) with prominent peaks labeled by sequential Fibonacci number pairs. (b) Transform with discrete fluctuations ($d=2.83$ Å, $\sigma=2.83$ Å).

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[2]B. M. Clemens and J. G. Gay, Phys. Rev. B **35**, 9337 (1987).