Hadronic Screening Lengths in the High-Temperature Plasma

Steven Gottlieb
Department of Physics, Indiana University, Bloomington, Indiana 47405

W. Liu and D. Toussaint
Department of Physics, University of California at San Diego, La Jolla, California 92093

and

R. L. Renken and R. L. Sugar
Department of Physics, University of California, Santa Barbara, Santa Barbara, California 93106
(Received 28 August 1987)

Using a hybrid molecular-dynamics simulation of lattice QCD, we measure the spatial screening lengths of local meson and baryon operators with two flavors of Kogut-Susskind fermions. In the high-temperature phase the meson and nucleon screening lengths are parity doubled and their inverses are comparable to the zero-temperature masses. We construct chiral projections for the meson and nucleon propagators which demonstrate the chiral symmetry in a way independent of the extraction of masses from the propagators.

PACS numbers: 12.38.Mh, 11.30.Rd, 12.38.Gc

QCD with two flavors of massless quarks has a chiral flavor $SU(2)_L \otimes SU(2)_R$ symmetry. If the symmetry were manifest, the isotriplet pseudoscalar π and the isosinglet scalar σ would be degenerate since they belong to the same $(\frac{1}{2},\frac{1}{2})$ representation of $SU(2)_L \otimes SU(2)_R$. Also, the nucleon would be either massless or degenerate with an opposite-parity partner.[1] Studies of lattice QCD have shown that for massless quarks chiral symmetry is restored at high temperature,[2] as indicated by the vanishing of $\langle \bar{\psi}\psi \rangle$ as the quark mass goes to zero. Recently DeTar and Kogut[3] have measured the spatial screening lengths for hadronic operators at finite temperature using four flavors of Kogut-Susskind fermions. This is one generalization to finite temperature of the zero-temperature hadron masses, and unbroken symmetries of the finite-temperature theory are reflected in degeneracies of the screening lengths. These screening lengths are not necessarily the same as the masses of the corresponding excitations defined by real-time correlation functions. However, for brevity we will refer to high-temperature masses, by which we mean inverse screening lengths.

We have measured mesonic and baryonic screening lengths with two flavors of dynamical Kogut-Susskind fermions on an $8^2 \times 24 \times 4$ lattice for quark masses of 0.1, 0.05, and 0.025. From our previous simulations of QCD thermodynamics[2] we know the value of $6/g^2$ for the crossover to high-temperature behavior for each of these quark masses at $N_\tau = 4$ and 6. (Even for quark masses for which we do not clearly see a phase transition, the crossover is sufficiently sharp to define a crossover value of $6/g^2$.) Most of our screening-length measurements were done at the crossover value of $6/g^2$ for $N_\tau = 6$. Since we were using $N_\tau = 4$, this corresponds to a temperature of $1.5T_c$, well into the high-temperature phase. We measured the spatial correlation functions corresponding to π, σ, ρ, A_1, and B mesons as well as nucleons of both parities. These operators can be measured with source operators at a single lattice point and so are easier to measure than "nonlocal" operators. We have also measured these hadrons' zero-temperature masses on $8^3 \times 24$ and $10^3 \times 24$ lattices at the same values of $6/g^2$, quark mass, molecular-dynamics time step, conjugate-gradient residual, and, in the case of the $8^3 \times 24$ lattices, x and y spatial sizes. This allows us to make comparisons in which many of the systematic errors cancel.

The reader is referred to Kluberg-Stern *et al.*,[4] Gilchrist *et al.*,[4] and Morel and co-workers[4] for the construction of the meson and baryon operators with Kogut-Susskind fermions. The meson and nucleon propagators are conventionally fitted by the forms

$$M(z) = A[e^{-mz} + e^{-m(N_z - z)}] + (-1)^z \tilde{A}[e^{-\tilde{m}z} + e^{-\tilde{m}(N_z - z)}], \tag{1}$$

$$B(z) = A[e^{-m_N z} + (-1)^z e^{-m_N(N_z - z)}] - (-1)^z \tilde{A}[e^{-m_{\tilde{N}} z} + (-1)^z e^{-m_{\tilde{N}}(N_z - z)}], \tag{2}$$

where the tildes indicate particles of opposite parity. The boundary conditions for the quark hopping matrix are antiperiodic in τ and periodic in the spatial directions, which makes the nucleon propagator antiperiodic in τ. Since $N_\tau = 4$, the contribution from the $\tau = 2$ slice vanishes from the antiperiodicity, and since we use only x, y, τ even in our nucleon propagators, only the $\tau = 0$ slice actually contributes to our high-temperature results.

The propagators at different values of z are strongly correlated, and this must be taken into account when we fit the

© 1987 The American Physical Society

TABLE I. Estimates for the π and σ masses and their differences in each phase. The bare quark mass is m_q and $\Delta m = m_\pi - m_\sigma$. Both positive- and negative-parity nucleon masses and their differences are also tabulated. The errors on the mass differences are statistical only, and include the effects of correlations between the two masses.

	High temperature			Low temperature		
m_q	m_π	m_σ	Δm	m_π	m_σ	Δm
0.10	1.086(2)	1.188(2)	0.102(2)	0.826(1)	1.250(7)	0.424(7)
0.05	0.993(2)	1.034(3)	0.041(2)	0.614(1)	1.103(14)	0.489(14)
0.025	0.937(3)	0.955(3)	0.018(1)	0.451(1)	0.949(15)	0.498(15)
m_q	m_N	$m_{\tilde{N}}$	Δm	m_N	$m_{\tilde{N}}$	Δm
0.10	2.38(3)	2.38(4)	0.005(30)	1.98(2)	2.22(7)	0.25(9)
0.05	2.42(8)	2.43(8)	0.014(30)	1.67(8)	2.20(28)	0.52(33)
0.025	2.19(6)	2.21(6)	0.018(10)	1.56(23)	2.24(51)	0.68(64)

propagators. Let M_z be the propagator found in the simulation and $M(z)$ be that calculated from Eq. (1). χ^2 and the covariance matrix C_{xy} are defined by

$$\chi^2 = \sum_{xy} [M(x) - M_x] \frac{C_{xy}^{-1}}{2} [M(y) - M_y], \tag{3}$$

$$C_{xy} = (\langle M_x M_y \rangle - \langle M_x \rangle \langle M_y \rangle)(N-1)^{-1}. \tag{4}$$

Here N is the number of data points and $\langle M \rangle = (1/N)\sum_{i=1}^{N} M_i$. With this definition χ^2 has its usual interpretation as an indicator of the goodness of fit. Similarly the errors on the parameters and the correlations among the parameters are described by the matrix of second derivatives of χ^2 with respect to the parameters. Since the parameters are correlated, the error on a combination of parameters such as a difference of two masses is not equal to the naive combination of the errors. When we quote mass splittings these correlations are taken into account in the errors. To estimate a mass or screening length, we fit the data for $z > z_{\min}$ and increase $z_{\min}$ from zero until the χ^2 becomes comparable to the number of degrees of freedom. We also require that the mass be reasonably constant as $z_{\min}$ increases.

The π and σ masses at low and high temperatures, as well as the mass splittings, are tabulated in Table I. As usual, we vary $6/g^2$ as we vary the quark mass so that the high-temperature results are always at $1.5T_c$.[5] (T_c is, of course, not necessarily constant in physical units as m_q varies.) The low-temperature results are obtained on an $8^3 \times 24$ lattice at the same values of $6/g^2$ as the high-temperature results. (We choose the $8^3 \times 24$ lattices for the low-temperature masses rather than the $10^3 \times 24$ lattices so that the smallest spatial size will be the same as in our high-temperature data.) As $m_q \to 0$ the zero-temperature pion mass goes to zero as $(m_q)^{1/2}$, but the high-temperature screening mass remains large. The high-temperature π and σ masses converge to a common finite value as the quark mass decreases, in contrast to the low-temperature masses. In Fig. 1 we plot the π and σ masses versus $6/g^2$ for fixed quark mass 0.05 and lattice size $8^2 \times 24 \times 4$. The large error bars on the σ mass at low temperatures reflect the difficulty of measuring this propagator.

When the chiral symmetry is unbroken the hadronic operators are conveniently written in terms of chiral projections such as $\bar{\psi}\Gamma(1 \pm \gamma_5)\psi$, where Γ is some Dirac matrix. Such a projection contains equal contributions from particles of both parities and the absence of one of the two chiralities in a propagator demonstrates the parity doubling of both masses and amplitudes in that channel. The remaining chiral symmetry for massless Kogut-Susskind fermions rotates the one-component Grassmann variables in opposite directions on odd and even lattice sites. On a lattice of spacing $2a$, these variables correspond to four flavors of four-component quarks. Following the analysis and notations of Kluberg-Stern *et al.* and Billoire *et al.*[4] this rotation is $\exp(i\theta\gamma_5 \times \gamma_5^*)$, where the first γ_5 operates on the Dirac index and the second on the flavor index of the four-component quark fields. (Although we have used the square root of the Kogut-Susskind fermion determinant in our con-

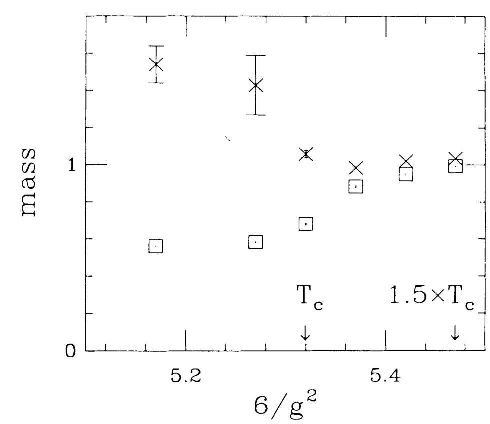

FIG. 1. π and σ masses in units of the lattice spacing vs temperature for quark mass of 0.05. Here $N_t = 4$ so that varying of $6/g^2$ is equivalent to varying of temperature. The squares are the π and the crosses the σ. The crossover temperature and 1.5 times the crossover temperature are marked with arrows.

figuration weighting to simulate two flavors, we can still construct the meson correlation functions for the original four flavors. Roughly speaking, we have two flavors of dynamical quarks but four flavors of valence quarks.) The mesonic correlation functions which are eigenvectors of this chiral rotation are obtained by insertion of $1\times 1 \pm \gamma_5 \times \gamma_5^*$ in the operators. In terms of the weight vectors on the lattice, this is a factor of $1\pm(-1)^{x+y+z+\tau}$, which projects out even or odd lattice sites, as expected from the transformation properties of the original Grassmann variables. Therefore we look at linear combinations of the usual local mesonic correlation functions[6]:

$$\begin{aligned} M_0^{\pm} &= M_{PS} \pm (-1)^z M_{SC} = \sum_{x,y,\tau} |M^{-1}(0,x)|^2[1\pm(-1)^{x+y+z+\tau}], \\ M_1^{\pm} &= M_{VT} \pm (-1)^z M_{PV} = \sum_{x,y,\tau} |M^{-1}(0,x)|^2[(-1)^x+(-1)^y+(-1)^\tau][1\pm(-1)^{x+y+z+\tau}], \end{aligned} \tag{5}$$

where M is the Kogut-Susskind fermion matrix. For the low-temperature masses z and τ are interchanged.

If the limit $m_q \to 0$ can be taken naively $M^{-1}(0,x)$ will vanish when x is an even site. This can be seen by our writing $M^{-1}(0,x)$ as $(1/M^\dagger M)M^\dagger\delta(0)$. $M^\dagger M$ only connects even sites to even sites and odd sites to odd sites. $M^\dagger$ has elements of order 1 connecting site 0 to its neighbors, which are odd sites, and a diagonal element $2m_q a$. Thus, unless $1/M^\dagger M$ blows up too fast as $m_q \to 0$, $(1/M^\dagger M)M^\dagger\delta(0)$ will vanish on even sites as $m_q \to 0$. In this case only the chiral projections containing $1-(-1)^{x+y+z+\tau}$ will survive. Since each chiral projection contains both parities this implies parity doubling in terms of the conventional correlation functions. Indeed, it implies that all particle masses in the channel as well as their amplitudes with the local source $\delta(0)$ are also doubled. When the chiral symmetry is spontaneously broken, the limit $m_q \to 0$ cannot be taken naively, the above argument fails, and the masses need not be doubled. Examination of the ratio of the two chiral projections provides a test of the parity doubling which is independent of the difficult and ambiguous process of estimating masses from the propagators.

In Fig. 2 we plot the ratio of the two chiral projections for the spin-0 mesons as a function of distance with three different quark masses at low and high temperatures. It can be seen that this ratio is going to zero with the quark mass at high temperature but remaining constant at low temperatures. For any finite m_q this ratio will approach 1 at large enough distance because the parity doubling is not exact.

In Fig. 3 we show a similar plot for the spin-1 mesons. Again it can be seen that the ratio is vanishing at high temperature. However, it is also sensitive to the quark mass at low temperature. To clarify this we plot in Figs. 4(a) and 4(b) the ratio as a function of quark mass at distances 1 and 2. Again, the ratio is clearly vanishing at high temperature but appears to be going to a finite limit at zero temperature.

If the nucleons are massive in a chirally symmetric phase, we will have parity doubling. In Table I we show the estimates for positive- and negative-parity nucleon masses and their differences in both phases, showing degeneracy at $1.5T_c$. These mass estimates were obtained

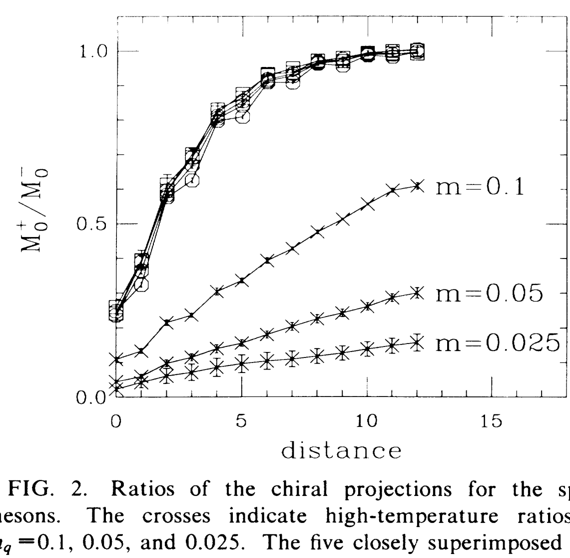

FIG. 2. Ratios of the chiral projections for the spin-0 mesons. The crosses indicate high-temperature ratios for m_q=0.1, 0.05, and 0.025. The five closely superimposed plots are low-temperature ratios. The squares are ratios with m_q=0.05 and 0.025 from an $8^3\times 24$ lattice. The octagons are for quark masses of 0.1, 0.05, and 0.025 on a $10^3\times 24$ lattice.

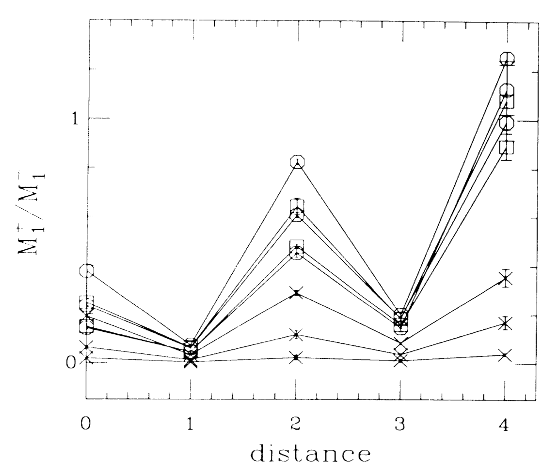

FIG. 3. Ratios of the chiral projections for spin-1 mesons. The meaning of the symbols is the same as in Fig. 2.

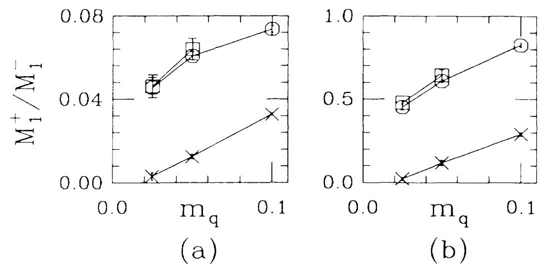

FIG. 4. Ratios for the spin-1 propagators as a function of quark mass at distances (a) 1 and (b) 2. The meaning of the symbols is the same as in Fig. 2.

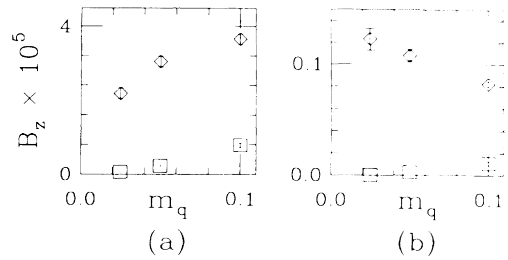

FIG. 5. Nucleon correlation functions at distances (a) 2 and (b) 4 vs quark mass. The lozenges are the high-temperature points and the squares the low-temperature points.

by our fitting the propagators with the form of Eq. (2), with one mass for each parity, using data with $z_{\min}=5$. While this is not the best way of extracting the nucleon mass, it is appropriate here because it treats both parities equally and can be consistently applied to both our high- and low-temperature data.

The nucleon correlation function involves $M^{-1}(0,x)$ only for $x+y+\tau$, even, so that if chiral symmetry is unbroken we expect it to vanish at even z distances. As with the mesons, this implies parity doubling of all masses and amplitudes in this channel. In Figs. 5(a) and 5(b) the high- and low-temperature nucleon propagators at distances 2 and 4 are plotted versus quark mass. These graphs show the vanishing of the even-distance propagators as $m_q \rightarrow 0$ in the high-temperature phase in contrast to the low-temperature phase where a spontaneous chiral-symmetry breaking remains at $m_q=0$.

The spatial correlation functions of color-singlet hadron operators at high temperatures show degeneracies corresponding to the restoration of chiral symmetry. The π screening mass is equal to the σ screening mass and comparable to the screening masses of other mesons. The nucleon screening length is also short in the chirally symmetric phase, and parity doubling is realized both for masses and amplitudes. These results must be understood in the context of our recent measurement of the response of the quark number to an infinitesimal chemical potential, which showed that the carriers of baryon number can be easily added to the high-temperature phase.[7] Thus the smallness of the screening lengths does not indicate that the masses of the elementary excitations will be large.

We would like to thank Roger Dashen, Carleton DeTar, Andreas Gocksch, Julius Kuti, and Pietro Rossi for helpful conversations. This work was supported in part by National Science Foundation Grants No. PHY-86-14185 and No. DMR83-20423, and by Department of Energy Grants No. DE-AT03-81ER40029 and No. DE-AC02-84ER-40125. We are grateful to E. I. DuPont de Nemours Co., Star Technologies Inc., and Xerox Corporation for their support. One of us (D.T.) is an Alfred P. Sloan Foundation fellow.

[1]L. McLerran, Rev. Mod. Phys. **58**, 1021 (1986).

[2]M. Fukugita and A. Ukawa, Phys. Rev. Lett. **57**, 503 (1986); M. Fukugita, S. Ohta, Y. Oyanagi, and A. Ukawa, Phys. Rev. Lett. **58**, 2515 (1987); R. V. Gavai, J. Potvin, and S. Sanielevici, Phys. Rev. Lett. **58**, 2519 (1987); E. V. E. Kovacs, D. K. Sinclair, and J. B. Kogut, Phys. Rev. Lett. **58**, 751 (1987); R. Gupta, G. Guralnik, G. W. Kilcup, A. Patel, and S. R. Sharpe, Phys. Rev. Lett. **57**, 2621 (1986); F. Karsch, J. B. Kogut, D. K. Sinclair, and H. W. Wyld, Phys. Lett. B **188**, 353 (1987); S. A. Gottlieb, W. Liu, D. Toussaint, R. L. Renken, and R. L. Sugar, Phys. Rev. D **35**, 3972 (1987).

[3]C. DeTar and J. Kogut, Phys. Rev. Lett. **59**, 399 (1987), and Phys. Rev. D **36**, 2828 (1987).

[4]H. Kluberg-Stern, A. Morel, O. Napoly, and B. Petersson, Nucl. Phys. **B220**, 447 (1983); J. Gilchrist, H. Schneider, G. Schierholz, and M. Teper, Nucl. Phys. **B248**, 29 (1984); A. Morel and J. Rodrigues, Nucl. Phys. **B247**, 44 (1984); A. Billoire, R. Lacaze, E. Marinari, and A. Morel, Nucl. Phys. **B251**, 581 (1985).

[5]S. A. Gottlieb, W. Liu, D. Toussaint, R. L. Renken, and R. L. Sugar, Phys. Rev. Lett. **59**, 1513 (1987).

[6]We use the notation of K. C. Bowler, C. B. Chalmers, R. D. Kenway, G. S. Pawley, and D. Roweth, Nucl. Phys. **B284**, 299 (1987).

[7]S. A. Gottlieb, W. Liu, D. Toussiant, R. L. Renken, and R. L. Sugar, "The Quark Number Susceptibility of High Temperature QCD," to be published.