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Ground-State Instability of a Random System

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The problem of finding the best path in a random medium is investigated. If the random medium is allowed to undergo slow drifts, the best path can be drastically altered. The scaling of the excited states is also discussed. A host of new exponents are found for the 2D problem. The implications for growing surfaces are also pointed out.

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The following question is asked: In a system with quenched random disorders, if for some reason these disorders are no longer constant but undergo very slow drifts, how (un)stable are the corresponding ground states? For a 2D spin-glass model, a partial answer was recently given by Bray and Moore.[1]

It is plausible that the quenched disorders are themselves dynamical variables, albeit on very long time scales and with very small amplitudes. The reasons may be, e.g., aging and deterioration, interactions among the disorders, feedback reactions from the "fast" dynamical variables, strain release, etc. Ground states of random systems are generally very susceptible to these small changes in the random environments, since there are states whose energies are very close to that of the ground state, but the configurations can be very different. Finding the ground state of a random system is an optimization problem. In a broader sense, a similar question can be asked: If the input constraints of an optimization problem are slightly varied as a result of, e.g., compromises (where they are allowed!), limited precision in specifying the constraints, etc., how much different would the final solution be?

In this Letter I apply the above ideas to a well-studied model: directed polymers in a random matrix. The problem can be reduced to the following: In a 2D square lattice each bond is given a random value (energy). Let one end of every path be fixed at the origin and all the paths be extended in the direction which is called t (Fig. 1). The constraint is that the paths cannot go backwards in t. The task is then to find the path with lowest energy (best) among all possible path configurations. The technique which I use in this study is called the zero-temperature transfer-matrix method, which has been discussed previously.[2,3] At the level t, the best path can end anywhere in x and the displacement was shown[2] to scale as $t^{2/3}$. If we specify also the endpoint at the level t, say, at x_0, then there is also an optimal path with two ends specified. In this manner to every point x at the level t there is associated a local optimal path that started from the origin. The best path is then chosen from all these local optimal paths. The ensemble of these was shown to have a ultrametric family-tree structure.[3]

Let us prepare two sets of random disorders. Denote collectively as **R** a set of random values over all the

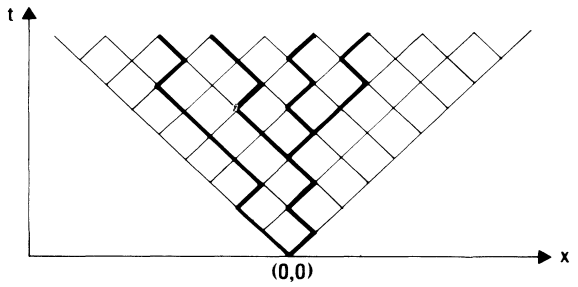

FIG. 1. In this square lattice each bond is given a random value; at the level t there are t points and each of them can be connected to the apex of the triangle in a unique optimal way. A few local optimal paths are shown; one path is the overall best path.

bonds on the lattice. Let $\mathbf{R}_{1,2}=\mathbf{R}_0+\Delta\mathbf{R}_{1,2}$, where the $\mathbf{R}_0$ are distributed uniformly between 0 and 1, while the $\Delta\mathbf{R}$ are between 0 and δ ($\delta \ll 1$). This procedure ensures that the two sets have an identical distribution, with large overlaps (strong correlation). In the two different random environments it is found that two corresponding best paths (located at x_1 and x_2, respectively) can be different. In Fig. 2, I present the mean displacement (average over $|x_1-x_2|$, including cases $x_1=x_2$). We see that the mean displacement scales as t^{α}, $\alpha=0.83 \pm 0.02$. The statistical errors are smaller than the size of the symbols; the quoted errors are only from a subjective assessment. Incidentally, I have also checked that in the finite-temperature version of Kardar,[4] the mean displacement resulting from two different temperatures rather than different sets of disorders scales as $t^{2/3}$. I have observed that the above displacement is linearly proportional to δ, for small values of δ. For δ increasing to a large value, say, 2, i.e., when the correlation between the two sets becomes small, α approaches $\frac{2}{3}$, a value that would apply when the two sets are completely independent. There appear to be two regions, for small and large δ, respectively. For small δ, when the correlation is strong, I believe that the exponents are universal. In the inset of Fig. 2 I present the probability that a jump actually happens ($x_1 \neq x_2$). It can be seen that a small perturbation of strength δ will probably destabilize the ground state. This probability increases very slowly with the length of the paths; in fact, it is consistent with a logarithmic law. For a wide range of δ (0.001–0.8), the results are qualitatively the same. This implies that although the destabilization can happen, there is also a resistance to it; i.e., there is a strong memory effect (in Hopfield's sense?) and it is losing only logarithmically with the length t. In any case, the events $x_1 \neq x_2$ do occur with finite probabilities.

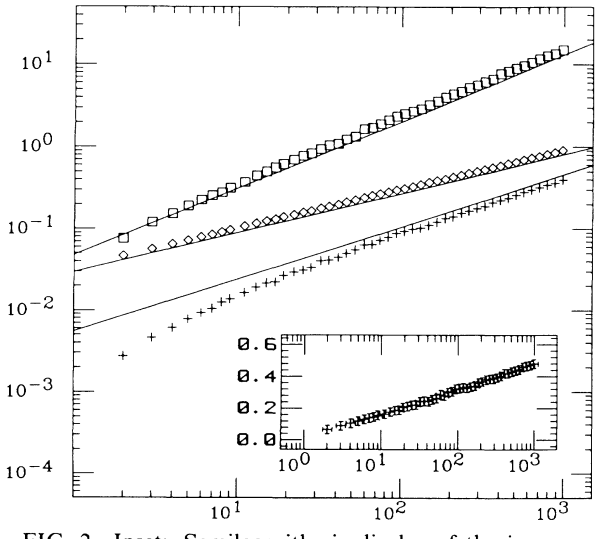

FIG. 2. Inset: Semilogarithmic display of the jump probability ($x_i \neq x_2$ events). Upper curve: The mean jump distance, and a line with a slope 0.83 guides the eye. Middle curve: The mean energy difference of the same path (old best path) in two different environments; the line has a slope of 0.48. Lower curve: The mean energy gain of the best path with respect to the old best path when both are in the new environment; the line has a slope 0.65.

We can understand the large value of α qualitatively in the following way. In Ref. 3 it is shown that the ensemble of the paths has a family-tree structure. If the random environment changes, it is unlikely for the best path to jump a small distance, since in its neighborhood the paths are strongly correlated, i.e., they have a long ancestry in common. Thus the relatives of the best path are unlikely to take advantage of the changes in the environment. That is, the best path is stable with respect to its own family and this is why there is a resistance to the jump. On the other hand, the other families of paths can win over the present best path in a new environment. They have little overlap with the original best path and it is more likely for them to accumulate enough energy gains, and, of course, they are usually located far away. We thus learn that the best path either stays where it is or jumps a long distance away. The above exponent is the result of these two competing factors. It is also illuminating to let $\mathbf{R}$ undergo a continuous random walk; I have observed that the best path has an intermittent behavior of the Levy flight type.

In a new environment the best path may still be a local optimal path. Since we have only the information of the end point x of an optimal path, we would like to know if all the parts of the best path stay in their original positions. In Fig. 2 I also plot the energy change of the same path in the different environments; it scales approximately as $t^{1/2}$. This implies that the local optimal path now overlaps with the old best path, at least predominantly. It is then natural to ask what is the energy advantage, in the new environment, to motivate a jump of the best path (when jumps do not occur, the advantage is zero, and this case is also counted). In Fig. 2 we see that the energy advantage scales against t with an exponent close to $\frac{2}{3}$. Unfortunately, I do not have a convincing argument at present to explain this exponent or to relate it to other exponents.

What are the possible requisites that make a local best path a would-be new best path? Is the candidate path in the old environment one of the lowest excited states, which are defined as those nearest to the best path in energy? Thus we are led to examine the lowest excited states. In the following analysis, I do not change the random environments. I first check the energies of the lowest excited states with respect to the ground-state energy. In Fig. 3 we find that the first two energy gaps decrease slowly with increasing path length. Then it seems that they approach separate constant values, though, numerically, one cannot exclude very small logarithmic

corrections. It is likely that other (low) excited states behave similarly and it appears that we are dealing with a problem with discrete spectra. Thermal fluctuations would have to overcome these finite energy barriers to destabilize the ground state. Next let us ask how far away these lowest excited states are located from the ground state. In Fig. 3 we see the mean displacements of the first two excited paths from the best path. They scale against the path length with an exponent very close to $\frac{1}{3}$. Intuitively, one can understand this in the following way: Immediate neighbors of the best path are too few to be chosen as lowest excited states; because of the family-tree structure, we expect that lowest excited states are among the relatives of the best path in an enlarged neighborhood. However, I cannot explain why it is with that exponent nor whether all the energy gaps should approach constants.

The above results allow us to define the family concept more precisely and I advance the following conjecture: All the local optimal paths are grouped into families; each family has its characteristic energy (i.e., differences of the energies are constant, and thus one energy value is enough to represent the family); the size of the family grows as $t^{1/3}$ and this in turn implies that the total number of the families grows as $t^{2/3}$, the total number of the local optimal paths being proportional to t. From Ref. 3 we learn that the family tree is hierarchical: Families are descendants of superfamilies and so on and, in fact, they all have the same ancestor (the origin). Thermal fluctuations will affect the best path in its own family, a stronger fluctuation (at a higher temperature) will probe the relatives of this family, and so on. I have performed some preliminary finite-temperature simulations in which I have calculated the relative fluctuation $\langle (x-\langle x\rangle)^2\rangle$, where the inner angular brackets denote the thermal average,[4] and then there is an average over independent samples. For strong disorders (or low temperatures), this quantity clearly deviates from $t^{1/2}$ with an exponent between $\frac{1}{3}$ and $\frac{1}{2}$. I do not know if the zero-temperature result $\frac{1}{3}$ would hold for finite temperatures or if there are long transients.

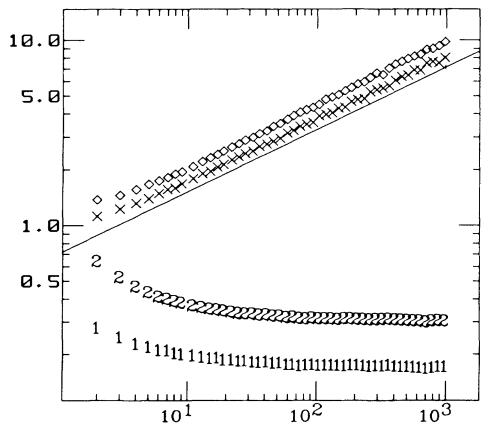

FIG. 3. The upper two curves are the mean distance of the first two excited-state paths from the ground-state best path (first and second going upwards); the line has a slope $\frac{1}{3}$ and it serves as a guide to the eye. The two lower curves are the first two corresponding energy gaps.

To summarize I note that the thermal fluctuations will likely influence the best path inside its family. The family size and the number of the families grows as $t^{1/3}$ and $t^{2/3}$, respectively. If the random disorders undergo slow drifts, to a new random environment slightly different from the old one, the best path has resistance to a jump since its family members or close relatives are unlikely to compete with it (family protection), or otherwise a completely different rival family which is normally far away wins, and thus a long jump of the best path occurs. On average, the jump distance scales against t with an exponent close to 0.83, and the energy advantage of the winning path with respect to the old best path scales with an exponent close to $\frac{2}{3}$.

The directed-polymer problem was shown to be equivalent to the Eden growing-surface problem.[5] The results of this Letter have direct consequences on the surface growth processes: Suppose that there are two surfaces growing simultaneously; the stochastical noises for the two surfaces are synchronized everywhere and at any time. Let us introduce small distortions in the synchronization (the communications are imperfect); after a long time we shall have two similar but different surfaces. If one asks how the relative distance between the highest peak in surface 1 and the highest peak in surface 2 scales against time, the above exponent will account for that. The exponent $\frac{1}{3}$ for the excited states governs the expansion of the spatial coherence region.

Another possible consequence of this study regards noise amplification in random systems. The directed-polymer problem in 2D is equivalent to the domain-wall problem of a random-bond Ising model. If the random bonds undergo small and slow drifts, the domain wall can change its position drastically. I expect that there will be large observable noises in measurements, e.g., electrical conductivities, magnetizations, and optical properties. Thus it is seen that these small and slow noises in the random backgrounds act as modulation forces and they are capable of inducing large observable noises. This may be especially true at the low-frequency ends; some recent studies[6] attribute the $1/f$ noises of random systems to slow random disorder modulations.

I expect that in more general random systems the question raised in this work can also be asked. Immediate extensions should be readily carried out for domain walls and directed polymers in higher dimensions, random-field domain structures, and spin-glasses. From the example analyzed in this work, we learn that, in general, instabilities caused by perturbations of the quenched random disorders and by thermal fluctuations

(e.g., by global changes of the temperature) are quite different. The position of the domain wall can be very different under a new set of random disorders. Careful analysis of this sort for the above-mentioned extensions and other optimizational problems would deepen our understanding of random systems.

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