Prediction of $B \to K^* \gamma$ as a Test of the Standard Model

N. G. Deshpande, P. Lo, and J. Trampetic[a]
Institute of Theoretical Science, University of Oregon, Eugene, Oregon 97403

and

G. Eilam and P. Singer
Department of Physics, Technion–Israel Institute of Technology, 32 000 Haifa, Israel
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The branching ratio for $B \to K^* \gamma$ is calculated as a function of the top-quark mass. We include QCD enhancement effects in the short-distance operators and the hadronic matrix elements are determined by use of the constituent-quark model. The branching ratio is in the accessible range of $\simeq 10^{-4}$.

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Historically, flavor-changing one-loop processes such as $K_L \to \mu\bar{\mu}$ and $K \to \pi e^+ e^-$ and K-$\bar{K}$ mixing have provided crucial information[1] on the charm mass. In this Letter we focus our attention on the flavor-changing process $B \to K^* \gamma$, which relies on a Glashow-Iliopoulos-Maiani cancellation between the top and charm quarks and is experimentally feasible. Further, unlike the processes involving K decays where long-distance contributions were of the same order of magnitude as short-distance contributions, we expect such ambiguities to be absent in heavy-quark systems. This is certainly correct for $B \to K e^+ e^-$, where, however, the rate is close to the limit of present experiments.[2] The process $B \to K^* \gamma$ has the advantage that the signature (monoenergetic photon) is clean and the branching ratio of order 10^{-4} is accessible to present experiments. Tests of electroweak theory in one loop are of interest in their own right, because they verify the gauge structure of the theory.

We expect the dominant mechanism for the decay $B \to K^* \gamma$ to be the quark subprocess $b \to s\gamma$. There are additional contributions from the nonleptonic transitions accompanied by photon emission from the B and K^*. In the case of $u\bar{b} \to s\bar{u}$ via W exchange, the diagram is suppressed because of Kobayashi-Maskawa angles.[3] There may also be a gluon-exchange nonleptonic transition related to the penguin diagram. In this paper we investigate the effect of $b \to s\gamma$.

The flavor-changing vertex $b \to s\gamma$ proceeds in one loop through exchange of u, c, and t quarks and W boson, and is given by[4,5]

$$V_\mu = G_1 \bar{s}(\gamma_\mu q^2 - q_\mu \mathbf{q}) b + G_2 \{\bar{s}\sigma_{\mu\nu} q^\nu b_R m_b + \bar{s}\sigma_{\mu\nu} q^\nu b_L m_s\}. \tag{1}$$

For a real photon G_1 does not contribute, and the dominant contribution to G_2 is from c and t quarks and is

$$G_2 = \frac{G_F}{\sqrt{2}} \left[\frac{e}{4\pi^2}\right] (V_{bc} V_{cs}^\dagger) \left[\frac{7}{12}\frac{m_c^2}{M_W^2} - F_2(m_t)\right], \tag{2}$$

where $F_2(m_t)$ is given in Table 2 of Ref. 5 and takes the values 0.03, 0.09, 0.21, 0.37, and 0.46 for $m_t = 20$, 40, 80, 160, and 240 GeV, respectively. As $m_t \to \infty$ the asymptotic value of $F_2(m_t)$ is $\frac{2}{3}$.

As noted by Vasanti[6] and shown explicitly by Shifman, Vainshtein, and Zakharov[7] (where $s \to d\gamma$ is considered), there is a large QCD correction for the G_2 term. This is because there is an accidental cancellation of $\ln(m_c/m_t)$ term in the lowest-order evaluation of G_2. When one-gluon corrections are included, the dominant part of the QCD-corrected part of G_2 is given by

$$\tilde{G}_2 = \frac{G_F}{\sqrt{2}} \left[\frac{e}{3\pi^3}\right] \alpha_s (V_{bc} V_{cs}^\dagger) \ln\left[\frac{m_t^2}{m_c^2}\right]. \tag{3}$$

In spite of a factor $4\alpha_s/3\pi$, $\tilde{G}_2/G_2$ is approximately 9 for $m_t \simeq 40$ GeV. We shall present our results with and without QCD correction for comparison. Previous work[2] on $B \to K^* \gamma$ has ignored QCD enhancement and determination of hadronic matrix elements.

It is straightforward to calculate the inclusive process $B \to \gamma + x$ where x contains no charm by equating it to $b \to s\gamma$. The matrix element for this decay is

$$M(b \to s\gamma) = \tilde{G}_2 m_b \bar{s}\sigma_{\mu\nu} q^\nu b_R \epsilon^\mu(q), \tag{4}$$

and the rate is

$$\Gamma(B \to \gamma + x(\text{no charm})) = \Gamma(b \to s\gamma) = (m_b^5 |\tilde{G}_2|^2/16\pi)(1 - m_s^2/m_b^2)^3. \tag{5}$$

The inclusive process is, however, difficult if not impossible to detect. We therefore shall discuss the exclusive process $B \to K^* \gamma$ in some detail. The exclusive decay $B \to K^* \gamma$ is expected to be a measurable fraction of the inclusive process as $B \to K\gamma$ is forbidden. In the absence of rigorous methods we have carried out a detailed study of the hadronic matrix element using the constituent quark model (CQM) (based on the Schrödinger equation with Coulomb plus linear potential) as a phenomenological model of QCD in the nonperturbative regime. This model has had considerable success in describing hadronic structure.[8] The same model was described and successfully applied to determine exactly the same type of hadronic matrix elements in recent papers by Grinstein, Wise, and Isgur (see Ref. 8).

The most general Lorentz structure of the matrix element of the operator $\bar{s}\sigma_{\mu\nu}q^\nu b_R$ between K^* and B states, with use of current conservation, is

$$(2k_0 2p_0)^{1/2}\langle K^*(k)\,|\,\bar{s}\sigma_{\mu\nu}q^\nu b_R\,|\,B(p)\rangle = i\epsilon_{\mu\nu\lambda\sigma}\epsilon^\nu(k)p^\lambda k^\sigma f_1(q^2) + \{\epsilon_\mu(k)(m_B^2 - m_{K^*}^2) - (p+k)_\mu[\epsilon(k)\cdot q]\}f_2(q^2), \quad (6)$$

where $q = p - k$. For a real photon $q^2 = 0$.

We shall evaluate the left-hand side of Eq. (6) in the rest system of the B quark. Note that the zero recoil of the K^* meson (i.e., $\mathbf{k}=\mathbf{0}$) corresponds to $q^2 = t_m = (m_B - m_{K^*})^2$, and is far from $q^2=0$, where we need to evaluate the form factors. The constituent quark model, unlike the bag model (which is static), permits one to evaluate the form factors with recoil. We also incorporate corrections due to relativistic effects. The results are

$$f_1(q^2) = \tfrac{1}{2}\sqrt{2}(1 + m_{K^*}^2/m_B^2)^{1/2} I(q^2), \quad f_2(q^2) = f_1(q^2)/2. \quad (7)$$

Here $I(q^2)$ computed in the constituent quark model is

$$I_{\rm CQM} = \int_{-\infty}^{+\infty} d^3p\,\phi_{K^*}(\mathbf{p} - m_d\mathbf{k}/\tilde{m}_s)\phi_B(\mathbf{p})\alpha(\mathbf{k}-\mathbf{p},\mathbf{p})[1 + \mathbf{p}\cdot(\mathbf{p}-\mathbf{k})/3m_b^2], \quad (8)$$

where $\tilde{m}_s = m_s + m_d$ and

$$\alpha(\mathbf{k}-\mathbf{p},\mathbf{p}) = [(E_s+m_s)(E_b+m_b)/4E_sE_b]^{1/2}, \quad E_s = [m_s^2 + (\mathbf{k}-\mathbf{p})^2]^{1/2}, \quad E_b = (m_b^2 + \mathbf{p}^2)^{1/2}. \quad (9)$$

The integrals are evaluated with the use of Gaussian wave functions which are used in the solution of the Schrödinger equation utilizing the variational method, as described in Ref. 8. We choose

$$\phi_M(\mathbf{p}) = (\pi\beta_M^2)^{-3/4}\exp(-\mathbf{p}^2/2\beta_M^2; \quad M = K^*, B, \quad (10)$$

in which β_M is the variational parameter. From the formulas (8) and (9) it is clear that we included recoil as well as the relativistic corrections. Recoil corrections are caused by the motion of the hadron as a whole.

There are two sources of relativistic corrections in Eq. (8). Those due to motion of quarks inside the potential given by $\mathbf{p}\cdot(\mathbf{p}-\mathbf{k})/3m_b^2$ are less than 1%. Corrections in the function α, Eq. (9), however, contribute a factor of $1/\sqrt{2}$ because $E_s \gg m_s$. One can then show that

$$I_{\rm CQM}(q^2) = \tfrac{1}{2}\sqrt{2}[2\beta_{K^*}\beta_B/(\beta_{K^*}^2 + \beta_B^2)]^{3/2}\exp[-m_d^2(t_m - q^2)/2\tilde{m}_s\tilde{m}_b(\beta_{K^*}^2 + \beta_B^2)], \quad (11)$$

where $\tilde{m}_s = m_s + m_d$, $\tilde{m}_b = m_b + m_d$. The exponential damping factor describes the recoil correction of the form factors $f_{1,2}$ if we set $q^2 = 0$. In the numerical evaluation of the integral $I_{\rm CQM}$ we choose the parameters[8] $m_u = m_d = 0.3$ GeV, $m_s = 0.55$ GeV, $m_b = 5$ GeV, $\beta_{K^*} = 0.34$ GeV, and $\beta_B = 0.41$ GeV, and find

$$f_1(0) \cong 0.25. \quad (12)$$

The smallness of the form factor $f_1(0)$ is mainly due to the sharp exponential damping factor because of recoil. Without recoil, i.e., $\mathbf{k}=\mathbf{0}$, $f_1 \approx 1/\sqrt{2}$. This model has been successful in predicting $f_\pm(q^2)$ form factors for K-π, D-π, D-K. This gives us confidence that the value $f_1(0) \cong 0.25$ is probably reliable. A more realistic calculation of $f_1(0)$ requires an understanding of nonperturbative effects of QCD responsible for binding the quarks into hadrons. Since this is lacking, our approach represents the best one can do at present.

The width for $B \to K^*\gamma$, with the use of the matrix element of Eqs. (6) and (7), is found to be

$$\Gamma(B \to K^*\gamma) = \frac{m_b^2\tilde{G}_2^2}{32\pi m_B^3}(m_B^2 - m_{K^*}^2)^3[f_1^2 + 4f_2^2]. \quad (13)$$

We find for the ratio of exclusive to inclusive process

$$R = \frac{\Gamma(B \to K^*\gamma)}{\Gamma(b \to s\gamma)} \cong \frac{m_b^3(m_B^2 - m_{K^*}^2)^3 f_1^2}{m_B^3(m_b^2 - m_s^2)^3} \approx 0.07. \quad (14)$$

To evaluate the branching ratio, we calculate the total width for Γ_B in pure spectator approximation using QCD-improved Hamiltonian, and find

$$\Gamma_B = (G_F^2 m_b^5\,|\,V_{cb}\,|^2/192\pi^3)[(2c_+^2 + c_-^2)(r_c + r_{cc}) + (2r_c + r_{c\tau})], \quad (15)$$

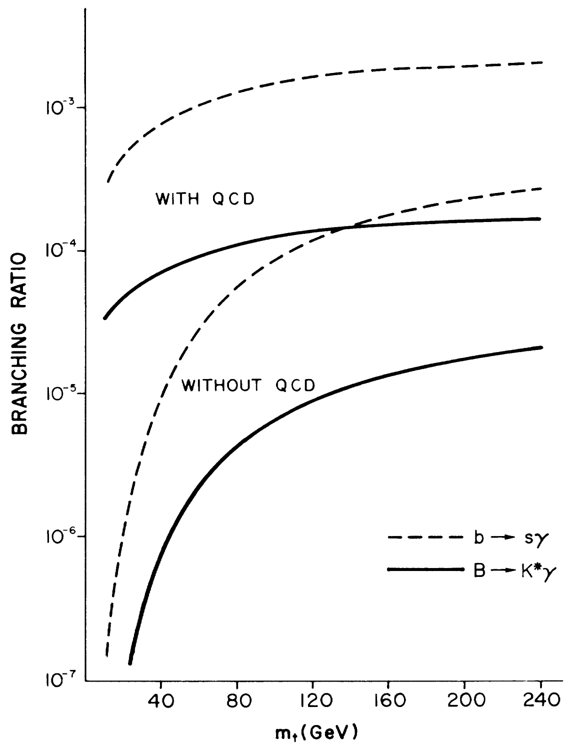

FIG. 1. Branching ratios for $B \to K^* \gamma$ and $b \to s\gamma$ as functions of m_t.

where $r_c = 0.447$, $r_{cc} = 0.119$, and $r_{c\tau} = 0.062$ are phase-space factors for one c, two c's, and $c\tau$ in final states. The factors $c_\pm$ are given by

$$c_- = [\alpha_s(m_b)/\alpha_s(M_W)]^{12/(33-2N_f)} = 1/c_+^2. \qquad (16)$$

In Fig. 1 we plot the branching ratio for $B \to K^* \gamma$ as a function of m_t. We have used $\alpha_s(m_b/2) = 0.33$ which corresponds to $\Lambda = 250$ MeV. We see that the QCD-enhanced branching ratio is close to the present measurable range of $\simeq 10^{-4}$. The value without QCD effects arises from use of G_2 of Eq. (2) and is lower by a factor of $\simeq 80$ for $m_t \approx 40$ GeV. For higher m_t values the ratio becomes smaller. The inclusive rate is also shown in Fig. 1. We would urge our experimental colleagues to look for these decays.

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Note added.—After the submission of this paper we received a paper by Bertolini, Borzumati, and Masiero,[9] in which they derive exactly the same conclusion for the inclusive rate $b \to s\gamma$. The numerical difference is due to their choice of α_s.

[a]On leave of absence from Rudjer Bošković Institute, Zagreb, Yugoslavia.

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