Suppression of Nonrenormalizable Terms in the Effective Superpotential for (Blown-Up) Orbifold Compactification

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It is shown that *all* the nonrenormalizable terms in the effective superpotential for *any* Abelian symmetric orbifold with at least (0,2) world-sheet supersymmetry as well as any blown-up (2,2) orbifold are exponentially suppressed by the size R of the compactified space, i.e., $\propto \exp(-R^2/\alpha')$.

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Orbifold compactifications[1–5] of the superstring theories are especially attractive because interactions on orbifolds can be calculated *exactly* at the string tree level.[6,7] Thus all the parameters of the tree-level effective superpotential can be determined exactly, i.e., including contributions which are perturbative as well as nonperturbative in the ratio $\sqrt{\alpha'}/R$, where α' is the string tension and R is the radius of the orbifold. (For example, the effects of world-sheet instantons are automatically incorporated.)

The left-right [(2,2)] symmetric orbifolds, i.e., those with spin and gauge connections identified, can also be blown up[6–11] into the corresponding Calabi-Yau manifolds. This is achieved by giving nonzero vacuum expectation values to the massless scalar fields associated with the orbifold singularities, i.e., the so-called blowing-up modes, whose potential is flat. Although perturbative in the blowing-up vacuum expectation values, the method enables one to obtain *explicit* values for parameters of the blown-up orbifolds, thus giving exact results at the string tree level.

Calculations[10] for the mass spectrum and Yukawa couplings, i.e., the terms of dimension ≤ 4, for the blown-up orbifolds agree with the general results of the world-sheet instanton calculations.[8] In particular, *all* the matter singlets acquire masses which are proportional to $\exp(-R^2/\alpha')$ while **27** and **27*** of E_6 do not pair up. Also, all the "moduli" remain massless as expected. On the other hand, Yukawa couplings of the form $h_{ija}\mathbf{27}_i\mathbf{27}^{*j}\mathbf{1}_a$ for any pair (i,j) are nonvanishing for some a while Yukawa couplings of the type $h_{ijk}\mathbf{27}_i \times \mathbf{27}_j\mathbf{27}_k$ are nonzero in general. Some of these Yukawa couplings are nonzero already in the field-theory limit, i.e., $\alpha'/R^2 \to 0$, while some become nonzero due to nonperturbative effects. The nonrenormalizable terms $(\mathbf{27}\,\mathbf{27}^*)^K$ (with $K \geq 2$) in the effective superpotential for Z_N orbifolds and their blown-up versions have been studied in Ref. 11. It was observed that for a large class of orbifolds and their blown-up versions *all* such terms are *absent*, thus questioning the mechanism[12] for generating an intermediate scale for such compactifications.

In this note I show that for *all* Abelian symmetric orbifolds with at least (0,2) world-sheet supersymmetry as well as blown-up (2,2) orbifolds, *all the nonrenormalizable terms* in the effective potential are, at most, exponentially damped by the size of the compactified space, i.e., $\propto \exp(-R^2/\alpha')$. Here R is the radius of the compactified space and α' is the string tension.

All such orbifolds possess the local conformal invariance[13–15] in the right-moving (r) sector. One can thus use the picture-changing formalism, with vertices having different ghost numbers for the bosonized right-moving superconformal ghost in different "pictures."[15,16] Tree-level amplitudes involve collections of vertices such that the total ghost number ϕ equals -2.[15] The simplest form of the vertex operator for a space-time fermion is the $-\frac{1}{2}$ picture, while that for a space-time boson is the -1 picture. The picture-changing formalism enables one to obtain vertices in other pictures. For example, the vertex for a space-time boson in the 0 picture is obtained in the following way[15]:

$$[V_B(z)]_0 = \lim_{w\to z} \{\exp(\phi) T_F(w) [V_B(z)]_{-1}\}. \tag{1}$$

Here $[V_B(z)]_{-1}$ is the corresponding vertex operator in the -1 picture and

$$T_F = T_F^{\rm int}(X^i, \bar{X}^{\bar{i}}, \psi^i, \bar{\psi}^{\bar{i}}) + \partial X^\mu \phi^\mu \tag{2}$$

is the world-sheet-supersymmetry generator[15]—the stress-energy tensor. Here X and ψ are the string bosonic and fermionic coordinates, respectively; the indices $(i,\bar{i}) = (1,2,3)$ and $\mu = (1,2,3,4)$ denote the three complex internal and the four space-time dimensions, respectively. Partial derivatives are with respect to the right-moving world-sheet coordinate z. It is crucial that for an orbifold model, $T_F^{\rm int}$ takes the explicit form

$$T_F^{\rm int} = \partial X^i \bar{\psi}^{\bar{i}} + \partial \bar{X}^{\bar{i}} \psi^i. \tag{3}$$

The right-moving $N=2$ superalgebra of (0,2) as well as (2,2) models incorporates a $U(1)_r$ current algebra, generated by $J_r = -i\sqrt{3}\partial H_r$. $H_r(z)$ is a free right-moving scalar field. Actually for orbifolds, $U(1)_r$ world-sheet symmetry of the r sector is enlarged to $[U(1) \otimes U(1) \otimes U(1)]_r$. Thus, instead of one conserved charge

$H_r \equiv \sum_{i=1}^{3}(H_i)_r$, there are three conserved charges, $(H_1)_r$, $(H_2)_r$, and $(H_3)_r$, which are classified[1] for *all* the Z_N orbifolds and are related to the matrix of the discrete rotation θ acting on the three compactified coordinates. $(H_i)_r$ charges, along with the explicit form of the T_F [see Eqs. (2) and (3)], in turn uniquely determine the r sector of the vertex operator for emission of massless states at the string tree level. For example, in the -1 and $-\frac{1}{2}$ pictures (emission of a massless boson and massless fermion, respectively, and belonging to the space-time chiral superfield with *positive chirality*) the r sector of the vertex operators are the following:

$$(V_B)_{-1} = \exp(-\phi)\psi^j \exp(ik_\mu X^\mu), \quad \text{untwisted sector}, \tag{4a}$$

$$(V_B)_{-1} = \exp(-\phi)\prod_i \sigma_i s_i \exp(ik_\mu X^\mu), \quad \text{twisted sector}, \tag{4b}$$

$$(V_F)_{-1/2} = \exp(-\phi/2) u \prod_i \exp[-(\hat{H}_i)_r/2]\psi^j \exp(ik_\mu X^\mu), \quad \text{twisted sector}, \tag{5a}$$

$$(V_F)_{-1/2} = \exp(-\phi/2) u \prod_i \sigma_i \exp[-(\hat{H}_i)_r/2] s_i \exp(ik_\mu X^\mu), \quad \text{twisted sector}, \tag{5b}$$

with $\mu, i, \bar{i}$ defined as before and u referring to the spinor of the four uncompactified dimensions. The bosonic twist fields σ_i and fermionic twist fields s_i take care of the emission of the massless state from the propagating string with the twisted boundary conditions for the bosonic X^i and the fermionic ψ^i coordinates, respectively.[7] Fermionic fields are presented in terms of the three bosonic $U(1)_r$ charges

$$\psi^j = \exp[i(\hat{H}_j)_r], \tag{6a}$$

$$s_j = \exp[ik_j/N(\hat{H}_j)_r]. \tag{6b}$$

The three separate charges $(H_j)_r$ should satisfy the constraint that $H_r = \sum_j (H_j)_r = \sum_j k_j/N = 1$.

The calculation of parameters of the effective superpotential in a particular theory reduces to the study of the corresponding amplitude of the massless states emitted from the string propagating in this particular background. It is most convenient to calculate[10,11] the following Yukawa-type n-point function in the orbifold background:

$$A = \langle V_{F_1} V_{F_2} V_{B_1} \cdots V_{B_{(n-2)}} \rangle. \tag{7}$$

Here V_{F_i} and V_{B_i} denote the vertices for the emission of the massless fermionic and bosonic modes, respectively [see Eqs. (4)-(6)]. This amplitude enables one to probe the parameters of the superpotential directly, unlike the amplitude for n bosons.[17]

With the explicit form of the vertices (4), (5), and (1), one can then evaluate the amplitudes in the particular background which must obey selection rules that the total ϕ charge equals -2 and $(H_i)_r$ charges should be separately conserved.

It has been shown[10,11] that in the amplitudes (7) which probe the terms of the superpotential, only the terms of $(V_B)_0$ with $H_r = 0$ contribute, i.e., only terms proportional to $\partial X^i \bar{\psi}^{\bar{i}}$ survive in such amplitudes in order to conserve the total H_r charge.[18] Note that for $V_{-1/2}$, V_{-1}, we have $H_r = -\frac{1}{2}$, 1, respectively. Then A assumes the following form:

$$A = \langle V_{-1/2} V_{-1/2} V_{-1} V_0 \cdots V_0 \rangle \propto \langle \partial_z X^i, \ldots \rangle_{\sigma^{J_1}, \ldots, \sigma^{J_n}}. \tag{8}$$

For the nonrenormalizable terms, i.e., $n \geq 4$, the amplitude (8) has $n-3 \geq 1$ vertices in the 0 picture and it is thus proportional to at *least one power* of $\partial_z X$ evaluated in the presence of the twist fields σ^J.[19] This part of the amplitudes can be evaluated[7] by separating the classical and the quantum part of the solution for ∂X's. Namely,

$$\langle \partial_z X, \ldots \rangle_{\sigma^{J_1}, \ldots, \sigma^{J_n}} = Z_{\text{qu}} \sum_{\partial_z X_{\text{cl}}} \partial_z X_{\text{cl}} \cdots e^{-S_{\text{cl}}}, \tag{9}$$

where $\partial_z X_{\text{cl}}$ denotes the classical solution for $\partial_z X$ in the presence of the twist fields σ^J, $S_{\text{cl}} = \int d^2 z\, (\partial_z X_{\text{cl}} \partial_{\bar{z}} \bar{X}_{\text{cl}} + \partial_{\bar{z}} X_{\text{cl}} \partial_z \bar{X}_{\text{cl}})$, and Z_{qu} is the quantum part of the twist correlation function *independent* of the size of the compactified space. Note that in (9) there are *no* factors proportional to $\partial_z X_{\text{qu}}$, since $\langle \partial_z X_{\text{qu}} \rangle = 0$.

The form of ∂X_{cl}'s is determined[7] as follows[20]:

$$\begin{aligned} \partial_z X_{\text{cl}}(z) &= \sum_{i=1}^{L-M-1} a^i \omega_K^i(z), \quad \partial_z \bar{X}_{\text{cl}}(z) = \sum_{j=1}^{M-1} \bar{b}^j \omega_{N-K}^j(z), \\ \partial_{\bar{z}} \bar{X}_{\text{cl}}(\bar{z}) &= \sum_{i=1}^{L-M-1} \bar{a}^i \bar{\omega}_K^i(\bar{z}), \quad \partial_{\bar{z}} X_{\text{cl}}(\bar{z}) = \sum_{j=1}^{M-1} b^j \bar{\omega}_{N-K}^j(\bar{z}). \end{aligned} \tag{10}$$

Here L is the number of twist fields σ^J; ω_K and ω_{N-K} are determined by the operator-product expansion[7] of ∂X with the twist fields. E.g., $\omega_K^i(z) = z^{i-1}\prod_{j=1}^{L}(z - z_j)^{-(1-k_j/N)}$, with $\sum_{j=1}^{L} k_j/N = M$ and $M = 1, \ldots, L-1$, and similarly for $\omega_{N-K}^i(z)$. The condition on k_j's arises from the point-group selection rules, i.e., Z_N symmetry of interactions. The coefficients a^i ($\bar{a}^i$) and b^j ($\bar{b}^j$) are particular linear combinations of the coset vectors $\mathbf{v}$ ($\bar{\mathbf{v}}$) which belong to a class of lattice vectors. They are determined by the global monodromy conditions[7,21]; i.e., by choosing $L-2$ independent "closed loops"[21] γ_i around which X ($\bar{X}$) acquires no phase, but it may be translated by particular coset vectors which depend on the type of twist fields which are encircled by the independent closed loops γ_i. Note that for the symmetric orbifold, i.e., when the right- and the left-moving internal coordinates are rotated in the same way, one sees that $\omega_K^i = \bar{\omega}_K^{i*}$ $(\omega_{N-K}^j = \bar{\omega}_{N-K}^{j*})$ as well as $a^i = \bar{a}^{i*}$ $(b^j = \bar{b}^{j*})$. So $S_{\rm cl}$ assumes the form $\sum_{ii'} a^i \Omega_K^{ii'} \bar{a}^{i'} + \sum_{jj'} b^j \Omega_{N-K}^{jj'} \bar{b}^{j'}$, where $\Omega_K^{ii'} \equiv \int d^2z\, \omega_K^i \bar{\omega}_K^{i'}$ and $\Omega_{N-K}^{jj'} \equiv \int d^2z\, \omega_{N-K}^j \bar{\omega}_{N-K}^{j'}$ are entries of strictly positive-definite matrices $\mathbf{\Omega}_K$ and $\mathbf{\Omega}_{N-K}$, respectively.[22] This in turn implies

$$S_{\rm cl} = 0 \Longleftrightarrow a^i \equiv 0 \text{ and } b^j \equiv 0. \tag{11}$$

In this case also all $\partial X_{\rm cl}$'s are identically equal to zero [see Eq. (10)].

Using (10) one sees that in the amplitude (9) the only terms that survive *are* exponentially damped, i.e., $\propto |\mathbf{v}|^{n-3} e^{-|\mathbf{v}|^2 O(1)} \leq (R/\sqrt{\alpha'})^{n-3} e^{-R^2/\alpha' O(1)}$ with $n > 3$.

For the *exact* string tree-level result, we needed only the (0,2) world-sheet supersymmetry; thus the result is valid not only for all the (0,2) Abelian as well as (2,2) Abelian orbifolds but also for the Calabi-Yau manifolds obtained by blowing up the (2,2) Abelian orbifolds. However, the above conclusion need not apply to the asymmetric orbifolds.[3,5] In this case the global monodromy condition may be different for the left- and the right-moving sectors. Therefore one can, in principle, satisfy the constraint $S_{\rm cl} = 0$, but $\partial_z X_{\rm cl} \neq 0$ thus making the amplitude (9) for the nonrenormalizable terms *not* exponentially damped.

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[17]Note that in this case one is probing the scalar potential, which is the mixture of the F and D terms.

[18]This in turn implies that the effective superpotential calculated in this way cannot be mimicked by a massless exchange of gauge or gravitational particles; the amplitudes of such exchanges would be proportional to k^2 which is absent in the amplitude (7).

[19]From now on we shall suppress the index i for internal coordinates.

[20]We suppress the dependence on z_i, the location of the vertices.

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[22]One can show by using the Schwartz inequality that $|\Omega_K^{ii'}|^2 \leq \Omega_K^{ii} \Omega_K^{i'i'}$ and similarly for Ω_{N-K}'s. The equality sign applies only when $i = i'$.