Observation of Nonclassical Effects in the Interference of Two Photons

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By measuring the joint probability for the detection of two photons at two points as a function of the separation between the points, we have demonstrated the existence of nonclassical effects in the interference of signal and idler photons in parametric down-conversion. In principle, the detection of one photon at one point rules out certain positions where the other photon can appear.

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It has long been known that an optical field can exhibit nonclassical interference effects of the fourth order in the absence of the more usual second-order interference. We wish to report the observation of such an effect. As an example, let the field produced by two (secondary) sources A and B be detected at some point x_1 in a distant plane (see Fig. 1). If we may simplify the situation so as to consider only a single-mode contribution of each source (a more realistic treatment is given by Mandel and co-workers[1,2]), we can express the positive-frequency part of the field $\hat{E}^{(+)}(x_1)$ in the form (Hilbert-space operators are identified by the caret)

$$\hat{E}^{(+)}(x_1)=\hat{a}_A e^{i\mathbf{k}_A\cdot\mathbf{r}_{A1}}+\hat{a}_B e^{i\mathbf{k}_B\cdot\mathbf{r}_{B1}}, \tag{1}$$

where $\mathbf{k}_A$ and $\mathbf{k}_B$ are wave vectors, $\mathbf{r}_{A1}$ and $\mathbf{r}_{B1}$ are displacements shown in Fig. 1, and $\hat{a}_A,\hat{a}_B$ are photon annihilation operators. The probability $\mathcal{P}_1(x_1)\delta x_1$ of photodetection at x_1 within some narrow range δx_1 is then given by[3]

$$\mathcal{P}_1(x_1)\delta x_1=K_1[\langle\hat{E}^{(-)}(x_1)\hat{E}^{(+)}(x_1)\rangle]\delta x_1=K_1[\langle\hat{a}_A^\dagger\hat{a}_A\rangle+\langle\hat{a}_B^\dagger\hat{a}_B\rangle+\langle\hat{a}_A^\dagger\hat{a}_B\rangle e^{i(\mathbf{k}_B\cdot\mathbf{r}_{B1}-\mathbf{k}_A\cdot\mathbf{r}_{A1})}+\text{c.c.}]\delta x_1, \tag{2}$$

where K_1 is a scale factor characteristic of the detector.

To make the calculation more explicit, we now suppose that A and B correspond to the signal and idler modes produced in the process of degenerate parametric down-conversion in a nonlinear medium. In this process a pump photon fissions into a signal and an idler photon, and the appropriate down-converted state is the two-photon Fock state $|1_A,1_B\rangle$. The expectations in Eq. (2) are then easily evaluated and the probability reduces to

$$\mathcal{P}_1(x_1)\delta x_1=2K_1\delta x_1, \tag{3}$$

so that there is no interference or any periodic variation of $\mathcal{P}_1(x_1)$ with x_1. If a similar measurement is made at x_2 with another detector, we again have

$$\mathcal{P}_2(x_2)\delta x_2=2K_2\delta x_2. \tag{4}$$

The fundamental reason for the absence of second-order interference is, of course, that the two down-converted photons have no definite phase relationship.

Nevertheless, a fourth-order interference effect, involving measurements with two detectors, exists. Thus we may choose to use two photodetectors at positions x_1 and x_2 to measure the joint probability $\mathcal{P}_{12}(x_1,x_2)\delta x_1 \times\delta x_2$ of detection at x_1 and x_2 within δx_1 and δx_2, in which case[3]

$$\mathcal{P}_{12}(x_1,x_2)\delta x_1\delta x_2=K_1K_2\langle\hat{E}^{(-)}(x_1)\hat{E}^{(-)}(x_2)\hat{E}^{(+)}(x_2)\hat{E}^{(+)}(x_1)\rangle\delta x_1\delta x_2. \tag{5}$$

With the help of Eq. (1) we obtain for the same two-photon state $|1_A,1_B\rangle$ [1,2,4-6]

$$\begin{aligned}\mathcal{P}_{12}(x_1,x_2)\delta x_1\delta x_2&=2K_1K_2\delta x_1\delta x_2\{1+\cos[\mathbf{k}_A\cdot(\mathbf{r}_{A2}-\mathbf{r}_{A1})+\mathbf{k}_B\cdot(\mathbf{r}_{B1}-\mathbf{r}_{B2})]\}\\&=2K_1K_2\delta x_1\delta x_2\{1+\cos 2\pi(x_1-x_2)/L\},\end{aligned} \tag{6}$$

where $L\approx\lambda/\theta$ is the spacing of the classical interference fringes that correspond to the geometry of Fig. 1, i.e., two waves of wavelength λ coming together at a small angle θ. Because of the cosine term, this probability exhibits interference with a relative depth of modulation $\eta=100\%$, and has certain nonclassical and nonlocal features, particularly when $|x_1-x_2|=(n+\frac{1}{2})L$ ($n=0, 1,2,\ldots$), in which case $\mathcal{P}_{12}(x_1,x_2)=0$. In essence there is an interference between two two-photon probability amplitudes, because the apparatus cannot distinguish be-

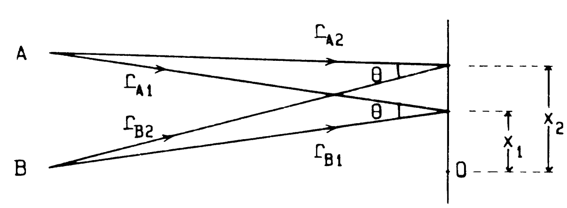

FIG. 1. The geometry of the interference experiment.

tween photons from A and B being detected at x_1 and x_2, respectively, or vice versa. Despite the fact that any one photon can be detected at any position x, we see that if the photon is detected at x_1, there are certain positions x_2 where the other photon can never be found. However, if x_1 were displaced by $\frac{1}{2}L$, the other photon could show up at previously forbidden positions, x_2. This would be so even if the two detections were disjoint, so that a type of nonlocal phenomenon is encountered.

Let us contrast this with the situation in classical optics. If the fields $E^{(+)}(x_1),E^{(+)}(x_2)$ in Eqs. (1) to (5) are classical c-number fields, and it is assumed that the sources are randomly phase so that they do not produce second-order interference effects, we readily find that Eq. (5) leads to[1]

$$\mathcal{P}_{12}(x_1,x_2)\delta x_1\delta x_2=2K_1K_2\delta x_1\delta x_2\left[1+\frac{2\langle|a_A|^2|a_B|^2\rangle}{\langle(|a_A|^2+|a_B|^2)^2\rangle}\cos\frac{2\pi(x_1-x_2)}{L}\right], \tag{7}$$

in which the "visibility" or relative modulation is always less than or equal to 50%. As a result, the joint probability $\mathcal{P}_{12}(x_1,x_2)\delta x_1\delta x_2$ can never vanish in classical optics, because there is a nonvanishing optical field at both positions x_1 and x_2.

Of course, in practice, measurements are not made at a point, but extend over noninfinitesimal regions Δx centered at x_1 and x_2, so that we ought to integrate the foregoing expressions over Δx. From Eqs. (3) and (4) the measured probabilities $P_1(x_1)$ and $P_2(x_2)$ are therefore related to $\mathcal{P}_1(x_1)\delta x_1$ and $\mathcal{P}_2(x_2)\delta x_2$ by

$$P_1(x_1)=2K_1\Delta x,\quad P_2(x_2)=2K_2\Delta x, \tag{8}$$

and from Eq. (6) the measured joint detection probability $P_{12}(x_1,x_2)$ is given by

$$P_{12}(x_1,x_2)=\int_{x_1-(1/2)\Delta x}^{x_1+(1/2)\Delta x}\int_{x_2-(1/2)\Delta x}^{x_2+(1/2)\Delta x}\mathcal{P}_{12}(x_1,x_2)dx_1dx_2=2K_1K_2(\Delta x)^2\left\{1+\left(\frac{\sin\pi\Delta x/L}{\pi\Delta x/L}\right)^2\cos\frac{2\pi(x_1-x_2)}{L}\right\}. \tag{9}$$

As a consequence of the finite detector width Δx, the relative modulation η has been reduced from 1 to $[\sin(\pi\Delta x/L)/(\pi\Delta x/L)]^2$. Once again, it follows with the help of Eq. (7) that the maximum η would be half as great for a classical field.

We wish to report an experiment in which this nonclassical effect has been observed in the interference of signal and idler photons produced in the degenerate parametric frequency splitting of light. The two-photon state $|1_A,1_B\rangle$ is created in the down-conversion of a pump photon, and an equation very similar to Eq. (6) can be derived from a more realistic treatment of the problem.[2]

An outline of the experiment is shown in Fig. 2. The light from an argon-ion laser oscillating on the 351.1-nm line falls on a 1.5-cm-long nonlinear crystal of $LiIO_3$, where some incident photons split into two half-frequency signal and idler photons, which emerge at angles of $\pm 3.3°$ to the pump beam. A beam stop deflects the latter, while two mirrors $M1$ and $M2$ cause the down-converted photons to come together at an angle $\theta \approx 2°$ and interfere in a plane distant 1.1 m from the crystal, after passing through an interference filter. Any interference pattern formed in this plane is magnified and reimaged by the lens shown, so as to make the fringe spacing $L\approx 0.34$ mm. Two movable glass plates, each of thickness $\Delta x \approx 0.14$ mm, collect the incoming photons at x_1 and x_2 edge on, and direct them to two counting photomultipliers, whose pulses, after some amplification and shaping, are fed to the start and stop inputs of a time-to-digital converter. Pulse pairs arriving within a 5-nsec interval are treated as "coincident" for the purpose of the measurement. When accidental coincidences

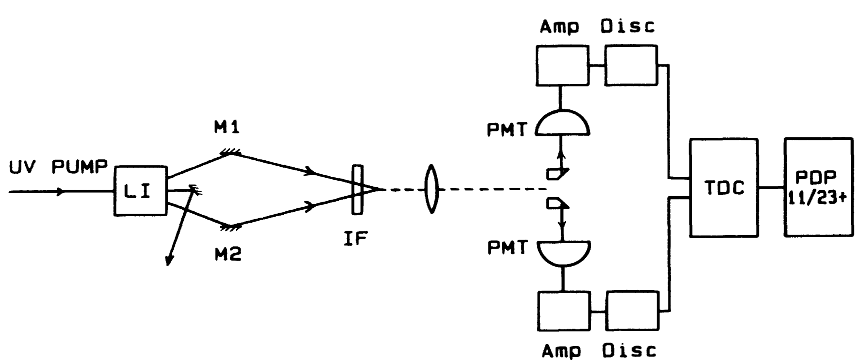

FIG. 2. Outline of the apparatus.

TABLE I. Experimental results. Accidental coincidence rate $\approx (18.2 \pm 0.3)$/h.

$\lvert x_1 - x_2 \rvert / L$	No. of coincidences	Measured coincidence rate (h^{-1})	Measured − accidental coincidence rate (h^{-1})
0.41	215 in 10 h	21.5 ± 1.5	3.3 ± 1.5
0.50	178 in 9 h	19.8 ± 1.5	1.6 ± 1.5
1.00	354 in 13 h	27.2 ± 1.4	9.0 ± 1.4

are subtracted out, the rate of coincidence counting provides a measure of the joint probability $P(x_1,x_2)$, up to a scale factor. To determine the expected number of accidentals, we treat the number of photon pairs registered with delays ranging from 35 to 75 nsec as accidental coincidences. When this number is multiplied by $\frac{5}{40}$, it yields the expected number of accidentals within the 5-nsec resolving time. The accidental coincidence rate determined in this way was (18.2 ± 0.3)/h, in reasonable agreement with what would be expected from the average counting rates 1003/sec and 1000/sec of the two photodetectors. The corresponding dark counting rates were 83/sec and 64/sec.

The numbers of coincidences registered in approximately 10-h-long counting intervals, and the corresponding counting rates, are shown in Table I for several different values $x_1 - x_2$. Because of the 0.14-mm thickness of the plates, the minimum value of $x_1 - x_2$ was 0.14 mm, corresponding to $(x_1 - x_2)/L = 0.41$, and the geometric factor $\eta = [\sin(\pi \Delta x/L)/(\pi \Delta x/L)]^2$ was 0.55. The full curve in Fig. 3 is a plot of $P(x_1,x_2)$ given by Eq. (9), with the scale factor K_1K_2 adjusted arbitrarily to give best agreement with the measured coincidence rates. The broken curve in Fig. 3 is the corresponding prediction from classical probability theory, when the relative modulation amplitude has its maximum value 0.275, and with the scale constant adjusted for best fit. It is apparent that the classical prediction provides a much poorer fit.

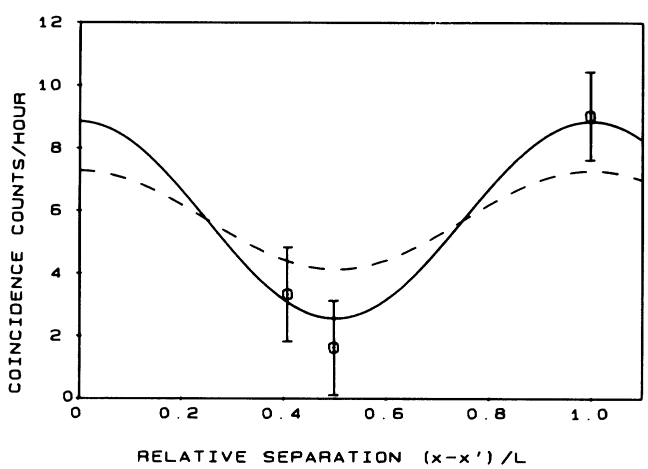

FIG. 3. Experimental results superimposed on the predictions of quantum theory given by Eq. (9) (full curve), and of the classical theory with maximum modulation (dashed curve).

From the differences between the experimental points and the classical and quantum theoretical predictions, we have calculated the value of χ^2, and have used this to test the hypothesis that one or the other theory is satisfied. We find that $\chi^2_{\mathrm{qu}} \approx 0.44$ and $\chi^2_{\mathrm{class}} \approx 4.9$. The probabilities of encountering this large a value of χ^2 by chance are $\mathrm{Prob}(\chi^2 \geq 0.44) \approx 0.92$, and $\mathrm{Prob}(\chi^2 \geq 4.9) \approx 0.18$. To within the statistical uncertainties, we have therefore demonstrated that classical probability is violated and quantum mechanics is obeyed in this interference experiment.

It is shown elsewhere[7] that a violation of one form of Bell's inequality should be demonstrable in a slightly different form of interference experiment.

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