Nonlinear Nonreciprocity and Directional Bistability in a Ring Resonator with a Quadratic Nonlinear Medium

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The mutual interaction of counterpropagating waves in a ring resonator containing a medium with second-order dispersive nonlinearity is studied. The theoretical treatment of nonreciprocal propagation is based upon the novel coupling of counterpropagating and mode-locked first- and second-harmonic waves. The experimentally observed transfer characteristics of a high-frequency model system reveal a directionally asymmetric bistability. This nonreciprocity can also be attributed in the time domain to the occurrence of dissipative structures, i.e., to solitons traveling within the resonator.

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The intense study of optical bistability in nonlinear optics has offered new facilities for all-optical signal processing devices.[1] The so-called nonlinear nonreciprocity[2–8] which has recently been investigated seems to be especially relevant for novel unidirectional devices. Up to now, it is known that in case of a saturable[2] or Kerr-type[3–8] nonlinear medium a cross interaction of counterpropagating waves occurs and, in consequence, the propagation constants for forward- and backward-traveling waves of different amplitudes are not equal. In connection with a passive ring resonator filled with such a nonlinear material, this nonlinear nonreciprocity may lead to directionally asymmetric bistability.[8]

Recently, we have shown that via second-harmonic generation a quadratic nonlinear medium can behave similar to a material with a $\chi^{(3)}$ nonlinearity which, for example, yields an amplitude-dependent index of refraction.[9,10] By use of a high-frequency transmission line as the experimental model system,[11] it has been demonstrated that this process also results in dispersive bistability in case of Fabry-Perot[9] and ring resonators.[10] The purpose of this Letter is to study nonlinear nonreciprocity and directional bistability in materials with a $\chi^{(2)}$ susceptibility, which provide one of the fastest electronic nonlinearities which are available. We present a novel and general theory, based on interacting harmonics, study for the first time the influence of the relative phases of forward and backward waves, and consider spatially limited systems. Experimentally, we report on the first observation of directionally asymmetric bistability. The presented system allows further new views into the phenomena of nonreciprocity because a key position is here taken by the propagation of solitons. The significance of the results to other fields is finally discussed.

The theory is based upon the following Boussinesq equation for the electric field E in the (x,t) laboratory system[11]:

$$E_{xx}-(1/u_0^2)E_{tt}=-(\chi/2u_0^2)(E^2)_{tt}-\kappa E_{xxtt}+aE_t-bE_{xxt}. \tag{1}$$

Here the subscripts denote partial derivatives; u_0 is the small-signal phase velocity at low frequencies; χ, κ, a, and b denote the parameters of nonlinearity, dispersion, and frequency-independent and frequency-dependent losses. We now examine the case of counterpropagating waves and intend to derive a nonlinear dispersion relation, i.e., a relation among phase constant, frequency, and field amplitude of forward- and backward-traveling waves. For that purpose and on the supposition of lossless second-harmonic generation $(a=b=0)$ the usual *Ansatz* of interacting first and second harmonics[12,13] is extended to the case of standing waves yielding

$$E(x,t)=\mathrm{Re}\left[\sum_{\mu,\nu=1}^{2}\hat{A}_{\mu\nu}(x)\exp\{i[(-1)^{\mu-1}k_\nu x+\phi_{\mu\nu}(x)+\nu\delta_{2\mu}\phi_0-\nu\omega t]\}\right], \tag{2}$$

where $i^2=-1$, $\delta_{2\mu}$ is the Kronecker symbol, the subscripts $\mu=1,2$ denote forward and backward waves, respectively, and $\nu=1,2$ designate first and second harmonics, with complex envelope functions $\hat{A}_{\mu\nu}(x)\times\exp[i\phi_{\mu\nu}(x)]$. ω is the fundamental angular frequency; $k_\nu=k(\nu\omega)$ are the phase constants of the first and second $(\nu=2)$ harmonics, where the small-signal dispersion law is given by

$$k(\omega)=\omega/[u_0^2(1-\kappa\omega^2)]^{0.5}. \tag{3}$$

Moreover, ϕ_0 is the initial phase difference between forward and backward fields. In the following it is assumed that the variation of the amplitudes along distances of the order of a wavelength is small, for example $|\hat{A}_{\mu\nu xx}|\ll k_\nu|\hat{A}_{\mu\nu x}|$, and that for each direction the first and second harmonics form a mode-locked stationary wave. As a result of the latter assumption, the phase velocities of these harmonics are equal[9,10] and thus there is no change in their relative phase difference yielding

$$2[k_1+(-1)^{\mu-1}\phi_{\mu 1x}]=k_2+(-1)^{\mu-1}\phi_{\mu 2x}. \tag{4}$$

On insertion of Eq. (2) into Eq. (1) and with the help of Eqs. (3) and (4) and an analogous procedure as described by Marburger and Felber,[14] the following coupled mode equations for counterpropagating harmonic waves can be deduced:

$$\hat{A}_{\mu 1x}=\tfrac{1}{4}\chi k_1[-\hat{A}_{\mu 1}\hat{A}_{(3-\mu)2}\sin 2\theta+\hat{A}_{(3-\mu)1}(\hat{A}_{\mu 2}-\hat{A}_{(3-\mu)2})\sin\theta], \tag{5a}$$

$$\hat{A}_{\mu 2x}=-\tfrac{1}{8}\chi k_2(\hat{A}^2_{(3-\mu)1}\sin 2\theta+2\hat{A}_{11}\hat{A}_{21}\sin\theta), \tag{5b}$$

$$\phi_{\mu 1x}=\tfrac{1}{4}(-1)^{\mu}\chi k_1[\hat{A}_{\mu 2}+\hat{A}_{(3-\mu)2}\cos 2\theta+\hat{A}_{(3-\mu)1}/\hat{A}_{\mu 1}(\hat{A}_{12}+\hat{A}_{22})\cos\theta], \tag{5c}$$

$$\phi_{\mu 2x}=\tfrac{1}{8}(-1)^{\mu}\chi k_2(1/\hat{A}_{\mu 2})(\hat{A}^2_{\mu 1}+\hat{A}^2_{(3-\mu)1}\cos 2\theta+2\hat{A}_{11}\hat{A}_{21}\cos\theta), \tag{5d}$$

with

$$\theta(x)=2k_1x+\phi_{11}(x)-\phi_{21}(x)-\phi_0. \tag{6}$$

First, from Eqs. (5) it can easily be seen that an averaging over a few spatial periods according to Ref. 14 would lead to a decoupling of forward and backward waves. The system of coupled equations (5) then collapses to two identical and independent systems as usually discussed; cf. Ref. 13. In that case, a nonlinear dispersion relation can be derived[9]; and on the assumption of a small nonlinear index $\chi\hat{A}_{\mu 1}/\kappa\omega^2$, the nonlinear contribution $\phi_{\mu 1x}$ of the phase constant of the first harmonic is proportional to $\hat{A}^2_{\mu 1}$, a result which is identical to the case of cubic nonlinear media. That means that via the second-harmonic interaction, quadratic nonlinear media cause a self-interaction similar to cubic nonlinear materials.

However, if in an experimental system the length of the nonlinear section is limited—for example, on the order of the wavelength—the above approximation is not valid. Then without averaging a net contribution remains and a cross effect between forward- and backward-traveling waves takes place. From our numerical calculations it is concluded that Eqs. (5) predict a pronounced oscillatory behavior of ϕ_{11x} and ϕ_{21x}, with a period of one-half of the wavelength of the fundamental frequency which is expected from the standing wave. Thus the cross interaction is based on the formation of a nonlinear index grating. As a further result, the interaction results in a clear difference of the phase velocities for two directions provided that the magnitudes of the amplitudes of the counterpropagating waves are different. On the other hand, this behavior can also be judged directly from Eq. (5c) by use of $\theta(x)=2k_1x$. It should, however, be noted that the wavelength is slightly enlarged by nonlinear interaction.[9,10] The numerical calculations also show that another interesting case arises if $\phi_0\neq 0$; then the position of the standing wave is shifted with respect to the nonlinear medium of finite length. Consequently, the average nonlinear phase shift is also altered by the phase offset ϕ_0. In conclusion of the theory, in case of a short medium, a second-order nonlinearity can also lead to a nonreciprocal propagation as has been discussed in the very beginning. Similar to the mechanism described by Kaplan,[8] it may then be possible that as a result of a kind of instability the amplitudes of the counterpropagating waves within a resonator become different even if the system is pumped by signals of equal amplitude. In the following these ideas are compared with first results from measurements with a high-frequency model system.

As represented in Fig. 1, we consider a bidirectional ring resonator partly containing the nonlinear medium. The incident beams of opposite directions are of equal amplitude $\hat{A}_{\rm in}$. $\hat{A}_{\mu\rm out}$ $(\mu=1,2)$ are the output amplitudes, measured at the fundamental frequency, of the forward and backward waves, respectively. Moreover, the relative phase of the two input signals can be controlled externally. Experimentally, we use a high-frequency model system where the ring consists of a short nonlinear electrical transmission line[11] and a linear section, i.e., a common 50-Ω coaxial cable; cf. Ref. 10. By means of directional couplers, both input signals are fed into the ring and in the same way the two output signals arising from the forward and backward waves inside the resonator can be measured.

In the first experiment to be described, the fixed frequency of the incident waves corresponds to the first natural mode of the resonator. In the symmetric case as shown in Fig. 2(a), i.e., if $\phi_0=0$, the output versus input amplitude characteristics of forward and backward propagating waves qualitatively exhibit the same behavior. The hysteresis cycles indicate the range of the input amplitude where bistability occurs. In case of a phase

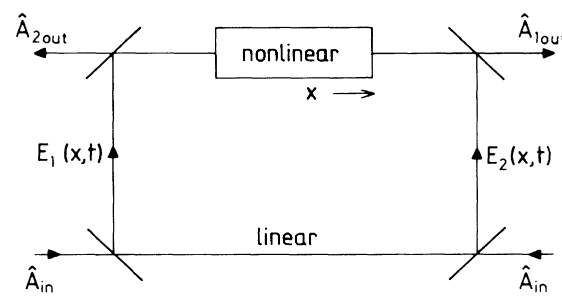

FIG. 1. Nonlinear bidirectional ring resonator where $E_\mu(x,t)$, $\mu=1,2$, denote the forward and backward waves, respectively (see also text).

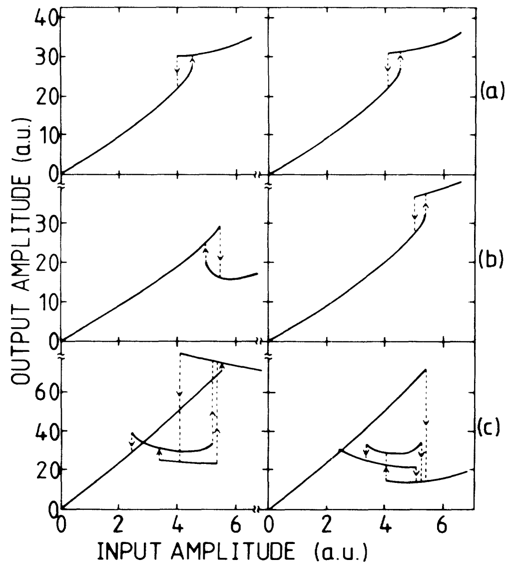

FIG. 2. Experimental output vs input amplitude characteristics of the forward (left column) and backward (right column) waves in case of the high-frequency resonator: (a) directionally symmetric bistability, $\omega/2\pi=48.0$ MHz, $\phi_0=0$; (b) asymmetric bistability, $\omega/2\pi=48.0$ MHz, $\phi_0=0.48\pi$; (c) asymmetric multistability, $\omega/2\pi=142.5$ MHz, $\phi_0=0.71\pi$.

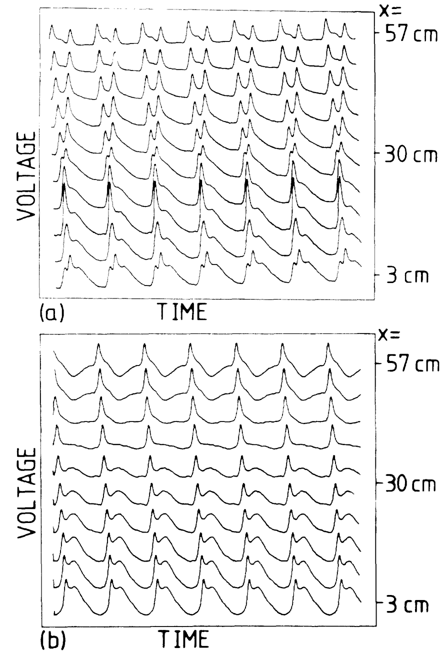

FIG. 3. Temporal wave forms as measured at different positions along the nonlinear part of the high-frequency ring resonator: (a) symmetric regime; cf. Fig. 2(a); (b) asymmetric regime; cf. Fig. 2(b). The traveling solitons can easily be detected by an oblique inspection of the figure.

difference of $\phi_0 \simeq \pi/2$, the situation is quite different; cf. Fig. 2(b). Apart from the fact that the output level is increased to higher values, the backward wave (right column) still shows a behavior as in Fig. 2(a). On the other hand, the output signal of the forward-traveling wave (left column) no longer jumps from a state of low to a state of high transmission, but now the hysteresis cycle is run in an opposite way. Thus these transfer curves portray a clear directionally asymmetric bistability.[8] In addition, Fig. 2(c) illustrates that this feature also occurs when the system is multistable. For that purpose we have measured the transmission properties by using the third natural mode which leads to a pronounced multistability as has previously been observed in a resonator; cf. Gasch, Wedding, and Jäger.[15] Now, in case of the bidirectional ring, the transmission is again asymmetric when appropriate control parameters are used [Fig. 2(c)]. Here up to four distinct states of transmission can be observed at a fixed input amplitude.

From unidirectional ring resonators with a quadratic nonlinear medium, it is known that in case of bistability and multistability the different states of transmission can be described by characteristic soliton modes.[10,15] Recently, we have shown that, in the presence of dissipation and above a threshold value, these stationary dissipative structures are parametrically generated and amplified by a pump wave.[16] In the following it is examined whether this idea is also applicable to standing waves and whether a detailed description of the interaction of counterpropagating waves inside the resonator can also be given in the time domain.

As an experimental result, Fig. 3 shows the evolution of the field measured at different positions along the nonlinear part of the bidirectional ring resonator. First, in the symmetric regime and considering the state of high transmission [cf. Fig. 2(a)], two counterpropagating solitons exist as is illustrated in Fig. 3(a). Apparently, the pulses pass through each other without losing their identities, thus indicating the fundamental properties of solitons.[11] Second, in Fig. 3(b) the case of directionally asymmetric bistability is studied where the output power of the forward wave is low and that of the backward wave is high [cf. Fig. 2(b)]. Clearly, a harmonic standing wave and a soliton moving to the left, i.e., into the direction of the backward wave, are observed. Hence it is found from the experiment that in this case two contradirectional harmonic pump waves are moving in the resonator where only one of them has generated a soliton. It is obvious that this situation leads to asymmetric transfer characteristics.

In summary, we have studied the interaction of counterpropagating waves in a medium with second-order nonlinearity. Theoretically, the *Ansatz* of first- and second-harmonic waves yields a set of coupled mode equations. As a result, it is found that in a medium of finite length, the standing wave gives rise to a cross interaction between forward- and backward-traveling waves. This leads to a spatially dependent phase shift[17] which can additionally be controlled by the relative phases of the input signals. It has yet to be verified whether an energy transfer can also occur as in the case of photorefractive media.[18-20] Application of this nonlinear nonreciprocity to a ring resonator brings first experimental evidence of directionally asymmetric bistability and multistability. That nonreciprocal behavior, as well as the switching in case of bistability, can also be interpreted on the basis of coherent structures, solitons, in a driven dissipative system. Thus the results of this Letter which also apply to Fabry-Perot resonators may be interesting for the fields of nonlinear fiber gyroscopes[6,7] and the soliton laser.[21] Further implications are foreseen for similar soliton-carrying systems such as Toda lattices and molecular chains and to the second-harmonic generation in resonators. Nonlinear nonreciprocity is especially interesting in integrated optics[22] and integrated microwave techniques,[23] where a second-order nonlinearity may easily be realized.

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