Zener Transitions and Energy Dissipation in Small Driven Systems

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We consider the dynamics of an electron in a finite one-dimensional system, subject to a uniform electric field (or an electron in a ring, subject to an emf induced by a time-dependent magnetic flux). In the presence of elastic scattering due to localized potentials the driving source does not supply energy to the system in the steady state. The dissipation in the presence of inelastic scattering is evaluated. The Ohmic resistance of the system depends crucially on the inelastic rate even in the weak-localization limit.

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Basic concepts in transport theory have to be revisited when submicron systems are considered. One must ask how dissipation and irreversibility arise in mesoscopic systems. Here we shall consider a simple model of an electron in a one-dimensional (1D) metallic loop[1,2] driven by a magnetic flux ϕ confined within the loop, which varies linearly in time.[3-5] We address the question (which has been the subject of some recent controversy) whether the mere fact that the system is biased by an external source is sufficient to pump in (and eventually to dissipate) energy, even when the electron experience only elastic scattering. Our treatment avoids some of the assumptions made earlier,[6] and accounts for interference effects, essential for a correct description of the time evolution of the system. We show that in the absence of inelastic scattering, the system eventually reaches a state where no additional energy can be pumped into it, regardless of the strength of the bias. Inelastic events are essential to bring about finite resistance, R. The value of R is strongly affected by the inelastic rate even when we consider a good conductor (i.e., when we consider the weak-localization regime). Our picture differs from the Landauer picture[7] of quantum resistance, according to which the value of the resistance (for a fixed inelastic-scattering length) is controlled by the *elastic* scattering.

Büttiker, Imry, and Landauer[1] showed that, when Zener transitions are neglected, no dissipation arises when the flux in the ring varies with time. It was shown later that thermal transitions may lead to dissipation,[8,9] and, alternatively, that Zener tunneling combined with inelastic effects[10] may be a possible mechanism (most importantly at low temperatures) for energy dissipation. In the absence of a coupling to a heat reservoir, Landauer[11] has demonstrated that Zener transitions can be undone by reversal of the external driving field. Consequently, the work done on the system by the driving source can be retrieved and no energy is being dissipated. Most recently Lenstra and van Haeringen[6] have argued that in the presence of a constant electromotive force around the ring, Zener transitions may lead to a nonvanishing dc current. This immediately implies that even in the absence of inelastic scattering (some elastic scattering is needed, though), energy is dissipated. Our results, summarized above, suggest quite a different picture.

Our description applies to a wide range of random potentials. An example is the Hamiltonian

$$\mathcal{H}=\frac{\hbar^2}{2M}\left(-i\frac{\partial}{\partial y}-\frac{\phi}{\phi_0}\right)^2+\sum_k V_k e^{iky},$$

which describes an electron in a ring. Here ϕ_0 is the quantum fluxoid and y is the coordinate along the ring. The electromotive force is $FL=-\partial\phi/\partial\tau$, where L is the circumference of the ring, F the electric field, and τ the time. The Fourier coefficients of the potential are independent random variables of mean zero with $V_{-k}=V_k^*$; $k=2\pi n/L$, where n is integer; this corresponds to a set of local potentials at random positions. In the case of an electron in one dimension driven by an electric field, the term ϕ/ϕ_0 is replaced by $\tilde{\tau}\equiv eFL\tau/2\pi\hbar$. One can now solve for the adiabatic single-electron levels, treating τ as a parameter (Fig. 1). These levels are periodic in $\tilde{\tau}$ with a period 1. The narrow gaps E_g (e.g., the energy difference between points c and d in Fig. 1) are randomly distributed as a function of the band index n, and for $n\approx n_F$ (n_F being the index of the Fermi level ϵ_F) the average value of E_g does not depend sensitively

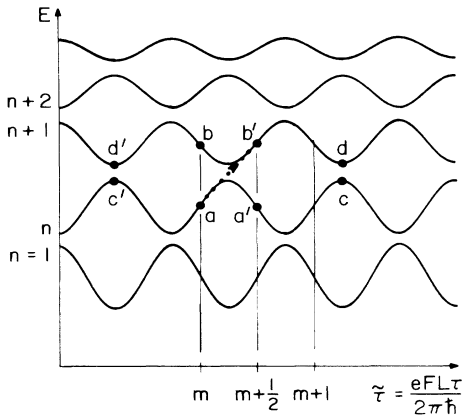

FIG. 1. The $(E,\tilde{\tau})$ phase space for a one-dimensional system: energy bands vs normalized time.

on n. The levels may be doubly occupied because of spin degeneracy. Zener transitions among the levels will take place because of the explicit dependence of $\mathcal{H}$ on time. If E_g is sufficiently small, one may consider Zener transitions only between adjacent levels. These transitions will take place mainly in the neighborhood of the narrow gaps.

For a weak elastic scattering, when

$$x^2 \equiv \frac{E_g^2}{(2\pi\hbar^2 e/ML)Fn_F} \ll 1,$$

one can use the sudden approximation to evaluate the complex reflection and transmission amplitudes, describing the Zener transitions. The transition amplitude from a to b', $B(a,b')$ (Fig. 1), is denoted by $t(n,n+1)$. Similarly, $B(b,a') \equiv t'(n,n+1)$; $B(a,a') \equiv r(n,n+1)$; $B(b,b') = r'(n,n+1)$. We found[12] $r(n,n+1) = ix_{n,n+1}$, where the indices of x refer to the energy gap between the bands n and $n+1$. The corresponding transmission amplitudes, given by $t = t' = (1-|r^2|)^{1/2}$, are real. These amplitudes satisfy time-reversal and unitarity requirements,[13] and are invariant under $m \to m+1$. Similar expressions are obtained for integer values of $\tilde{\tau}$.

The current in the system is given by

$$\langle I(t)\rangle = \partial\langle E(t)\rangle/\partial\tilde{\tau}, \tag{1}$$

where angular brackets denote a quantum-mechanical expectation value. If an electron follows adiabatically one particular energy band, its contribution to the dc current (the average slope of the band) is zero. In the following we shall consider the evolution of the system in the $(E,\tilde{\tau})$ phase space. We shall see that the occupation of the energy bands reaches a saturation ("localization in the energy direction"), which means that in the steady state $\overline{\langle E(t)\rangle} = \text{const}$. From Eq. (1) it then follows that $\overline{\langle I(t)\rangle} = 0$. Therefore, for large values of $\tilde{\tau}$ the average work done on the system is zero; this corresponds to a dissipationless situation.

We shall now relate the probability amplitudes at time $\tilde{\tau} + \frac{1}{4}$ (at points a', b' for instance) to the amplitudes at a time $\tilde{\tau} - \frac{1}{4}$. When the slope of the band at a point is positive, the corresponding amplitude is denoted by $A_\uparrow$. Conversely, we use the notation $A_\downarrow$. We then have

$$\begin{aligned} A_\uparrow(\tilde{\tau}+\tfrac{1}{4},n) &= tA_\uparrow(\tilde{\tau}-\tfrac{1}{4},n-1)\exp[-i(E_n+E_{n-1})(\pi/eFL)] + r'A_\downarrow(\tilde{\tau}-\tfrac{1}{4},n)\exp[-iE_n(2\pi/eFL)], \\ A_\downarrow(\tilde{\tau}+\tfrac{1}{4},n) &= tA_\downarrow(\tilde{\tau}-\tfrac{1}{4},n+1)\exp[-i(E_n+E_{n+1})(\pi/eFL)] + rA_\uparrow(\tilde{\tau}-\tfrac{1}{4},n)\exp[-iE_n(2\pi/eFL)]. \end{aligned} \tag{2}$$

We have included here phase factors $-\hbar^{-1}\int E\,dt$ associated with the time evolution of the wave function. Here E_n is the average adiabatic energy of the nth band. We now take the Fourier transform of Eqs. (2) with respect to $\tilde{\tau}$. We note that, since the coefficients t, r, r' and the phase factors are τ independent in our discrete time, there is no coupling between different ω components. After some algebra,[12] we can relate the amplitudes at the $(n+1)$th level to those at the nth level by the following transfer matrix:

$$\begin{pmatrix} A_\uparrow(n+1,\omega) \\ A_\downarrow(n+1,\omega) \end{pmatrix} = \begin{pmatrix} 1/\theta_n & (\rho_n/\theta_n)^* \\ \rho_n/\theta_n & 1/\theta_n^* \end{pmatrix} \begin{pmatrix} A_\uparrow(n,\omega) \\ A_\downarrow(n,\omega) \end{pmatrix}, \tag{3}$$

where

$$\theta_n(\omega) = t(n,n+1)\exp[+i(E_{n+1}+E_n)(\pi/eFL)+i\omega],$$

and

$$\rho_n(\omega) = -r(n,n+1)\exp[+iE_{n+1}(2\pi/eFL)+i\omega].$$

The above transfer matrix obeys unitarity and time-reversal symmetries. θ_n and ρ_n are random because of the randomness of the corresponding r's and t's, and also because of the pseudorandomization that arises from the phase factors.[14] Thus, the problem has been mapped onto that of electron localization in one dimension,[15] with θ and ρ being the effective transmission and reflection amplitudes. The localization length is given by

$$\xi = [\overline{\ln(1/\theta)}]^{-1} = [\overline{\ln(1/t)}]^{-1}, \tag{4}$$

and is independent of ω. Thus, ξ is also the *one-dimensional* localization length in the energy direction.[16,17] Substituting our expression for r, we obtain (for $|r| \ll 1$) $\xi \simeq 2/\bar{x}^2$, where ξ is the number of levels over which the localized wave function is extended. The evolution in time of an initially narrow wave packet (in the action space) is ballistic until a width $\sim\xi$ is reached. The time required for this process is $\tau_\xi \simeq (\delta\tau)\xi$, where $\delta\tau$ is the time associated with a single oscillation in the $(E,\tilde{\tau})$ space. For $\xi \simeq 100$, $FL = 10^{-7}$ V, we obtain $\tau_\xi \simeq 10^{-9}$ s.[18] These numbers will change drastically when a multichannel ring is considered,[12] because ξ is dramatically increased; the analysis presented below of dissipation in such systems for $\tau_{in} < \tau_\xi$ remains valid.

Dissipation occurs in the presence of inelastic events. At times smaller than τ_{in} (the time scale over which the electron loses coherence because of inelastic events) the electron wave packet is coherent. For times τ such that $\tau_{in} < \tau, \tau_\xi$, it propagates diffusively in the energy direction (on length scales shorter than ξ). Any gain in energy due to this diffusion will eventually be dissipated by, for example, the emission of phonons (followed by the "collapse" of the wave packet). The energy loss of a single electron is

$$\Delta E(\tau_{in}/\delta\tau)^{1/2} \simeq (2\pi^2\hbar^2 n_F/ML^2)(\tau_{in}/\delta\tau)^{1/2}$$

(where ΔE is the typical energy spacing near ϵ_F). The number of levels around E_F (and therefore the number

of electrons) involved in the collapse is $\simeq \tau_{in}/\delta\tau$. Thus, the total energy dissipated at time τ_{in} is

$$\Delta E(\tau_{in}/\delta\tau)^2 \simeq (e^2F^2n_F/2M)\tau_{in}^2.$$

The dissipation *increases* with τ_{in} since for $\tau_{in} < \tau_\xi$ more electrons can be excited to higher energy bands as τ_{in} is increased. The resistance R of the system can be defined as the ratio between $(FL)^2$ and the dissipated power. We find

$$R \approx 2ML^2/e^2n_F\tau_{in}.$$

For $\tau_{in} > \tau_\xi$ (cf. Thouless[19]),

$$R \approx (MF^2L^4/\pi^2\hbar^2n_F\xi^2)\tau_{in}.$$

Note that we have not made any strong assumptions concerning the rate of elastic scattering.

To test the localization in the phase space numerically, we have considered a uniform interband spacing, with no randomness in either t or r. In Fig. 1 there are several paths that contribute to $B(q,p)$. The path $B_1 \equiv B(qu_1u_2p)$ involves a factor tr; the path $B_2 \equiv B(qu_1v_2p)$ involves a factor rr', etc. We also have additional phase factors [cf. Eq. (2)]: B_1 contains an extra factor of $e^{i\alpha}$ compared with B_2. The phase α depends on the energy and the time scale. Figure 2 shows $|B(i,f)| \equiv |B(j,l)|$ as a function of l. More cases have been studied by us, and we have concluded that for $\alpha/2\pi > 1$ and irrational the system is indeed pseudorandomized. Thus, ξ is given by Eq. (4), and is not sensitive to the value of α (provided pseudorandomization is satisfied).

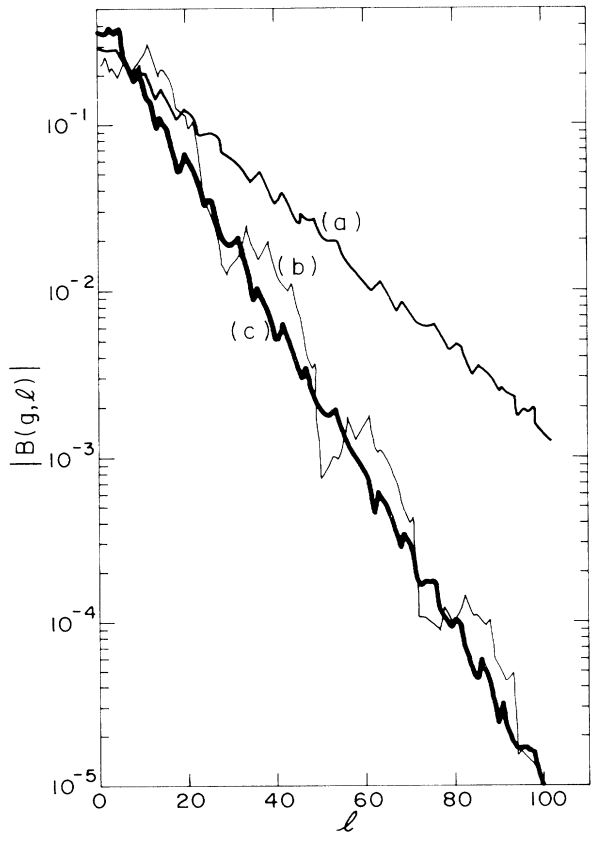

FIG. 2. $|B(j,l)|$ vs l. The resulting amplitude is the sum over all contributing paths. Here $\alpha = s$. Curve a, $j=3$, $r=0.32i$; curve b, $j=7$, $r=0.44i$; curve c, $j=3$, $r=0.44i$.

We have also considered the case in which the phase accumulated between consecutive transmission or reflection events may be neglected ($\alpha \simeq 0, 2\pi, \ldots$). This may correspond either to an unphysical situation in which very few energy bands near ϵ_F contribute, or to special resonant transmission conditions (cf. Fig. 2). Details of the analysis of this case will be given elsewhere.

In conclusion, we have shown that in the presence of elastic scattering the wave function of the driven ring is localized in the energy-band picture. We have discussed the transient leading to the localized situation and demonstrated how the collapse of the wave function due to phase-smearing events leads to energy dissipation.

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[1]M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983).

[2]Y. Imry, in *Directions in Condensed Matter Physics*, edited by G. Grinstein and G. Mazenko (World Scientific, Singapore, 1986).

[3]D. Lenstra and W. J. Van Haeringen, J. Phys. C **14**, 5293, L819 (1981).

[4]D. Lenstra and W. J. Van Haeringen, Physica (Amsterdam) **183B**, 26 (1985).

[5]This problem can be mapped onto that of an electron in 1D with certain time-dependent boundary conditions.

[6]D. Lenstra and W. J. Van Haeringen, Phys. Rev. Lett. **57**, 1623 (1986); P. Lenstra, H. Ottevanger, W. J. Van Haeringen, and A. G. Tijhuis, Phys. Scr. **34**, 438 (1986); see also R. Landauer, Phys. Rev. Lett. **58**, 2150 (1987).

[7]R. Landauer, IBM J. Res. Dev. **1**, 223 (1957), and Philos. Mag. **21**, 863 (1970).

[8]R. Landauer and M. Büttiker, Phys. Rev. Lett. **54**, 2049 (1985).

[9]M. Büttiker, Phys. Rev. B **32**, 1846 (1985).

[10]Y. Gefen, E. Ben-Jacob, and A. O. Caldeira, to be published.

[11]R. Landauer, Phys. Rev. B **33**, 6497 (1986).

[12]Details will be given elsewhere.

[13]See, e.g., P. W. Anderson, D. J. Thouless, E. Abrahams, and D. S. Fisher, Phys. Rev. B **22**, 3519 (1980).

[14]For $FL=10^{-7}$ V, $\epsilon_F=1$ eV, and $n_F=10^6$, $(E_{n_F}-E_{n_F-1})2\pi/eFL\simeq 40\gg 2\pi$. Thus, the phase mod$(2\pi)$ can be considered to be random for all practical purposes.

[15]See, e.g., D. J. Thouless, in *Physics in One Dimension,* edited by J. Bernasconi and T. Schneider, Springer Series in Solid-State Science, Vol. 23 (Springer-Verlag, Berlin, 1981), p. 306.

[16]A closely related work by S. Fishman, D. R. Grempel, and R. E. Prange [Phys. Rev. A **29**, 1639 (1984); see also D. L. Shepelyansky, Phys. Rev. Lett. **56**, 677 (1986)] has demonstrated that the wave function of a periodically kicked rotor may also be localized in the phase space. Note, however, the difference between their dynamics and ours. In particular, our Hamiltonian is not invariant under translation in time, but rather under $-i(\partial/\partial y)\rightarrow -i(\partial/\partial y)+1$, $\tilde{\tau}\rightarrow\tilde{\tau}-1$.

[17]For different dependence of ξ on the energy, as well as for other energy spectra, exponential localization may not be obtained [see, e.g., C. M. Soukoulis, J. V. José, E. N. Economou, and Ping Sheng, Phys. Rev. Lett. **50**, 764 (1983)]. In fact, F. Bentosela, R. Carmona, P. Duclose, B. Simon, B. Souillard, and R. Weder, Commun. Math. Phys. **88**, 387 (1983), have shown that under quite general conditions an electron subject to an electric field is delocalized. In our case delocalization may take place as the probability of Zener events increases with the energy for the high-lying levels. An almost localized wave packet will then "leak" towards high energies, resulting in exponentially small corrections to the results presented here.

[18]Note that the discussion in the first paper of Ref. 6 missed the localization effect because of the neglect of cross interference terms. Also in any numerical analysis one has to go to $t\gg\tau_\xi$ to see this effect.

[19]D. J. Thouless, Phys. Rev. Lett. **39**, 1167 (1977).