QCD Enhancement of Radiative B Decays

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We study the decay $b \to s\gamma$. We show that the QCD corrections enhance by more than an order of magnitude the prediction for $B(b \to s\gamma)$, for $m_t < m_W$. The implications for the standard model with three generations and the possible experimental signatures in the inclusive and exclusive modes are discussed.

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Rare B decays may become one of the most important classes of tests of the standard model (SM) in the near future. Experimentally, in addition to expected improvements at the Cornell Electron Storage Ring, the advent of the upcoming machines (the Stanford Linear Collider, LEP at CERN, the Tevatron at Fermilab, and even more, the Superconducting Super Collider) offers promising prospects for rare B physics. Apart from a large $B\bar{B}$ pair-production cross section, new detection techniques, such as vertex detectors, and the relatively large B lifetime should be decisive in improving the existing bounds on rare B decays by some orders of magnitude. Theoretically, rare B physics involves flavor-changing neutral currents (FCNC) and thus constitutes a new test of higher-order corrections to the SM. It is presumably a much cleaner test than the analogous rare kaon decays since it does not suffer appreciably from uncertainties due to long-distance effects.[1] Moreover, it has been pointed out that new physics, in particular low-energy supersymmetry,[2] can play a major role in these processes.

In this Letter we consider the B radiative decays in the context of the SM. We show that the dipole transition $b \to s\gamma$ (at the quark level) is drastically affected by QCD corrections, in particular for a relatively light top quark ($m_t < 60$ GeV). The reason for this unexpectedly important role played by QCD corrections in a system as heavy as a B meson is that the typical m_q^2/m_W^2 Glashow-Iliopoulos-Maiani suppression of the one-loop magnetic transition is turned into a milder $\ln(m_t^2/m^2)$ suppression in the two-loop contributions with gluon exchange (m is the typical hadron mass of the process, here $m \sim 5$ GeV). For instance, for $m_t = 45$ GeV, $B(b \to s\gamma)$ is 1.4×10^{-4} when QCD corrections are taken into account; this is 16 times larger than the branching ratio computed without strong corrections. This enhancement is of decisive importance for the bounds on m_t in the SM with three generations, as we shall explain below.

The amplitude for $b \to s\gamma$ in the SM, without the QCD corrections, is

$$A(b \to s\gamma)_{\rm SM} = \{\epsilon_\mu \bar{s} i\sigma^{\mu\nu} q_\nu (m_b P_R + m_s P_L) b\} \frac{G_F}{2\sqrt{2}} \frac{e}{2\pi^2} \{V_{ts}^* V_{tb}[F_2(x_t) - F_2(x_u)] + V_{cs}^* V_{cb}[F_2(x_c) - F_2(x_u)]\}, \quad (1)$$

where $x_j = m_j^2/m_W^2$, $j = u,c,t$, with $P_{R,L} = (1 \pm \gamma_5)/2$, and the V's being elements of the Kobayashi-Maskawa (KM) matrix [its unitarity has been used in (1)]. The function $F_2(x)$ can be found in Inami and Lim,[3] Eq. (B.3). The essential feature of (1) is the presence of the Glashow-Iliopoulos-Maiani mechanism which leads to the cancellation of contributions of order $G_F e/\pi^2$, resulting in an extra power suppression of the type $(m_i^2 - m_u^2)/m_W^2$ from $F_2(x_i) - F_2(x_u)$ $(i = c,t)$. QCD corrections now play a major role: Indeed, taking gluons into account (Fig. 1) removes this Glashow-Iliopoulos-Maiani power suppression and turns it into a "mild" logarithmic suppression, i.e., factors m_t^2/m_W^2 are replaced by $\ln(m_t^2/m^2)$. The computation of the QCD corrections to the operator $\bar{s} i\sigma^{\mu\nu} q_\nu (m_b P_R + m_s P_L) b$ in the leading-logarthmic approximation in first order in α_s can be immediately derived from the results of Kogan and Shifman.[4] A previous work evaluated the QCD corrections which are obtained by summing up all the leading-

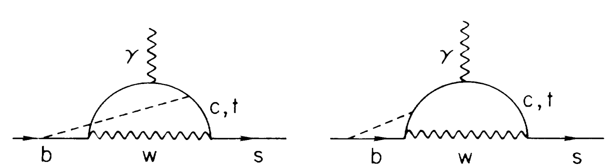

FIG. 1. Two-loop diagrams responsible for the logarithmic QCD corrections to the electroweak transition $b \to s\gamma$. The dashed line denotes the gluon propagator.

logarithmic contributions in a four-quark model, Eq. (44) of Shifman, Vainshtein, and Zakharov.[5] It is not difficult to accomplish an extension of Eq. (44) of Ref. 5 to the six-quark case. Indeed, the integration from m_t to m_W can be exactly obtained from the integration from m_c to m_W of Ref. 5 by simply replacing the coefficient $b \equiv 11 - 2N_f/3$ that appears explicitly in Eq. (44) with its value for the six-flavor case. However, one must be careful in using Eq. (44) for the region between the hadronic mass (in our case $m \sim 5$ GeV) and the intermediate scale m_t since the integration of the renormalization-group equations up to m_c in Ref. 5 was carried out with the explicit value of b corresponding to three flavors. Although this implicit b dependence can be readily retraced in Eq. (44), it is important to notice that the analysis of Ref. 5 takes advantage of the smallness of the difference in mass scales of ordinary and charmed hadrons, which is certainly not a good approximation when m_c is replaced by m_t. In any case, we have checked numerically that for light m_t, $m_t \simeq 25-30$ GeV, where the integration from the intermediate scale to m_W yields the leading contribution and the analysis of Ref. 5 can be reliably applied, the difference between the sum to all orders in α_s and the first order in α_s is only a few percent. Therefore, we are confident that taking the leading-logarthmic approximation at the first order in α_s even for larger m_t, but $m_t < m_W$, can be a reliable procedure. In passing, it is worthwhile to remark that the coefficient of the first-order α_s logarithmic expansion does not depend on the number of flavors, which enter only at higher orders. With the neglection of the small charm contribution, the inclusion of these QCD corrections modify (1) as follows:

$$A(b \to s\gamma)_{\text{QCD corr}} = \{\epsilon_\mu \bar{s} i \sigma^{\mu\nu} q_\nu (m_b P_R + m_s P_L) b\} \frac{G_F}{2\sqrt{2}} \frac{e}{2\pi^2} V_{ts}^* V_{tb} \left\{ F_2(x_t) + \frac{4}{3}\frac{\alpha_s}{\pi} \ln \frac{m_t^2}{m^2} \right\}. \quad (2)$$

The strong coupling α_s should be evaluated at a typical mass scale in the loop; we take $\alpha_s = 0.15$. In Table I we show that the QCD correction factor $(4/3\pi)\alpha_s \ln(m_t^2/m^2)$ always dominates over $F_2(x_t)$. Obviously, the dependences on m_t^2 and $\ln m_t^2$ of $F_2(x_t)$ and the QCD factor, respectively, explain why the QCD enhancement increases for smaller values of m_t. In Table I we report the values of the QCD factor even for $m_t \gtrsim m_W$, although, strictly speaking, the aforementioned procedure applies only to the case $m_t < m_W$. As we shall see below, we have reason to believe that this extrapolation can be somewhat trusted.

The amplitude (2) yields the width

$$\Gamma(b \to s\gamma)_{\text{QCD corr}} = \frac{\alpha G_F^2}{128\pi^4} m_b^5 \left(1 - \frac{m_s^2}{m_b^2} \right)^3 \left(1 + \frac{m_s^2}{m_b^2} \right) |V_{ts}^* V_{tb}|^2 \left\{ F_2(x_t) + \frac{4}{3}\frac{\alpha_s}{\pi} \ln \frac{m_t^2}{m^2} \right\}^2. \quad (3)$$

We compute $B(b \to s\gamma)$ by making use of the semileptonic decay $b \to ce\bar{\nu}$. In this way we can take advantage of the equality $|V_{ts}| \simeq |V_{bc}|$ which is very accurate in the 3×3 Kobayashi-Maskawa matrix. Thus,

$$B(b \to s\gamma) = \frac{\Gamma(b \to s\gamma)}{\Gamma(b \to ce\bar{\nu})} B(b \to ce\bar{\nu}), \quad (4)$$

where for $B(b \to ce\bar{\nu})$ we use the averaged experimental value[6] 11.65%.

The one-loop QCD corrections to $b \to ce\bar{\nu}$ have been evaluated by Cabibbo and Maiani[7] and Cortes, Pham, and Tounsi.[7] One obtains,[7,8]

$$\Gamma(b \to ce\nu)_{\text{QCD corr}} = \frac{G_F^2 m_b^5}{192\pi^3} \rho(m_c/m_b, 0, 0) |V_{bc}|^2 \left\{ 1 - \frac{2\alpha_s(m_b^2)}{3\pi} f(m_c/m_b, 0, 0) \right\}, \quad (5)$$

where the phase-space factor ρ is 0.447, $f(m_c/m_b, 0, 0) = 2.41$, and we take $\alpha_s(m_b^2) = 0.23$, which corresponds to $\Lambda_{\text{QCD}} = 200$ MeV.

We plug (3) and (5) into (4) to yield $B(b \to s\gamma)$ with the inclusion of the QCD corrections. The drastic effect of these corrections is now evident. From Table I we see that for $m_t = 25$ GeV the ratio $B_{\text{corr}}/B_{\text{uncorr}} = 42.6$ and for $m_t = 60$ GeV this is still equal to 11.6. In Fig. 2 we plot $B(b \to s\gamma)$, for both the QCD-corrected and -uncorrected[9] values, versus m_t. The fact that the curve tends to become flatter for values of m_t above 60 GeV increases our confidence in the extrapolation that we have made for $m_t \gtrsim m_W$.

The process $b \to s\gamma$ (at the quark level) gives rise to the experimental signature of a jet with strange, non-charmed particles recoiling against a hard photon. The

TABLE I. Comparation of the values of $B(b \to s\gamma)$, with or without QCD corrections, for different values of m_t.

m_t (GeV)	$F_2(x)$	$\frac{4}{3}\frac{\alpha}{\pi}\ln\left(\frac{m_t^2}{m^2}\right)$	B_{uncorr}	B_{corr}
25	0.040	0.205	1.42×10^{-6}	6.05×10^{-5}
30	0.054	0.228	2.59×10^{-6}	8.01×10^{-5}
45	0.099	0.280	8.72×10^{-6}	1.45×10^{-4}
60	0.144	0.316	1.84×10^{-5}	2.13×10^{-4}
80	0.200	0.353	3.56×10^{-5}	3.08×10^{-4}
100	0.250	0.381	5.56×10^{-5}	4.01×10^{-4}

contributions to this inclusive decay from radiative corrections to the annihilation graph are likely to lead to $B(b \to s\gamma) < 10^{-4}$. Therefore, at least barring accidental cancellations between the contributions coming from penguin and annihilation diagrams, we can conclude from Fig. 2 that an upper bound of 10^{-4} for the inclusive decay $b \to s\gamma$ would imply the severe constraint $m_t < 35$ GeV. On the other hand, a search with negative results at the level of 5×10^{-5} could cause serious trouble for the SM with three generations since for $m_t > 25$ GeV we have $B(b \to s\gamma) > 6\times10^{-5}$ (we recall that for values of m_t as low as 25 GeV we have checked explicitly that the contributions at first order in α_s and at all orders in α_s in the leading-logarithmic approximation coincide within a few percent).

The major source of background comes from the decays of B mesons into charmed particles of the type $B \to D^*\gamma$ or $B \to D+\gamma+X$ followed by the decay of the D mesons into kaons. At the inclusive level, unless one succeeds in discriminating the presence of charmed hadrons in the final state, one must rely on efficient energy cuts on the emitted photons. To discuss this point, we must consider the exclusive modes which are associated to $b \to s\gamma$. The channel $B \to K\gamma$ is forbidden, and so we are left with the decays into $\gamma+$ resonances of the kaon: $K^*(892)$, $K^{**}(1400)$, and $K^{***}(1800)$. The evaluation of the relative importance of these exclusive channels is quite controversial. For $B \to K^*\gamma$ the estimates range from 97% of the total[10] to 30%–50%.[11] It is, however, likely that because of the relatively high energy of the "emitted" s quark, the fraction of the $K^*\gamma$ mode is somewhat lower.[12] Experimentally there already exists an upper bound on $B \to K^*\gamma$, $B(B \to K^*\gamma) < 1.8\times10^{-3}$,[13] and it is likely to be improved by almost an order of magnitude by the end of next year.[14] If we consider, for instance, the estimate that $B \to K^*\gamma$ represents 30% of the total, pushing $B(B \to K^*\gamma)$ down to $<10^{-4}$ would imply $m_t < 80$ GeV (Fig. 2) and SM with three generations would predict $B(B \to K^*\gamma) > 2\times10^{-5}$. If, on the other hand, the $K^*\gamma$ channel is negligibly smaller than the $K^{**}\gamma$ or $K^{***}\gamma$ channels, then one could try to measure the inclusive decay $b \to s\gamma$, taking into account that whereas in $B \to K^*\gamma$ and $B \to K^{**}\gamma$ the monochromatic photon has an energy of 2.56 and 2.45 GeV, respectively, for $B \to D^*\gamma$ we have $E_\gamma = 2.25$ GeV and thus an efficient photodetector with a 10% resolution or better could distinguish part of the inclusive $b \to s\gamma$ decay from the decays into charmed particles.

In conclusion, we think that the QCD-corrected expectations for $b \to s\gamma$ in the SM with three generations, which we have presented in this Letter, strongly encourage an experimental effort for the detection of radiative B decays in the upcoming machines.

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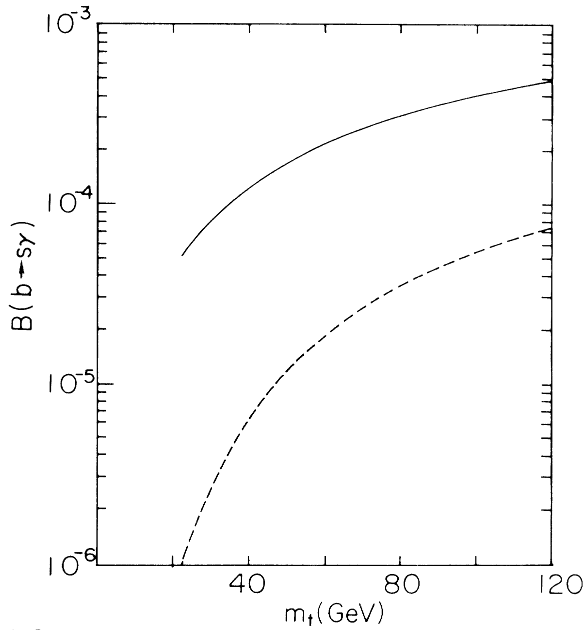

FIG. 2. Branching ratio for the inclusive process $b \to s\gamma$ as a function of m_t, with (solid line) or without (dashed line) the inclusion of QCD corrections.

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