Simple Predictive Model for Flavor Production in Hadronization

C. D. Buchanan and S. B. Chun
Department of Physics, University of California, Los Angeles, Los Angeles, California 90024
(Received 6 July 1987)

We have found a simple powerful *Ansatz*, interpreting the Lund symmetric-fragmentation function as a universal production density, which predicts multiplicities and momentum and P_T^2 distributions of the many meson and baryon flavors observed in e^+e^- collisions at $E_{\text{c.m.}} \simeq 30$ GeV almost perfectly with only two "natural" parameters of the Lund symmetric-fragmentation function. The model depends only on hadron masses and not on quark-level suppression factors such as s/u or qq/q. At a minimum, it provides an easy basis for the prediction of particle production rates and distributions, and may provide insight into the actual production mechanism.

PACS numbers: 13.65.+i, 13.87.Fh

Figure 1 depicts the general situation for an e^+e^- annihilation in which the virtual photon couples to a primary quark-antiquark pair. This in turn produces a "color field" (to use QCD language) from which virtual $q\bar{q}$ pairs are produced as the system evolves into the observed hadrons. The salient experimental features are as follows: (1) The hadrons have limited P_T relative to the primary $q\bar{q}$ axis, (2) production of high-mass hadrons is suppressed, (3) spin counting is important (e.g., ρ/π production is enhanced by a factor of 3), and (4) there is a longitudinal distribution favoring low momenta. Presumably the P_T and high-mass suppression may occur either on the quark level of the virtual $q\bar{q}$ pairs or on the final-hadron level. In addition, in the QCD description, the self-coupling of the many emitted soft virtual nonperturbative gluons typically leads to a narrow high-field color flux tube between the primary quark and antiquark.

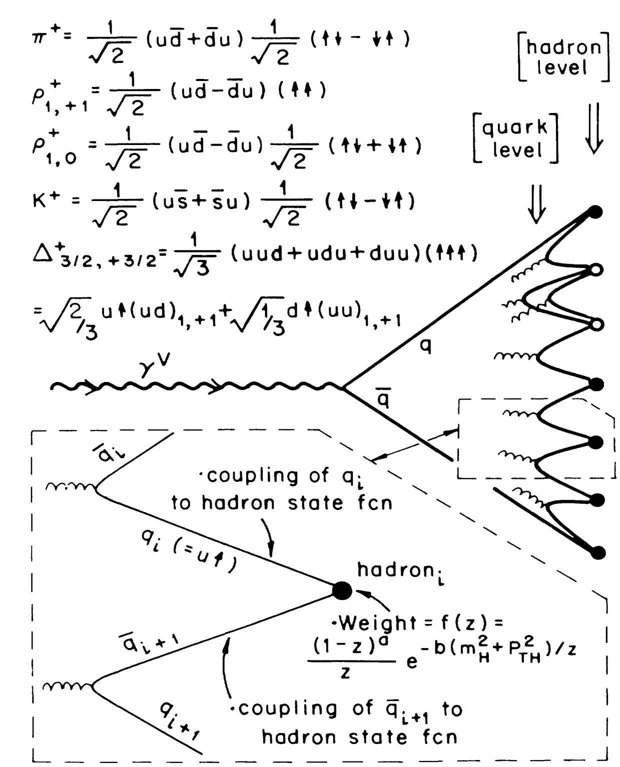

FIG. 1. Evolution of a $q\bar{q}$ color-field system into a chain of observed hadrons.

The very successful Lund model[1] presumes that the narrow color flux tube is well approximated by a relativistic string. In 1982, the Lund group proved the important theorem[2] that with the assumption of (a) a relativistic string of constant tension, (b) a central rapidity plateau, and (c) an iterative implementation in which the results are statistically left-right symmetric in terms of picking the string end of the next hadron, then the only allowable form of the fragmentation function for a single hadron flavor is

$$f(z) \propto [(1-z)^a/z] \exp[-b(m_H^2+P_{TH}^2)/z], \tag{1}$$

where $z \equiv (E+P_\parallel)_{\text{hadron}}/(E+P_\parallel)_{\text{quark}}$, m_H is the hadron mass, P_{TH} and $P_\parallel$ are the hadron transverse and longitudinal momenta, b is a universal constant related to the inverse of the string tension, and a is a constant which could be flavor dependent. Thus, it was anticipated that Eq. (1), known as the Lund symmetric fragmentation function (LSFF), would be a good approximation to the longitudinal z distributions of the various directly produced hadrons. It has, in fact, met with considerable success: the general momentum spectra of the observed hadrons, the very high-momentum particles[3] and the D^* distribution[4] which presumably contain the primary quark with little resonance decay dilution, the tendency for higher-mass hadrons to have higher momenta, and the baryon-antibaryon short-range anticorrelation recently reported by Aihara *et al.* (TPC/Two-Gamma Collaboration).[5] Following the tradition of Field and Feynman,[6] the Lund model is presently implemented by one's first picking the quark P_{Tq}^2 with a Gaussian distribution such that typical hadron P_{TH}'s are $\simeq$350–450 MeV/c, then using quark-level (qq/q for baryon/meson, s/u for strange particles, etc.) and hadron-level (vector/all for vector mesons) suppression parameters to pick the hadron identity, and finally inputting these into Eq. (1) to pick z and thereby ensure the left-right symmetry. Typically, 6–10 effective parameters are involved.

Our modification of the Lund approach arises from (a) the idea that perhaps all the mass and P_T suppression should occur on the observable hadron level and not on the unobservable intermediate quark level and (b) noting that the LSFF, if used as a production density in m_H and P_{TH}^2 as well as z, gives both a "natural" hadron-level suppression to heavy-mass states (see Fig. 2) and a "natural" P_{TH}^2 spectrum from $\exp(-bP_{TH}^2/z)$ of typically 350–500 MeV. In essence, our normalization condition on the LSFF is

$$1 = N\sum_H \int \frac{(1-z)^a}{z} \exp[-b(m_H^2 + P_{TH}^2)/z]\,dz\,dP_{TH}^2$$

with a universal normalization constant N for all possible hadrons, whereas the present Lund recipe implies $\int f(z)dz = 1$ for *each* hadron species after selection of the hadron flavor by use of the various suppression factors. We note that the original derivation of the LSFF was made with use of only one hadron mass; at present in the extrapolation to many masses, there is not yet a theoretical preference for either interpretation.

The most likely visualization behind our approach would seem to be that the primary $q\bar{q}$ color-field system undergoes a global low-Q^2 nonperturbative transition in which it must project quantum mechanically as a whole onto a chainlike set of hadrons of discrete known masses. In this context, our use of the LSFF would seem to represent a phase-space-like suppression (somewhat resembling the phase-space suppression of QCD cluster models[7]) involving the conversion of local energy stored in the color field into hadron mass and P_T^2. We note that, though the soft-gluon self-coupling presumably creates the narrow color flux tube, in our approach there is no dependence of the virtual $q\bar{q}$ production on the gluon mass (or virtuality) spectrum; we presume an energy density in the color field sufficient to provide whatever virtual $q\bar{q}$ pairs are needed, subject only to the hadron-level suppression from the LSFF.

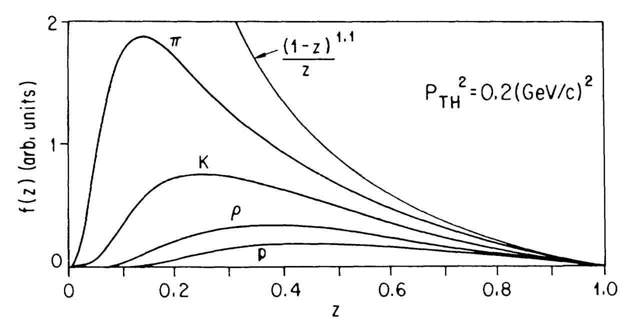

FIG. 2. The Lund symmetric fragmentation function. Note that a "natural" heavy-mass suppression is suggested.

In practice our *Ansatz* is implemented by an iterative adaptation of the Lund JETSET program[8] in which the Lund technique for choosing P_T^2, m_H, and z is replaced by ours, but all other features of the Lund implementation are preserved. Since there are no "arbitrary" suppression factors in our model, we must use the exact Clebsch-Gordan coefficient coupling from the virtual $q\bar{q}$ pairs to the final-hadron-state functions. Though baryons are more complicated objects, we have followed the same simple prescription for them as well.

The iteration procedure is illustrated in the inset of Fig. 1, where q_i is a spin-up up quark ($u\uparrow$). Some possible hadron state functions are also shown. Thus, $u\uparrow$ couples to π^+ with strength $\frac{1}{4}$, ρ^+ with total strength $\frac{3}{4}$, K^+ with $\frac{1}{4}$, and $\Delta^+_{3/2,+3/2}$ with $\frac{2}{3}$. We note that spin counting is automatic; e.g., vector to pseudoscalar meson production is enhanced by 3/1. Also, from the many baryonic spin and flavor states, we find a baryon-to-meson enhancement of $\simeq 3.2$. A further necessary iterative feature is that when an $s\bar{s}$ or diquark-antidiquark pair is created (rather than a $u\bar{u}$ or $d\bar{d}$ pair), *two* hadrons in the chain are heavier and both thereby provide suppression to the global probability of the overall chain. This phenomenon is simply handled by our weighting the production of the hadron at one stage by the appropriate weights from the next few stages. In practice, two additional stages are enough. In this way, what may in fact be a global transition can be treated rigorously in an iterative implementation. At present, no other correlations are used between stages in the iteration procedure; our studies indicate that such effects are typically $\lesssim 10\%$.

Thus, overall in our model's iterative implementation, the probability of creating a hadron$_i$ (see Fig. 1) with m_H, P_{TH}^2, and z is simply proportional to the product of (a) the square of the Clebsch-Gordan coefficient coupling the "input" quark q_i to the hadron$_i$, (b) the LSFF for that m_H, P_{TH}^2, and z, and (c) the relative weight of the next two stages for the "output" quark q_{i+1}.

We divide our comparisons with data into three parts: multiplicities per event, distributions in $X_E \equiv E_{\text{hadron}}/E_{\text{quark}}$, and P_T^2 distributions. To make the best possible multiplicity comparisons with data, we have compiled approximate "world averages" adjusted to $E_{\text{c.m.}} = 29$ GeV [appropriate to the SLAC storage ring PEP] for various flavored hadrons.[9] The uncertainty is typically 5%–10% for $\pi^\pm$, π^0, $K^\pm$, K^0, p, and Λ; 10%–20% for the vector mesons; and 30%–70% for the hyperons. The $D^{*\pm}$ and D^0 data have been corrected to the 1986 Particle Data Tables branching fractions. There also are uncertainties in model implementation which are typically $\simeq 20\%$ from such sources as decay-table uncertainties, finite-$E_{\text{c.m.}}$ effects, etc.

In Fig. 3, we compare our model predictions with the "world average" measured multiplicities for various had-

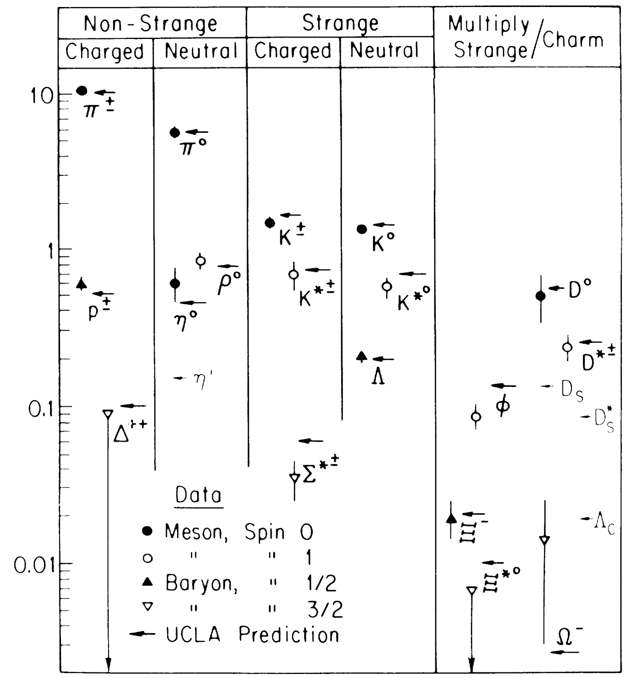

FIG. 3. Comparison of "world-average" experimental multiplicities (Ref. 9) with the predictions of our model for various hadron flavors created in e^+e^- interactions at $E_{\mathrm{c.m.}} = 29$ GeV. Antiparticles are included in all cases.

rons. Using Lund JETSET version 6.2[8] with matrix element calculation of 0–2 emitted gluons, we find best-tuned values of $a = 1.1$ and $b = 0.75$ $(\mathrm{GeV}/c)^{-2}$. Though the multiplicities vary by 3 orders of magnitude, we find very good agreement with the data, typically within 20% or 1.5σ. [We find similar predictions with $a \simeq 0.5$ and $b \simeq 0.85$ $(\mathrm{GeV}/c)^{-2}$ using JETSET version 6.3[8] with a gluon shower cutoff of $Q_0 = 1$ GeV/c.] The only hints of systematic effects are the suggestion that our model might overpredict strange-meson production by about 15% and that it might somewhat overpredict spin-$\frac{3}{2}$ baryon production, though the latter is not yet well determined experimentally.

In Fig. 4, we compare our predictions with experimental X_E distributions for characteristic hadron flavors.[9] To facilitate the shape comparison, we have normalized the data distributions to the world-average multiplicities. We see no indication of any systematic shape effects in these comparisons. It is particularly significant that the LSFF can predict the $D^{*\pm}$ distribution, which dominantly comes directly from a primary quark. In Fig. 5, we display the P_T^2 distributions predicted by our model for $|y| < 1$, for $\pi^\pm$ and protons.[10] Again, the agreement is good.

Because of the rigidity of our model, it makes rather specific predictions (good to about $\pm 10\%$–20%) when expressed in terms of the language of adjustable Lund-

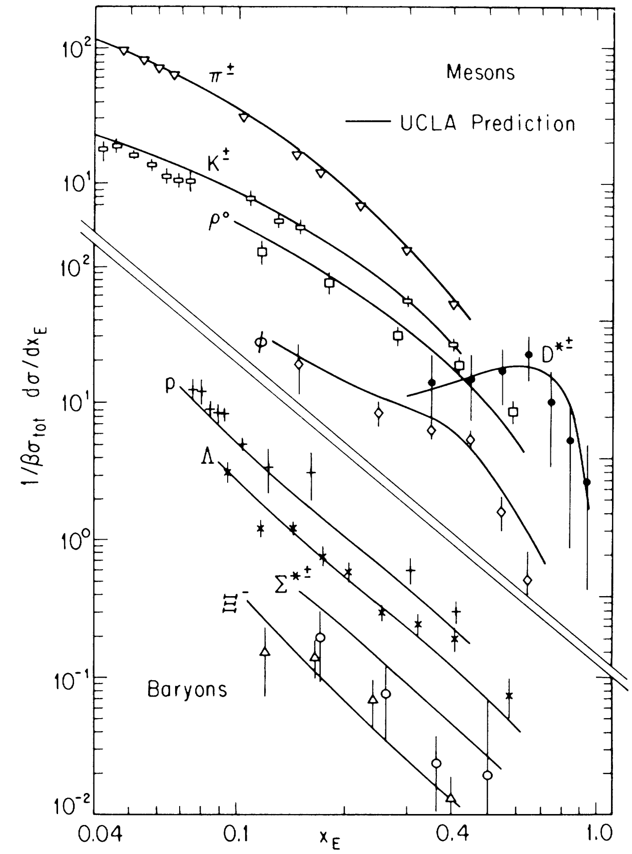

FIG. 4. Comparison of experimental X_E distributions with the predictions of our model. The $\pi^\pm$, $K^\pm$, $D^{*\pm}$, p, and Λ data are from TPC/Two-Gamma Collaboration, ρ^0 and Ξ^- from MARK II Collaboration, and $\Sigma^{*\pm}$ from HRS Collaboration (Ref. 9).

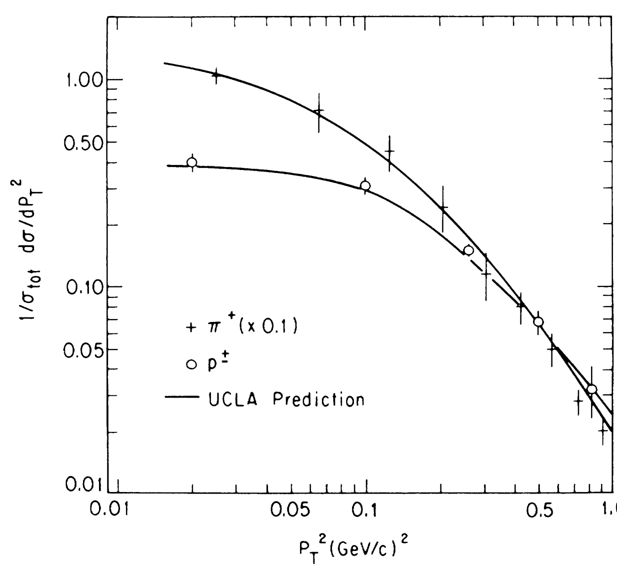

FIG. 5. Comparison of experimental distributions in P_T^2 relative to the sphericity axis for $\pi^\pm$ and protons with rapidity ≤ 1.0 (Ref. 10).

model parameters. We predict, beginning with a light (strange) [charmed] quark, that qq/q (the baryon/meson ratio) $\simeq 0.082$ (0.069) [0.042] and $s/u \simeq 0.34$ (0.38) [0.38] for meson formation. Vector/all $\simeq 0.39$ (0.49) [0.63] in formation of a light-quark (strange) [charmed] meson. The baryon parameters vary considerably with the specific flavor: We find values for $\delta \equiv (su/ud)(s/d)^{-1}$ of 0.6–1.1 and for $\alpha \cong \frac{1}{3}(qq)_1/(qq)_0$ of 0.15–0.30.[11] Further, our model predicts that as $z \to 1$, where all observed hadrons directly contain the primary quark, the $\pi/K/p$ ratio is ~62%/25%/13%. Future PEP data should sharpen these tests considerably. Also c/u (analogous to s/u for virtual $c\bar{c}$ production) is $\simeq 10^{-4}$ to 10^{-5} in our model. This may be measurable at the Stanford Linear Collider or the CERN collider LEP.

There is an interesting additional degree of freedom which probably *ought* to be implemented for baryons. Thus far, we have treated a diquark as a single topological unit (see, e.g., Fig. 1) in which the baryon and antibaryon share two virtual $q\bar{q}$ pairs (the diquark-antidiquark pair). However, because a baryon contains three partons, baryon production has additional "popcorn" degrees of freedom[12] unavailable to mesons in which the baryon and antibaryon share one or no virtual $q\bar{q}$ pairs, with one or more mesons "in between" them. Within the framework of our model, it appears that these topologically new degrees of freedom should *add* to the overall baryon production. As a beginning approximation to adding "popcorn," we have tried simply increasing the baryon production weights arbitrarily by factors of 2–3 and readjusting the values of a and b to keep the $\pi^\pm$ and proton multiplicities constant. Interestingly, this procedure slightly lowers the production rates of intermediate-mass mesons such as kaons, ρ, K^*, ϕ and of higher-mass hyperons and somewhat improves the overall agreement with the data.

In conclusion, we have found a simple, elegant *Ansatz* combining exact Clebsch-Gordan state-function coupling with the Lund symmetric fragmentation function as a production density function using only two "natural" parameters. Given only a particle's mass, spin, and quark content, the model predicts production rates and distributions for e^+e^- interactions at $E_{\rm c.m.} \simeq 30$ GeV quite well. The model will be carefully tested by the TPC/Two-Gamma Collaboration data from the upcoming PEP high-luminosity era. Future elaborations on the model will include (a) the addition of "popcorn" (see previous paragraph), (b) the addition of local P_T compensation (e.g., with use of $-\mathbf{P}_T$ of one hadron as the central $\mathbf{P}_T$ value of the next), and (c) studies of how to shift systems with different $E_{\rm c.m.}$'s. The simplicity of our model and its good agreement with experimental results suggests that the suppression of heavy-mass states and the generation of P_T may in fact occur on the observable level of produced hadrons and not on the unobservable intermediate quark level.

We gratefully acknowledge very useful and stimulating discussions with A. Ali, B. Andersson, H.-U. Bengtsson, N. Byers, H. Evans, W. Hofmann, P. Oddone, T. Sjöstrand, A. Soni, M. Strauss, H. Yamamoto, and R. Yeung. We are especially grateful to H.-U. Bengtsson who created the multiple-step weighted iterative implementation technique. The work was supported by the U.S. Department of Energy under Contract No. DE-AA03-76SF00034.

[1] B. Andersson *et al.*, Phys. Rep. **97**, 33 (1983).

[2] B. Andersson *et al.*, Z. Phys. C **20**, 317 (1983).

[3] D. Bender *et al.*, Phys. Rev. D **31**, 1 (1985).

[4] H. Aihara *et al.* (TPC/Two-Gamma Collaboration), Phys. Rev. D **34**, 1945 (1986); H. Yamamoto *et al.* (DELCO Collaboration), Phys. Rev. Lett. **54**, 522 (1985); M. Derrick *et al.* (HRS Collaboration), Phys. Lett. **146B**, 261 (1984); W. Bartel *et al.* (JADE Collaboration), Phys. Lett. **146B**, 121 (1984); M. Althoff *et al.* (TASSO Collaboration), Phys. Lett. **126B**, 493 (1983); J. M. Yelton *et al.* (DELCO and MARK II Collaboration), Phys. Rev. Lett. **49**, 430 (1982).

[5] H. Aihara *et al.*, Phys. Rev. Lett. **57**, 3140 (1986).

[6] R. D. Field and R. P. Feynman, Nucl. Phys. **B136**, 1 (1978).

[7] B. Webber, Nucl. Phys. **B238**, 492 (1984); T. Gottschalk and D. Morris, California Institute of Technology Report No. CALT-68-1365, Nucl. Phys. (to be published); see also M. G. Bowler, Z. Phys. C **22**, 155 (1984).

[8] T. Sjöstrand, Comput. Phys. Commun. **39**, 347 (1986); T. Sjöstrand and M. Bengtsson, Comput. Phys. Commun. **43**, 367 (1987).

[9] "World compilations": H. Yamamoto, in *Proceedings of the Twelfth International Symposium on Lepton and Photon Interactions at High Energies, Kyoto, Japan, 1985*, edited by M. Konuma and K. Takahashi (Nissha Printing Co., Kyoto, 1986), p. 50, Ref. 20; D. H. Saxon, Rutherford Appleton Laboratory Report No. RAL-86-057, 1986 (to be published); also, Ref. 4; W. Bartel *et al.* (JADE Collaboration), Phys. Lett. **104B**, 325 (1981); S. R. Klein *et al.* (MARK II Collaboration), Phys. Rev. Lett. **58**, 644 (1987); S. Abachi *et al.* (HRS Collaboration), Phys. Rev. Lett. **58**, 2627 (1987). We have not presently included data from $E_{\rm c.m.} \simeq 10$ GeV, because of concern for possible finite-$E_{\rm c.m.}$ effects.

[10] H. Aihara *et al.*, Phys. Rev. Lett. **55**, 1047 (1985).

[11] For comparison, a preliminary analysis of the "world-average" multiplicities in Fig. 3 yields approximate experimental Lund-model parameter values of $qq/q = 0.09 \pm 0.01$, $s/u = 0.29 \pm 0.02$, vector/all $= 0.41 \pm 0.05$ (0.46 ± 0.05) $[0.62 \pm 0.08]$, $\delta = 0.7 \pm 0.3$, and $\alpha = 0.05 \pm 0.04$. H.-U. Bengtsson *et al.*, University of California, Los Angeles, Report No. UCLA-87-006, 1987 (to be published).

[12] A. Casher *et al.*, Phys. Rev. D **20**, 179 (1979); B. Andersson *et al.*, Phys. Scr. **32**, 374 (1985).